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THE
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CONTENTS

NUMBER I

PAGE

The Absorption Spectrum of Iodine Vapor at High Temperatures. E. J. Evans 1

The Moon's Theoretical Spectrographic Velocity. Kurt Laves 17

Observations of the Sun on May 18 and 19, 1910. Frederick Slocum 24

On Some Methods and Results in Direct Photography with the 60-Inch Reflecting Telescope of the Mount Wilson Solar Observatory. G. W. Ritchey 26

The General Circulation of the Mean and High-Level Calcium Vapor in the Solar Atmosphere. Charles E. St. John . . 36

On the Velocity of the Sun's Motion through Space as Derived from the Radial Velocity of *Orion* Stars. J. C. Kapteyn and Edwin B. Frost 83

On the Average Parallax of the Stars of the Fourth Type as Compared with That of Stars of Other Types. J. C. Kapteyn 91

NUMBER II

Solar Disturbances and Terrestrial Temperatures. W. J. Humphreys 97

Formulae for the Spectral Series for the Alkali Metals and Helium. R. T. Birge 112

Two Solar Prominences. Frederick Slocum 125

Remarks on Wilsing and Scheiner's Memoir on the Temperature of 109 Stars. J. Wilsing 130

Studies on the Emission of Gases, II. H. Konen and W. Jungjohann 141

Series in the Spectra of Calcium, Strontium, and Barium. F. A. Saunders 153

The Brightness of Halley's Comet as Measured with a Selenium Photometer. Joel Stebbins 179

Minor Contributions and Notes: Variable Radial Velocities of Two Stars in the *Taurus* Stream. Storrs B. Barrett 183

229340

NUMBER III

PAGE

THE MEASUREMENT OF THE LIGHT OF STARS WITH A SELENIUM PHOTOMETER, WITH AN APPLICATION TO THE VARIATIONS OF *Algol*. Joel Stebbins 185

SECONDARY STANDARDS OF WAVE-LENGTHS, INTERNATIONAL SYSTEM, IN THE ARC SPECTRUM OF IRON. H. Kayser, Ch. Fabry, and J. S. Ames 215

STANDARDS OF THIRD ORDER OF WAVE-LENGTHS ON THE INTERNATIONAL SYSTEM. H. Kayser 217

ON THE APPARENT PERIODICITY IN THE SPACING OF THE SATELLITES OF SOME OF THE MERCURY LINES. G. F. Hull 226

PROBABLE ERRORS OF RADIAL VELOCITY DETERMINATIONS. J. S. Plaskett 230

THE COLLIMATION OF THE CORRECTING LENS. J. S. Plaskett . . 243

ON THE REVERSAL OF SPECTRAL LINES. H. Konen 249

MINOR CONTRIBUTIONS AND NOTES: The International Union for Cooperation in Solar Research, F. 258

NUMBER IV

THE DISPERSION OF METALS IN THE INFRA-RED SPECTRUM. L. R. Ingersoll 265

THE ABSORPTION SPECTRUM OF BROMINE AT HIGH TEMPERATURES. E. J. Evans 291

MEASURES ON NINETEEN NEW SPECTROSCOPIC BINARIES. Oliver J. Lee 300

THE ABSORPTION IN THE RED OF THE ACETATE, NITRATE, AND SULPHATE OF COBALT. Frank L. Cooper 309

MINOR CONTRIBUTIONS AND NOTES: Giovanni Virginio Schiaparelli, A., 313; On the Temperatures of Stars, C. G. Abbot, 319; Note on Bell's Paper on Star Colors, R. T. A. Innes, 321.

REVIEWS: The Scientific Papers of Sir William Huggins, Sir William and Lady Huggins (E. B. F.), 323; Acht Vorlesungen über theoretische Physik, Max Planck; Die Einheit des physikalischen Weltbildes, Max Planck (E. B. F.), 325; The Natural History and Scientific Book Circular No. 145 (R. W. Willson), 326; Tables du Bulletin de la Société belge d'astronomie (1895–1909), Auguste Collard (S. B. B.), 326; Geographical Essays, William Morris Davis (Frederick Slocum), 327.

PAGE

NUMBER V

THE MAGNETIC SEPARATION OF ABSORPTION LINES IN CONNECTION WITH SUN-SPOT SPECTRA. P. Zeeman and B. Winawer . . . 329

MASS-RATIOS IN THE SYSTEMS OF *Krüger 60* AND *Castor*. Henry Norris Russell 363

PHOTOGRAPHIC DETERMINATIONS OF STELLAR PARALLAX MADE WITH THE YERKES REFRACTOR. I. Frank Schlesinger 372

THE SOLAR ROTATION FROM THE MOTION OF THE FACULAE ON THE DISK (1906–1908). Stanislas Chevalier 388

REVIEWS: Physik der Sonne, E. Pringsheim (Frederick Slocum), 400; Les théories modernes du soleil, J. Bosler (Frederick Slocum), 402; The Spectroscope and Its Work, H. F. Newall (E. B. F.), 402.

INDEX TO VOLUME XXXII 404

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXXII JULY 1910 NUMBER 1

THE ABSORPTION SPECTRUM OF IODINE VAPOR AT HIGH TEMPERATURES

By E. J. EVANS

The influence of temperature on the absorption spectrum of iodine vapor is of great interest, because iodine is one of those substances which dissociate at high temperatures. Konen[1] investigated the absorption of the vapor at temperatures up to 800° C., and found that the absorption lines weaken with increase of temperature. The subject was afterward studied by Friederichs,[2] who found that the bands finally disappeared at 900° C. Konen also investigated the emission spectrum of heated iodine vapor, and discovered that it consisted of bands coinciding in position with those of the absorption spectrum. This result was confirmed by the later researches of L. Puccianti,[3] who considers the emission to be a true temperature emission. Fredenhagen[4] has also investigated the emission spectrum of iodine vapor, and claims to have shown that iodine vapor of homogeneous temperature can emit only a continuous spectrum. He also verified Konen's work on the effect of temperature on the absorption spectrum. The object of the present research is to investigate the effect of pressure on the temperature of disappearance of the absorption bands, and to see whether there is any connection between the disappearance of the bands, and the dissociation of the iodine vapor. Planck in his work on *Thermodynamics* gives an equation by means of which the dissociation of iodine vapor at any temperature and pressure can be calculated. If the vapor dissociates giving $n_1 I_2$ molecules and $n_2 I_1$ molecules $\frac{n_2^2}{n_1(n_1+n_2)}=a_1 e^{-\frac{b'}{\theta}}\frac{\theta}{p}$, where θ and p represent the absolute temperature and pressure respectively, and

---

[1] *Wiedemanns Annalen*, **65**, 257–286, 1898.

[2] *Zeitschrift für wissenschaftliche Photographie*, **3**, 154–164, 1905.

[3] *Atti della Reale Accademia dei Lincei*, **14**, 1° semestre, 84–89, 1905.

[4] *Physikalische Zeitschrift*, **8**, 89–91, 1907.

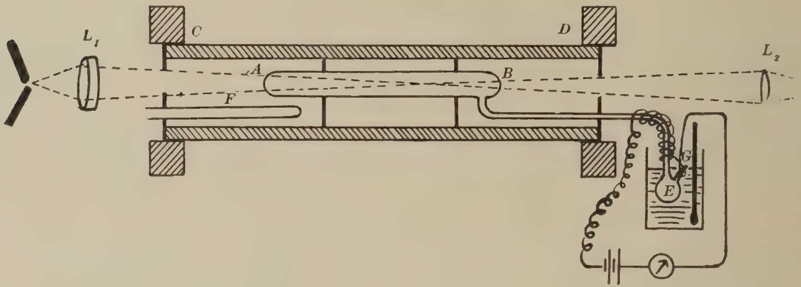

FIG. 1

a_1 and b' are constants, which can be evaluated if the vapor density of iodine at different temperatures and pressures is known. When p is measured in mm, the values of a_1 and b' deduced from the work of Meier and Crafts are 9375 and 14,690 respectively. If the temperature and pressure of the vapor are known, the degree of dissociation represented by $\frac{n_2}{2n_1+n_2}$ can be readily determined.

EXPERIMENTAL ARRANGEMENT

The apparatus employed is shown in the above diagram. The iodine vapor was heated in the quartz tube *AB*, provided with a side tube, bent round as shown in the figure, and ending in the bulb *E*, to which another side tube had been fused. The side tubes and the bulb were also made of quartz. The quartz tube was fixed inside the carbon tube *CD* by means of two carbon rings, and the carbon tube was heated by passing a current of 200 to 300 amperes through it. The current was led into the carbon tube by means of large graphite blocks provided with two openings in which the carbon tube made a tight fit. The heating tube was well packed with graphite to insulate the heat. The quartz tube was 20 cm long, and 1.4 cm external diameter, and the carbon heating tube was 40 cm long and 4 cm internal diameter. The tube *AB* was always placed in the center of the electric furnace, as the temperature was nearly uniform over a considerable length near the center of the furnace. The tube was also opaque except for the ends *A* and *B*, which were made of clear quartz to allow the light from the electric arc to pass through. Since the ends of the tube were curved, it was necessary to use a strong source of light. This was attained by allowing the light from the positive pole of the electric arc, after it had been concentrated by the large condensing lens L_1, to pass through the quartz tube. The light which had passed through was brought to a focus on the slit of a concave grating of 1 meter radius and 15,000 lines to the inch by means of the convex lens L_2. In this way a bright spot of light was focused on the slit of the grating spectroscope, and the first-order spectrum was observed.

MEASUREMENT OF PRESSURE

A quantity of iodine was introduced into the bulb *E*, and the side tube *G* connected with a Fleuss pump. The silica tube *AB* was evacuated to a pressure of 1 or 2 mm of mercury, and the small side tube *G* sealed in the oxyhydrogen flame. The iodine vapor will diffuse into the silica tube, and finally the pressure of the vapor in every part of the apparatus will be the vapor pressure of the iodine at the temperature of the coldest part of the apparatus. The bulb *E* containing the iodine was immersed in an oil bath, whose temperature could be regulated by means of a coil of manganin wire, which was electrically heated and placed immediately underneath the oil bath. This heating coil, covered with a thin layer of asbestos, rested at the bottom of a larger concentric beaker partially filled with oil. The temperature of the oil bath was read on a mercury thermometer immersed in the fluid. By regulating the current flowing through the heating coil, it was possible to keep the temperature of the bath constant to within 0°2 C. during the course of an experiment. As the vapor pressure of the iodine was to be deduced from the temperature of the bath, it was necessary to make the bath the coldest part of the apparatus. The length of the side tube inside the furnace was always hotter than the bulb, and to satisfy the above condition, it was only necessary to insure a higher temperature for that part of the tube between the end of the furnace and the bath. That was accomplished by winding the tube with thin aluminium wire, and regulating the current flowing through it by means of a wire-frame resistance. Preliminary experiments were made with the same wire wound round a silica tube of larger diameter, and the temperature inside the tube was measured for various currents. In each experiment on the absorption spectrum, the current flowing through the aluminium wire was adjusted to give a temperature within the tube higher than that of the oil bath. Nernst[1] gives the following equation from which the vapor pressure of iodine at any absolute temperature T can be evaluated:

$$\log p = \frac{-14609}{4.571\,T} + 1.75 \log T - \frac{0.0143\,T}{4.571} + C.$$

In the above equation p represents the pressure in atmospheres, and $C = 3.925$.

The following table gives the values of the pressure experimentally observed, and also the pressure calculated from the above equation.

$T-273°$	p (observed)	p (calculated)
−21°	0.000004	0.000005
19	.00024	.00024
60	.0067	.0050
85	.0267	.0222
137	.263	.263

In the present work the pressure was always determined by means of the above equation, but the results can always be corrected for any inaccuracy in the formula, for the bath temperature is always given.

MEASUREMENT OF TEMPERATURE

The temperature of the quartz tube was measured by a *Pt-Rh* thermo-couple whose ends were connected to a galvanometer reading directly in degrees centigrade. To protect the couple from furnace gases it was placed inside the quartz tube F closed at one end. The tube passed through a circular opening in a piece of asbestos, which was pressed against the end of the carbon heating tube, and the open end of this protecting tube was outside the furnace. The accuracy of the thermo-couple at high temperatures was tested by determining the melting-point of copper. The melting-point observed did not differ from the accepted value by more than 10° C., and consequently the high temperatures recorded by the thermo-couple were considered to be practically correct. The quartz tube *F* containing the couple was pushed well underneath the tube *AB* containing the vapor. It was found that the temperature over the length occupied by the quartz tube *AB* was practically uniform; the difference between the temperature at *A* and in the middle of the tube being about 25° C., when the temperature in the middle of the furnace was about 1000° C.

---

[1] "Applications of Thermodynamics to Chemistry," *Silliman Lectures*, 1906.

ADJUSTMENTS AND MODE OF WORK

The quartz tube having been placed at the center of the furnace, the line from the positive pole of the arc through the axis of the tube *AB* was adjusted to pass through the slit of the concave grating. Two asbestos disks with holes punched in them were pushed against the ends of the heating tube. The line between the centers of the holes passed through the axis of the quartz tube and the light from the positive pole of the arc could therefore pass through the quartz tube. The lenses L_1 and L_2 were placed so that their centers were on the line passing through the axis of the tube *AB* and their positions along this line were adjusted until the image on the slit of the concave grating was the brightest possible. The oil bath was now heated to the required temperature, and sufficient current was sent through the aluminium coil surrounding the tube outside the furnace. While the temperature of the bath was rising to the required value, a current was sent through the carbon tube to raise its temperature to 300° C., and the temperature was maintained at this value until the bath had reached the desired temperature. The desired bath temperature having been reached, the current passing through the heating coil was adjusted to keep the temperature constant. The temperature of the furnace was then increased, and visual observations of the absorption spectrum were made. The temperature at which the absorption spectrum had completely disappeared was carefully noted. The increase in the temperature of the furnace was adjusted to take place gradually, especially near the point of disappearance of the spectrum.

Over a considerable range of temperature the absorption bands were very weak, and consequently difficult to observe. It is therefore difficult to fix the point of disappearance very accurately. In these experiments the eyepiece used for the visual observations was slowly moved backward and forward, for it is much easier to see a faint object when the eyepiece is moving than when it is stationary. In a few cases photographs of the absorption spectrum were taken, and the weakening and disappearance of the lines at high temperatures were demonstrated.

DIFFICULTIES

The temperature of disappearance of the absorption spectrum for comparatively large pressures (about 12 cm) was found to be above 1300° C., but this temperature could not be accurately determined because the clear quartz ends of the tube containing the heated vapor became opaque at these high temperatures. A tube which had been three times heated to 1200° C. would allow very little light to pass through, when used for the fourth time. The quartz tubes do not seem to become very porous even at high temperatures, for during an experiment on the absorption spectrum of iodine at a pressure of 10 cm of mercury the tube collapsed at a temperature of 1380° C.

Another point requiring some comment is the character of the light falling on the slit at the higher temperatures. In addition to the light from the arc, the lens L_2 concentrates on the slit radiation from the sides of the carbon tube, the carbon rings supporting the quartz tube, and from the quartz tube itself. The light from the latter sources was scarcely noticeable until a temperature of 800° C. was reached, and even then it was not strong enough to produce a visible spectrum. When the temperature of the furnace was 1000° C., the radiation from it was sufficient to produce a visible spectrum, and the intensity of this spectrum increased as the temperature of the furnace increased. The light from the arc, after passing through the quartz tube, was therefore superposed on the radiation from the carbon and quartz tubes. The absorption spectrum of the iodine was readily seen at these high temperatures, and the light from the furnace did not increase the experimental difficulties.

In the observed absorption spectrum there were always present certain lines due to the arc, but they were easily distinguished from the iodine absorption lines.

EXPERIMENTAL RESULTS

The experimental results obtained are collected in the accompanying table. The temperatures (C.) of disappearance of the absorption spectrum for various bath temperatures are given. The first two series of experiments were carried out with a quartz tube 30 cm long, but in the other experiments the tube was shortened to 20 cm. The temperature of the vapor was more uniform in the last two series of experiments.

Series I		Series II		Series III		Series IV	
Temp. of Bath in Degr. C.	Temp. of Disappearance of Spectrum (Degr. C.)	Temp. of Bath	Temp. of Disappearance of Spectrum	Temp. of Bath	Temp. of Disappearance of Spectrum	Temp. of Bath	Temp. of Disappearance of Spectrum
16°	400°	18°	540°	18.15	540°	....	....
43.7	700	46.3	700	48.6	760	48°.7	770°
60	780	62	800	63.2	850	69.5	930
....	....	85	1080	81.2	1060	84.2	1090
....	....	100	1200	100	1200	....	....
110–140	1330	...	....	128 C.	Spectrum had not disappeared at 1250° C.	120	Spectrum had not disappeared at 1300° C.

The temperature of the oil bath in the last experiment in Series I varied from 110° at the bottom of the bath to 140° at the top. The beaker containing the oil was heated by a coil of manganin wire wound round its curved surface, and this method of heating was found to give different temperatures along the axis of the beaker. In all subsequent experiments the oil bath was heated from below, and the large variations of temperature along the axis of the beaker were avoided. In the particular experiment considered, the vapor pressure of the iodine was deduced on the supposition that the temperature of the bulb was 110° C. In the first experiment, when the temperature of the bath was the ordinary temperature of the room, both heating coils were unnecessary. The absorption lines were much weakened at 300° C., and finally disappeared at about 500° C. The temperature of the quartz tube was kept constant at 500°, while the oil bath was being heated to 49°.

The iodine absorption lines made their appearance, and increased in intensity as the vapor pressure of the iodine in the tube increased. When the temperature of the bath had reached 49° C., the current passing through the heating tube was increased, and the absorption spectrum observed. The intensity of the lines was found to diminish with increase of temperature, and at 680° C. was very faint. The absorption spectrum finally disappeared at 760° C. On cooling from 800° C. the spectrum reappeared at 740° C., and increased in intensity on further cooling.

Similar experiments were carried out when the temperatures of the bath were 63.2, 81.2, 100.0, and 128° C. In the last case (pressure of iodine vapor being about 14 cm) very little light from the arc penetrated the iodine vapor until a temperature of 900° C. was reached. At this temperature a faint reddish light made its appearance on the slit, but no absorption spectrum could be seen. At 1000° C. the red end of the spectrum crossed by thick dark lines was observed, and when the temperature of the vapor had reached 1100° C. the characteristic absorption spectrum of iodine, showing lines in red, yellow, green, and blue-green, made its appearance. On raising the temperature, the intensity of the absorption lines diminished, but at 1250° C., the highest temperature to which the iodine vapor was raised, there still remained a decided absorption spectrum. It was concluded from the appearance of the spectrum at 1250° C. that the vapor would have to be heated above 1400° C. before the disappearance of the spectrum could be observed.

THE ABSORPTION SPECTRUM OF A CONSTANT MASS OF IODINE VAPOR

In the previous experiments, the pressure of the iodine in the quartz tube was deduced from the temperature of the bath, and the equation expressing the relation between the vapor pressure and temperature. In the following experiment the pressure of the vapor in the tube was variable, and the value of that pressure when the spectrum vanished was deduced from the gas law $PV=R\theta$. A known weight of iodine was placed in the quartz tube, which had been previously dried. The quartz tube was provided with a short side-tube, which was connected to a Fleuss pump, and the apparatus was evacuated to a pressure of 2 mm. The side tube was sealed off in the oxyhydrogen flame. The silica tube containing the known weight of iodine was then fixed inside the furnace, and the behavior of the absorption lines with increase of temperature studied. The lines were readily visible at 1000° C., and also at 1100° C. In the latter case the intensity was less. When a temperature of 1225° C. was reached, the lines were very faint, and the absorption finally disappeared at 1280° C. If we make the assumption that the iodine vapor is monatomic at this temperature, the pressure of the iodine vapor can be deduced from the equation

$$p=\frac{R\theta}{V}\frac{M}{m},$$

where

$R=8.26\times10^7$,
θ=absolute temperature=1553°,
V=volume of tube at temperature θ=24.3 cc,
M=mass of vapor=0.0015 grams,
m=molecular weight of iodine=127.

The value of p deduced from the above equation is 4.73 cm. In the calculation the volume of the tube has been taken to be the same as the volume at ordinary temperatures. The coefficient of expansion of quartz is, however, very small, and the correction to be applied can be neglected in this calculation. The calculation shows that at a pressure of 4.73 cm the iodine absorption spectrum disappears at 1280° C. On examining the results of the constant pressure experiments, it is found that the spectrum disappeared at 1200° C. when the temperature of the bath was 100° C. (pressure of iodine vapor 3.7 cm), and at 1330° C. when the temperature of the bath was 110° C. (pressure of iodine vapor 6.17 cm). The agreement between the results is good, considering the difficulty of making an exact estimate of the pressure in the experiment just described.

EXPERIMENTS ON IODINE AT LOW PRESSURE

When the iodine bulb is at the ordinary temperature of the room (vapor pressure being about 1.8 mm), the characteristic iodine spectrum is easily observed. The faint lines are not seen, but the stronger ones are easily recognized. It was therefore interesting to estimate the smallest vapor pressure of the iodine that could be detected by its absorption spectrum. The bulb containing the iodine was immersed in a freezing mixture of ice and salt, whereby a temperature of $-22°$ C. was obtained. The vapor pressure of iodine corresponding to this temperature was less than 0.003 mm, and the light from the positive pole of the arc after passing through the quartz tube showed no trace of absorption. On leaving the freezing mixture to stand in the atmosphere, its temperature gradually increased, and the absorption spectrum was observed from time to time. No trace of absorption could be detected until the pressure of the iodine vapor in the tube was about 0.029 mm. The temperature of the bulb was $-1°$ C. Later another experiment was carried out with the same tube, and the same result was obtained. Finally a tube of two-thirds the capacity was employed, and a similar experiment performed. In this case the absorption spectrum could be detected when the temperature of the bulb had reached 1° C. The pressure of the iodine vapor in the tube was about 0.036 mm. If we assume that the iodine vapor in these experiments consists of diatomic molecules, it is possible to make an estimate of the minimum quantity of iodine that can be detected by its absorption spectrum.

Let M = mass of iodine in a tube of volume V.
Let m = molecular weight of iodine vapor.
Then $M = \frac{Vpm}{R\theta}$ where $R = 8.26 \times 10^7$ and $m = 254$.

Experiments with First Tube

$V = 31$ cc, $\theta = 290$, $p = .0000384$ atmosphere.
$M = 12.6 \times 10^{-6}$ grams.

Experiments with Second Tube

$V = 21.5$ cc, $\theta = 290$, $p = .000047$ atmosphere.
$M = 10.8 \times 10^{-6}$ grams.

The above experiments show that a quantity of iodine of the order 12×10^{-6} grams can be detected by its absorption spectrum.

DISCUSSION OF RESULTS

The experimental results show that the disappearance of the absorption spectrum takes place at a higher temperature when the pressure is increased. It is therefore interesting to calculate the degree of dissociation of the iodine vapor for each pressure and temperature, and thus determine the percentage of the iodine molecules dissociated when the absorption spectrum had disappeared. The results of the calculation are shown in the following table.

Temp. of Bath in Degrees C.	Vapor Pressure mm	Temperature of Disappearance of Spectrum	Percentage Dissociation (Meier and Crafts)	Percentage Dissociation (Victor Meyer)	Maximum Degree of Dissociation Observable by Spectroscope	Remarks
18°.15	0.168	540° C.	37	....	21%	
48.6	1.76	760	70	....	85	
63.2	4.6	850	74	97 at 840°	94	
81.2	13.8	1060	89	98 at 1030°	97.4	
100	37.2	1200	90	....	99.0	
110	61.7	1330	93	....	99.3	Temperature of disappearance probably too low
120	96.5	Spectrum had not disappeared at 1300°	87	....	....	Tube collapsed at 1380° C.
128	137	Spectrum present at 1250°	80	....	....	Hundreds of lines present at 1250° and tube would have to be heated to 1400° C. before absorption would vanish
Weighed quantity of iodine in tube	47	1280°	92.6	....	99.2	

The percentage of dissociation for each temperature of disappearance is found in the fourth column. These values were deduced from Planck's equation, in which the constants a' and b' were calculated from the vapor density determinations of Meier and Crafts. The vapor density of iodine has also been investigated by Victor Meyer, and his results differed widely from those of Meier and Crafts. The percentage of dissociation at 840° C. and 1030° C., and under atmospheric pressure, can be readily deduced from his experimental results, and from the relation $\frac{n_2^2}{n_1(n_1+n_2)}\ a\ \frac{1}{p}$ the degree of dissociation corresponding to any pressure can be calculated. The results of such a calculation are given in the fifth column. In the sixth column the degree of dissociation observable by the spectroscope has been calculated. In the calculation it was assumed that 12×10^{-5} grams of iodine in the diatomic state were always present when the spectrum disappeared. The above value was derived from the experiments previously described, and it is important to remember that it is only an estimate. Also the mass of I_2 necessary for detection may be a function of the temperature. In discussing the connection between the disappearance of the absorption spectrum and dissociation, it is advisable to leave out the first experiment, as the pressure of the iodine vapor in the tube was only $\frac{1}{7}$ mm, and the quantity of iodine present in the tube at 500° C. was comparable with the amount that could be detected at the ordinary temperature.

Confining our attention to the other results, it is seen that the temperature at which the absorption spectrum vanishes increases with the pressure, and the degree of dissociation at the vanishing point is always over 70 per cent. For the higher pressures the percentage of dissociation is above 90. Therefore, basing our calculations on the vapor density determinations of Meier and Crafts, it follows (for pressures above 3 cm) that the absorption spectrum disappears when more than 90 per cent of the I_2 molecules have been dissociated. On the other hand, the values of the dissociation calculated from the vapor density determinations of Victor Meyer seem to show that the iodine absorption spectrum vanishes when practically all the molecules are dissociated.

Friederichs gave the following explanation of the disappearance of the absorption bands. The bands were ascribed to the molecules I_2, and the absorption spectrum vanished when the emission accompanying dissociation and recombination was balanced by the absorption of the molecules still in the diatomic state. This hypothesis may possibly explain all the experimental results, if we admit that the emission from the heated iodine vapor can balance the absorption of the light emitted by the positive pole of the arc. It seems natural to ascribe the absorption bands to the diatomic molecules, for the spectrum is always present when the iodine vapor is in the undissociated state. On heating the vapor to a high temperature no new absorption spectrum appeared. It therefore follows that the monatomic iodine molecule must either be colorless, or give the same absorption spectrum as the diatomic molecule, unless it is assumed that its absorption is balanced by a corresponding emission at all temperatures and pressures. The latter assumption is improbable.

The results obtained in the present research may be explained on the assumption that the monatomic iodine molecule shows no absorption in the region of the visible spectrum. If this hypothesis be true, the absorption spectrum should vanish when practically all the I_2 molecules had been dissociated. The values of the dissociation calculated from the results of Meier and Crafts bear out this explanation to a certain extent, for high values of the dissociation accompany the vanishing of the absorption spectrum. If the values calculated from the vapor density determinations of Victor Meyer are accepted, the hypothesis and experimental results agree as well as could be expected. It is difficult to decide between the vapor density values, for both sets of results were obtained with a Victor Meyer apparatus, which is not suitable for determining the vapor density of dissociating substances. Assuming that in the present experiments the temperature of disappearance of the absorption lines has been estimated 60° C. too low, the degree of the dissociation of the vapor calculated on the basis of Meier and Crafts's results would still be below 95 per cent. As an illustration, the case of iodine vapor at a pressure of 13.6 mm will be considered. If it be assumed that the spectrum vanished at 1120° C. instead of 1060° C. the calculated degree of dissociation would be 93 per cent.

In attempting an explanation of the vanishing of the absorption spectrum, it is obviously necessary to take into account the existence of the emission spectrum, especially as both spectra consist of lines having identical wave-lengths. A few experiments were made to see whether the emission spectrum could be detected with the apparatus employed. The light emitted by the furnace at a temperature of 850° (when the pressure of iodine vapor in the silica tube was 0.46 cm) was examined visually, and no trace of a banded emission spectrum could be seen. Also the light emitted by the furnace made scarcely any impression on a photographic film placed at the focus of the grating, after an exposure of 40 minutes. It is important to note that the iodine vapor under the above conditions gave no absorption spectrum. A photograph of the absorption spectrum after an exposure of twenty minutes was obtained on the same film, when the temperature of the iodine vapor was lower. Several attempts were made to detect the emission spectrum of the iodine vapor at different temperatures (600° to 1200°), but in every case no trace of a banded emission spectrum could be seen. The experimental arrangement was not suitable for the observation of the emission spectrum, as the ends of the silica tube were curved, and the distance of the focusing lens L_2 from the middle of the tube was about 80 cm. Only a very small fraction of the light emitted by the iodine vapor would be concentrated on the slit of the concave grating.

Again, if the emission is assumed to be a true temperature emission, the banded emission spectrum of the heated vapor would be very difficult to detect. The silica tube and the iodine vapor inside were heated to a nearly uniform temperature, and the radiation from the interior would correspond to that of a black body at the same temperature. It would therefore be difficult to account for the disappearance of the absorption spectrum, unless it was assumed that the I_1 molecule gave no absorption within the limits of the visible spectrum.

Konen investigated the effect of temperature on the emission spectrum of iodine vapor, and found that the emission at first increased with rise of temperature, then became constant, and finally diminished. If the I_1 molecule be regarded as contributing nothing to the emission spectrum, it may be possible to account for the maximum observed by Konen, on the assumption that the emission spectrum is due to the I_2 molecule, and is a true temperature emission following Kirchhoff's law. With increase of temperature the number of I_2 molecules emitting light diminishes, but the amount of light emitted by a given number of I_2 molecules increases, and the maximum in the emission spectrum would occur when the diminution in the emission due to dissociation was balanced by the increase due to rise of temperature. A maximum would also be expected if the emission spectrum be due to the dissociation and recombination of the iodine molecules. It is difficult to determine when the maximum should occur, because the intensity of the light emitted by a given number of molecules during the act of dissociation and recombination may be a function of the temperature.

The experimental results given in this paper show that the temperature of disappearance of the absorption spectrum increased with increase of pressure of the iodine vapor. On the assumption that the disappearance is due to the formation of colorless I_1 molecules, the explanation is simple, for increase of pressure at constant temperature diminishes the percentage of dissociation. The number of I_2 molecules present when the pressure is increased is sufficient to show the characteristic iodine absorption spectrum, and to cause the disappearance of the absorption a higher temperature is necessary. If the iodine emission spectrum be due to temperature alone, it is difficult to account for the experimental result, on the assumption that the emission balances the absorption. The laws of temperature-radiation show that the ratio of the emissive power to the absorptive power is a function of the temperature and the wave-length. It therefore follows that if the emission balances the absorption at any temperature, the balance should not be disturbed when the pressure is increased, as long as the temperature is kept constant. In addition, it is difficult to see how the emission spectrum of iodine vapor at 1200° C. could ever balance the absorption spectrum obtained by passing the light from the positive pole of the electric arc through the heated vapor. On the assumption that the emission spectrum of iodine is not a true temperature emission, it is difficult to test the hypothesis which regards the disappearance of the absorption spectrum as being due to the presence of a compensating emission spectrum. From results already achieved, the effect of temperature on the bromine absorption spectrum appears to be similar to its effect on iodine.

SUMMARY OF RESULTS

a) The absorption spectrum of iodine disappears at high temperatures, and the temperature of disappearance increases as the pressure is increased.

b) The degree of dissociation of the vapor when the absorption spectrum vanishes is high. The values calculated from the vapor density determinations of Victor Meyer show that the absorption spectrum vanishes when nearly all the diatomic molecules are dissociated into monatomic molecules. On the other hand, the values of the degree of dissociation calculated from the vapor density determinations of Meier and Crafts show that from 80 to 93 per cent of the diatomic molecules are dissociated when the absorption spectrum disappears.

c) It may be possible to account for the disappearance of the absorption spectrum, on the assumption that the monatomic molecules produced by dissociation show no absorption within the limits of the visual spectrum. The vapor density determinations in the case of iodine are probably inaccurate, and it is difficult to test the hypothesis quantitatively.

d) It appears impossible to account for the disappearance of the absorption spectrum on the assumption that it is balanced by a corresponding emission spectrum, if it is assumed that the emission is a true temperature emission.

I am indebted to Professor Schuster for the interest he has taken in the work, and also to Professor Rutherford for providing the necessary facilities to carry it out.

PHYSICAL LABORATORY
MANCHESTER UNIVERSITY
March 1910

THE MOON'S THEORETICAL SPECTROGRAPHIC VELOCITY

By KURT LAVES

The moon's theoretical spectrographic velocity in the line of sight is obtained by astrophysicists by using Professor Campbell's method of computation.[1] The interpolation between the Nautical Almanac values is somewhat cumbersome and in the majority of cases the accuracy of the theoretical values obtained is greatly in advance of that of the observed values. This paper aims at a reduction of the labor of computation by proposing the use of some short tables which are computed for this purpose. These tables are based upon the elliptical values of the elements of moon and earth. This, at first sight, will perhaps appear as a step in the wrong direction, since it is likely that the observed value of the moon's spectrographic velocity before many years will be accurate to 0.01 km. Such an advance is not yet at hand instrumentally,[2] and, if it were, the values obtained by means of these tables would be of advantage as a check on the results obtained by Campbell's procedure. In the majority of cases the velocity computed by means of these tables will prove to be of sufficient accuracy. In order that the accuracy be tested, an inquiry into the effects of the perturbative terms has been made and the results are embodied in this paper.

1. Poincaré,[3] in discussing the spectroscopic observations of *Jupiter* by Deslandres, has given the fundamental equation upon which rests the computation of the velocity in the line of sight of the moon or any planet. Let S, E, M represent the positions of sun, earth, and moon, and call the distance $SE=D_1$, $ME=D_2$, and $SM=\Delta$. A luminous vibration of period τ leaves the sun's surface at the time t; it will reach the earth at the time $\frac{t+\Delta+D_2}{V'}$, where V' is the velocity of light. The next vibration will leave the sun's

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[1] See paper of Professor Campbell in this *Journal*, **11**, 141–143, 1900.

[2] See note by Professor Frost in the May number, p. 377.

[3] *Comptes Rendus*, **120**, 420, 1895.

surface at the time $t+\tau$ and will strike the earth's surface at the time $t+\tau+\frac{\Delta+D_2}{V'}+\left(\frac{\frac{d\Delta}{dt}+\frac{dD_2}{dt}}{V'}\right)\cdot\tau$. The apparent period of the vibration will therefore be $\left(\frac{\frac{d\Delta}{dt}+\frac{dD_2}{dt}}{V'}\right)\cdot\tau$. Calling E the angle at the earth between D_1 and D_2, and σ the parallactic angle at S, we have

$$\Delta^2=D_1^2+D_2^2-2D_1D_2\cos E\,. \tag{1}$$

From this equation we obtain:

$$\frac{d\Delta}{dt}+\frac{dD_2}{dt}=\frac{dD_1}{dt}\cdot\cos\sigma+\frac{dD_2}{dt}\,[1-\cos(E+\sigma)]+D_2\cdot\frac{dE}{dt}\cdot\sin(E+\sigma)\,. \tag{2}$$

The velocity in the line of sight V, due to the orbital motion of moon and earth, is given by the right-hand expression of the last equation. Campbell uses the following notation:

$$\left.\begin{aligned}&\frac{dD_1}{dt}=V_1 &\qquad& -\frac{dD_2}{dt}\cdot\cos E=V_3\\ &\frac{dD_2}{dt}=V_2 && D_2\cdot\frac{dE}{dt}\cdot\sin E=V_4\\ &-v_d=\text{component of } V \text{ due to the earth's rotation.}\end{aligned}\right\} \tag{3}$$

Using this notation, we have:

$$\left.\begin{aligned}V+v_d=V_1\cos\sigma+V_2+V_3\cdot\cos\sigma+V_2\sin E\sin\sigma+V_4\cdot\cos\sigma+\\ D_2\cdot\frac{dE}{dt}\cdot\cos E\sin\sigma\end{aligned}\right\} \tag{4}$$

By putting $\sin\sigma=0$ and $\cos\sigma=1$, Campbell obtains:

$$V+v_d=V_1+V_2+V_3+V_4\,. \tag{5}$$

While the approximation $\cos\sigma=1$ introduces in $V_1\cos\sigma$ an error of the order 3.10^{-6} km, we find that the neglect of $D_2\frac{dE}{dt}\cos E\sin\sigma$ may introduce an error of the order 7.10^{-3} km. It is desirable on this account to retain $E+\sigma$ in the computation of V. Since we have

$$\frac{D_2}{D_1}=\frac{\sin\sigma}{\sin(E+\sigma)}\,,$$

we derive from it a value of σ by the formula:

$$\tan(\sigma+E/2)=\frac{\pi_2+\pi}{\pi_2-\pi}\tan E/2\,, \tag{6}$$

where π is the horizontal equatorial parallax of the sun and π_2 that of the moon. For an accurate determination of $V+v_d$ we compute V_3 and V_4 from the formulae:

$$\left.\begin{aligned} V_3&=-\frac{dD_2}{dt}\cdot\cos(E+\sigma)\\ V_4&=D_2\frac{dE}{dt}\cdot\sin(E+\sigma)\end{aligned}\right\}\qquad(7)$$

We next proceed to derive the values for V_1, V_2, V_3, V_4 on the assumption that the elliptic values of the elements of the orbits of moon and earth give a sufficient approximation. From the polar equation of the ellipse

$$r=\frac{p}{1+e\cos v},$$

we obtain readily:

$$\frac{dr}{dt}=K\cdot e\sin v\,,\qquad(8)$$

where $K=\frac{c}{p}$, c being the constant of areas, p the parameter of the ellipse.

Similarly it follows that:

$$\frac{rdv}{dt}=K(1+e\cos v)=\frac{K\cdot p}{\rho_0}\sin\pi\,,\qquad(9)$$

where ρ_0 is the equatorial radius of the earth. The true anomaly of the earth is connected with the longitude λ of the sun by the equation

$$v=\lambda-\pi\,,$$

where π is the longitude of the sun when the earth is at the perihelion.

Since the Nautical Almanac gives λ for every day, it proved to be preferable to compute V_1 for the argument λ. Table A gives V_1, from 5° to 5° for the entire circumference. In the case of the moon, it is best to compute V_2 and V_4 by using the parallax as an argument. Since V_4 contains the lateral component of the moon's velocity relatively to the sun, we have to apply a correction to the absolute lateral component, as given by formula (9). This correction may be considered a constant, from the following consideration. In the spherical triangle formed by the sun, the moon, and the ascending node of the moon's orbit on the ecliptic, we have:

$$\cos E=\cos\beta_2\cos(\lambda_2-\lambda)\,.$$

By differentiation we get

$$\left.\begin{aligned} D_2 \sin E\frac{dE}{dt} &= D_2 \sin (\lambda_2-\lambda) \cos \beta_2 \, \frac{d(\lambda_2-\lambda)}{dt} + \\ & D_2 \sin \beta_2 \cos (\lambda_2-\lambda)\frac{d\beta_2}{dt} \end{aligned}\right\} \quad (10)$$

Since we do not retain terms which are smaller than 0.01 km, it follows that the second term of the right-hand member can be neglected. From the first equation we have

$$\cos \beta_2 \sin (\lambda_2-\lambda) = \sin E\sqrt{1-\frac{\sin^2 \beta}{\sin^2 E}}\,.$$

Again, the term under the radical can be put equal to unity, for the reason quoted above. Thus we obtain

$$V_4 = D_2 \sin E\frac{dE}{dt} = D_2 \cdot \frac{d(\lambda_2-\lambda)}{dt} \cdot \sin E\,. \qquad (11)$$

We find that $D_2 \cdot \frac{d\lambda}{dt}$ is practically a constant and equal to 0.089 km, so that $D_2\frac{d(\lambda_2-\lambda)}{dt}$ can be obtained by diminishing $\frac{D_2 d\lambda_2}{dt}$ by 0.09 km. It will be granted that the computation of the component velocities V_1, V_2, V_3, V_4 is made very rapid by the use of the columns in Tables A and B.

2. We next proceed to show to what extent the perturbative terms will change the elliptical values of the component velocities. The perturbation of the radius vector of the earth's orbit is less than 0.0000400 of its value,[1] so that we may entirely disregard the perturbations in V_1. In the case of the moon, conditions are quite different; it will be seen from what follows that certain terms will reach a value of 0.01 km. Taking into account only terms of the first and second order in the expression of the radius vector D_2, we have:[2]

$$\left.\begin{aligned} \frac{1}{D_2} = \frac{k^2(m_1+m_2)}{c_2^{\,2}}\Big\{ & 1-\tfrac{3}{4}k^2-\tfrac{1}{2}m^2+e_2 \cos (c\lambda_2-\alpha)-\tfrac{1}{4}k^2 \cos 2(g\lambda_2-\gamma) \\ & +m^2 \cos [(2-2m)\lambda_2-2\beta] \\ & +\tfrac{15}{8}m \cdot e_2 \cdot \cos [(2-2m-c)\lambda_2-2\beta+\alpha] \\ & +\text{higher terms}\Big\} \end{aligned}\right\} \quad (12)$$

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[1] *Annales de l'observatoire impériale de Paris*, **4**, sec. 6, Table III.

[2] See, for instance, H. Godfrey, *An Elementary Treatise of the Lunar Theory*, London, 1871.

The quantities appearing on the right side have the following meaning:

k^2 = Gauss's constant

m_1, m_2 = masses of earth and moon

c_2 = constant of integration in the equation

$$\left(D_2{}^2\frac{d\lambda_2}{dt}\right)^2=c_2{}^2+2\int T\cdot D_2{}^3 d\lambda$$

where T is that component of the perturbative force which lies in the plane of the orbit and is at right angles to D.

$k=\tan I$, where I is the inclination of the moon's orbit to the ecliptic,

e_2 = the eccentricity of the moon's orbit,

m = the ratio of the mean motions of moon and sun, in longitude,

a, γ = the moon's longitudes of the apse and of the node,

β = the sun's longitude at the time $t=0$,

c, g = quantities, which differ from unity by a small quantity of the first order.

From the expression for $\frac{1}{D_2}$ we obtain by differentiation:

$$\left.\begin{aligned}\frac{dD_2}{dt}=K_2\{e_2\sin(\lambda_2-a)-\frac{k^2}{2}\sin(\lambda_2-\gamma)+2m^2\sin[(2-2m)\lambda_2-2\beta]\\+\tfrac{15}{8}me_2\sin[(2-2m-c)\lambda_2-2\beta+a]+\ .\ .\ .\}\end{aligned}\right\}\tag{13}$$

K_2 expressed in kilometers is 1.024 km; the elliptic term K_2e_2 $\sin(\lambda_2-a)$ will *in maximo* amount to 0.06 km. For an approximate estimate of the coefficients of the second-order terms we put $m=\frac{1}{13}$, $e_2=\frac{1}{20}$, $k=\frac{1}{12}$ and obtain:

$$\left.\begin{aligned}\frac{dD_2}{dt}=\overset{\text{km}}{0.05}\sin(\lambda_2-a)-\overset{\text{km}}{0.003}\sin(\lambda_2-\gamma)+\overset{\text{km}}{0.01}\sin[(2-2m)\lambda_2-2\beta]\\+\overset{\text{km}}{0.02}\sin[(1-2m)\lambda_2-2\beta+a]+\ .\ .\ .\ .\end{aligned}\right\}\tag{14}$$

While the reduction-term is entirely negligible we see that both the variation and the evection terms will affect the second decimal place of the radial velocity. The lateral component of the moon's motion enters into the components V_4. To obtain $D_2\cdot\frac{d\lambda_2}{dt}$ we start from the equation

$$D_2\frac{d\lambda_2}{dt}=c_2\sqrt{1+2\int\frac{T\cdot D_2{}^3}{c^2}d\lambda_2}\ \frac{1}{D_2}.\tag{15}$$

The expression under the square-root sign is to be given as a trigonometric series, and after that it is to be multiplied by the series for $\frac{1}{D_2}$ as given above. Limiting our attention to the terms not higher than the second, it can be shown that we obtain:

$$\left.\begin{array}{l} D_2\frac{d\lambda_2}{dt}=K_2\{1-\frac{3}{4}k^2-\frac{1}{2}m^2+e_2\cos(\lambda_2-\alpha)-\frac{1}{4}k^2\cos 2(\lambda_2-\gamma)+ \\ \frac{7}{4}m^2\cos[(2-2m)\lambda_2-2\beta]+\frac{15}{8}me_2\cos[(2-2m-c)\lambda_2-2\beta+\alpha]+\dots\} \end{array}\right\}(16)$$

TABLE A

λ Long. of Sun	V_1	λ Long. of Sun	V_1	λ Long. of Sun	V_1	λ Long. of Sun	V_1
	km		km		km		km
0°	+0.49	95	+0.05	185	−0.50	275	−0.05
5	+ .50	100	+ .01	190	− .50	280	− .01
10	+ .50	105	− .03	195	− .50	285	+ .03
15	+ .50	110	− .08	200	− .49	290	+ .08
20	+ .49	115	− .12	205	− .49	295	+ .12
25	+ .49	120	− .16	210	− .47	300	+ .16
30	+ .47	125	− .20	215	− .46	305	+ .20
35	+ .46	130	− .24	220	− .44	310	+ .24
40	+ .44	135	− .28	225	− .42	315	+ .28
45	+ .42	140	− .31	230	− .39	320	+ .31
50	+ .39	145	− .35	235	− .36	325	+ .35
55	+ .36	150	− .38	240	− .33	330	+ .38
60	+ .33	155	− .40	245	− .30	335	+ .40
65	+ .30	160	− .43	250	− .26	340	+ .43
70	+ .26	165	− .45	255	− .22	345	+ .45
75	+ .22	170	− .47	260	− .18	350	+ .47
80	+ .18	175	− .48	265	− .14	355	+ .48
85	+ .14	180	− .49	270	− .10	360	+ .49
90	+ .10						

Assuming the values above we obtain:

$$\left.\begin{array}{l} D_2\cdot\frac{d\lambda_2}{dt}\overset{\text{km}}{=}1.024+0.051\cos(\lambda_2-\alpha) \\ \qquad -0.008-0.001\cos(\lambda_2-\gamma)+0.010\cos[(2-2m)\lambda_2-2\beta] \\ \qquad +0.012\cos[(1-2m)\lambda_2-2\beta+\alpha]+\dots \end{array}\right\}(17)$$

The terms in the first line of the right-hand member give the elliptic values, while the terms in the line below show the perturbative terms. A glance at (15) and (17) will at once show whether or not it will be desirable to substitute the short method proposed in this paper for the more lengthy interpolation used exclusively heretofore.

Tables A and B here appended will furnish directly V_1, V_2, and $D_2\frac{d\lambda_2}{dt}$. When we multiply $-V_2$ by cos E and $D_2\frac{dE}{dt}$ by sin E, the values of V_3 and V_4 are obtained.

TABLE B

Moon's Parallax	V_2	$D_2\frac{d\lambda_2}{dt}$	$D_2\frac{dE}{dt}$	Moon's Parallax	V_2	$D_2\frac{d\lambda_2}{dt}$	$D_2\frac{dE}{dt}$
	km				km		
54′ 0″	0.01	0.970	0.88	57′ 20″	0.06	1.028	0.94
10	.01	972	88	30	.06	030	94
20	.02	975	89	40	.06	033	94
30	.03	977	89	50	.06	036	95
40	.03	980	89	58′ 0	.05	039	95
50	.04	984	90	10	.05	042	95
55′ 0	.04	986	90	20	.05	045	96
10	.04	988	90	30	.05	047	96
12	.04	991	90	40	.05	050	96
30	.05	994	91	50	.05	053	96
40	.05	997	91	59′ 0	.05	056	97
50	.05	1.000	91	10	.04	059	97
56′ 0	.05	003	91	20	.04	062	97
10	.05	007	92	30	.04	065	98
20	.05	010	92	40	.03	068	98
30	.06	012	92	50	.03	071	98
40	.06	015	93	60′ 0	.02	075	99
50	.06	018	93	10	.02	078	99
57′ 0	.06	021	93	20	.01	1.081	0.99
10	.06	025	94				

If the moon's parallax is decreasing, V_2 is positive; if the moon's parallax is increasing, V_2 is negative.

Example.[1]—To compute V of the moon for 1900, January 9, $4^h\ 30^m$ Mt. Hamilton sidereal time, by both methods:

Campbell	Laves
$V_1 = +0.06$ km	$+0.06$ km
$V_2 = +0.04$	$+0.06$
$V_3 = +0.02$	$+0.03$
$V_4 = +0.86$	$+0.85$
$-v_d = +0.16$	$+0.16$
$V = +1.14$ km	$+1.16$ km

Measures of a spectrogram for the observed time gave, as the mean observed velocity, +1.46 km.

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[1] *Astrophysical Journal*, 11, 143, 1900.

OBSERVATIONS OF THE SUN ON MAY 18 AND 19, 1910

By FREDERICK SLOCUM

For comparison with the observations of the sun made in the eastern hemisphere during the transit of the head of Halley's comet, and in connection with the auroras, and other terrestrial magnetic phenomena seen in various places on May 18 and 19, it seems worth while to put on record the following observations of the sun.

Plate I, Figs. 1 and 2, are direct photographs of the sun taken with the 40-inch refractor (stopped down to aperture of 5 inches = 13 cm), May 18 and 19, 1910. The most conspicuous feature on the sun is the group of spots near the center of the disk, an enlarged view of which on May 18 is shown on Fig. 2, Plate II. Fig. 1, Plate II, shows the same region as photographed with the Rumford spectroheliograph in the light of the H_2 line of calcium on the same date.

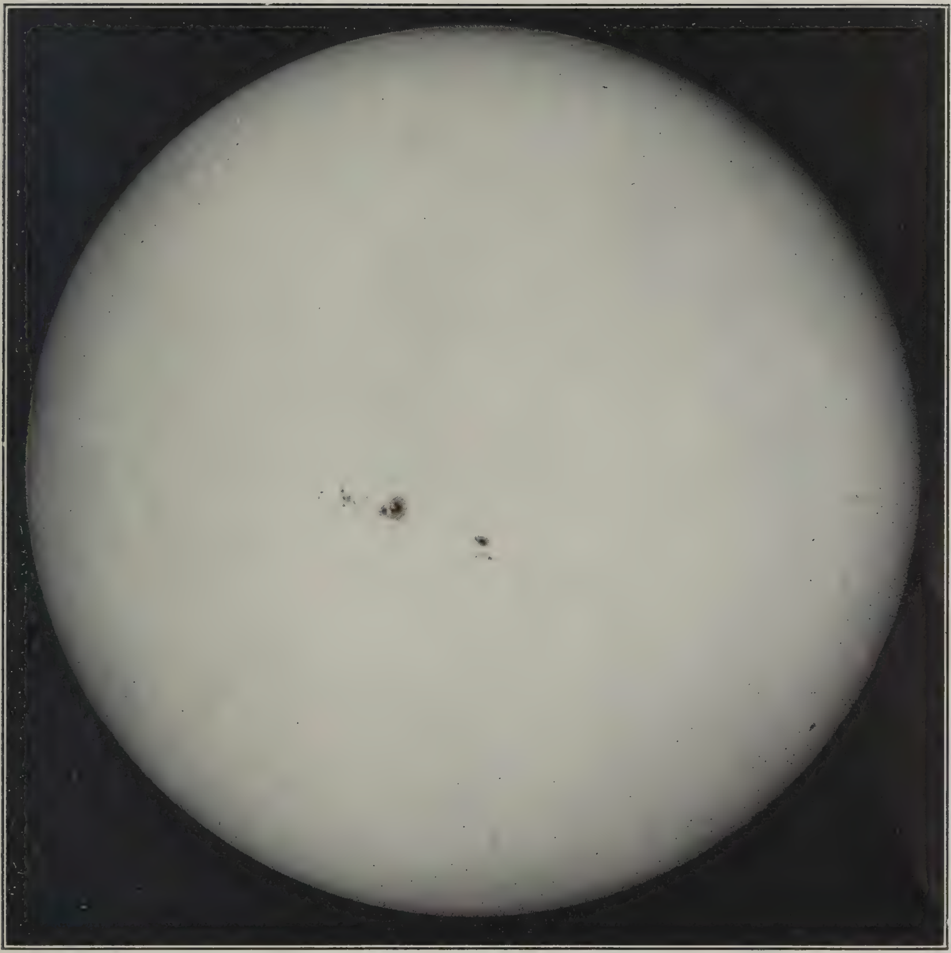
This group developed some time between April 28 and May 11 on the invisible side of the sun. The leader spot, A, of the group was first brought into view by rotation on May 12, and the follower, B, appeared the next day. On May 14, A seemed much larger than B, but both were still so foreshortened that a study of details was difficult. When next seen, on May 18, A had diminished in size, while B had greatly increased. The statistics in the following table were obtained from a plate exposed on May 18 at $8^h\ 42^m$ G.M.T.

Length of group of spots 310,000 km.
Length of area of calcium flocculi 360,000 km.
Spot A: Latitude −10°.9. Longitude 64°.
    Umbra: Length 12,000 km, width 8000 km.
    Penumbra: Length 30,000 km, width 16,000 km.
Spot B: Latitude −11°.2. Longitude 52°.5.
    Umbra (triple): Extreme length 30,000 km, width 16,000 km.
    Penumbra: Length 50,000 km, width 38,000 km.

A comparison of the direct photographs of the 18th and 19th shows considerable activity. Changes in the location and number of small spots, and changes in the arrangement and intensity of the bridges in the large spots may be seen. The calcium spectrohelio-

PLATE I

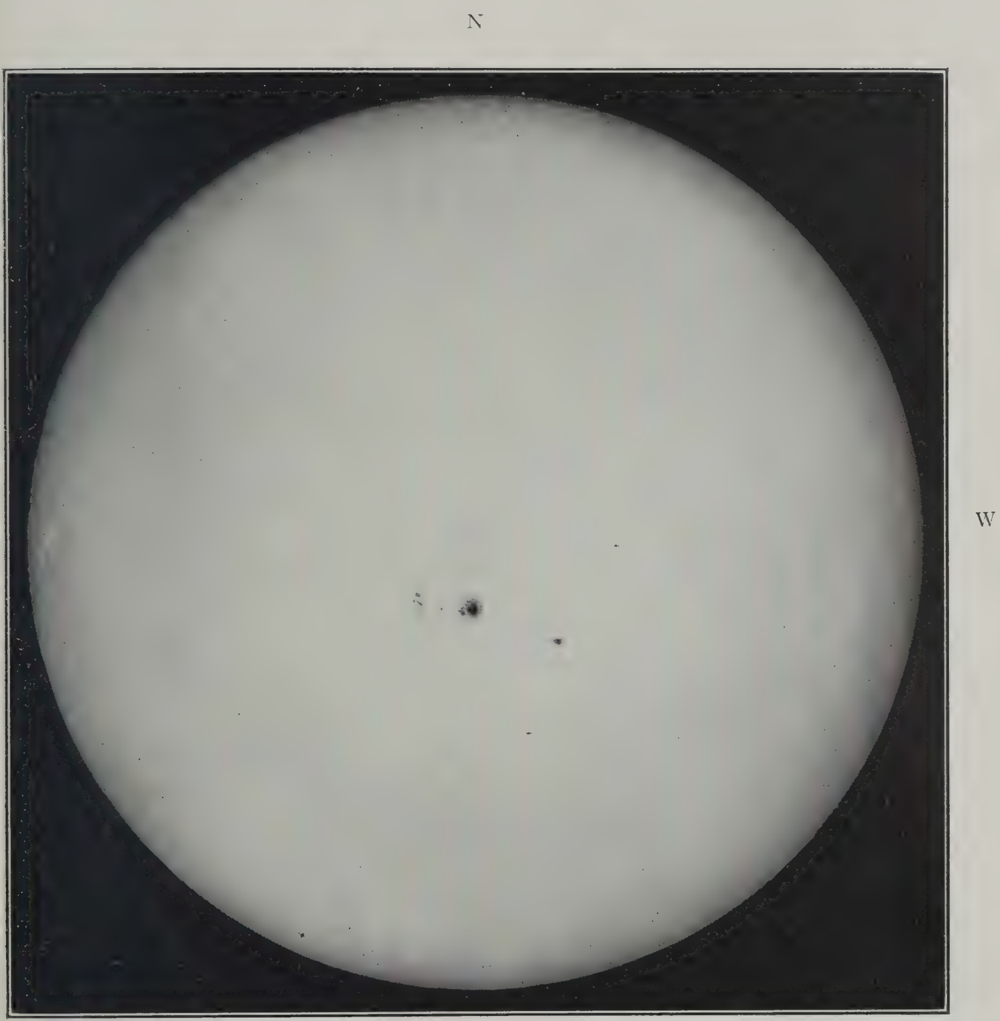
I



PHOTOGRAPH OF THE SUN, MAY 18, 1910, AT 10^{h} 34^{m} G.M.T. TAKEN WITH FORTY-INCH REFRACTOR

Two-thirds of original size

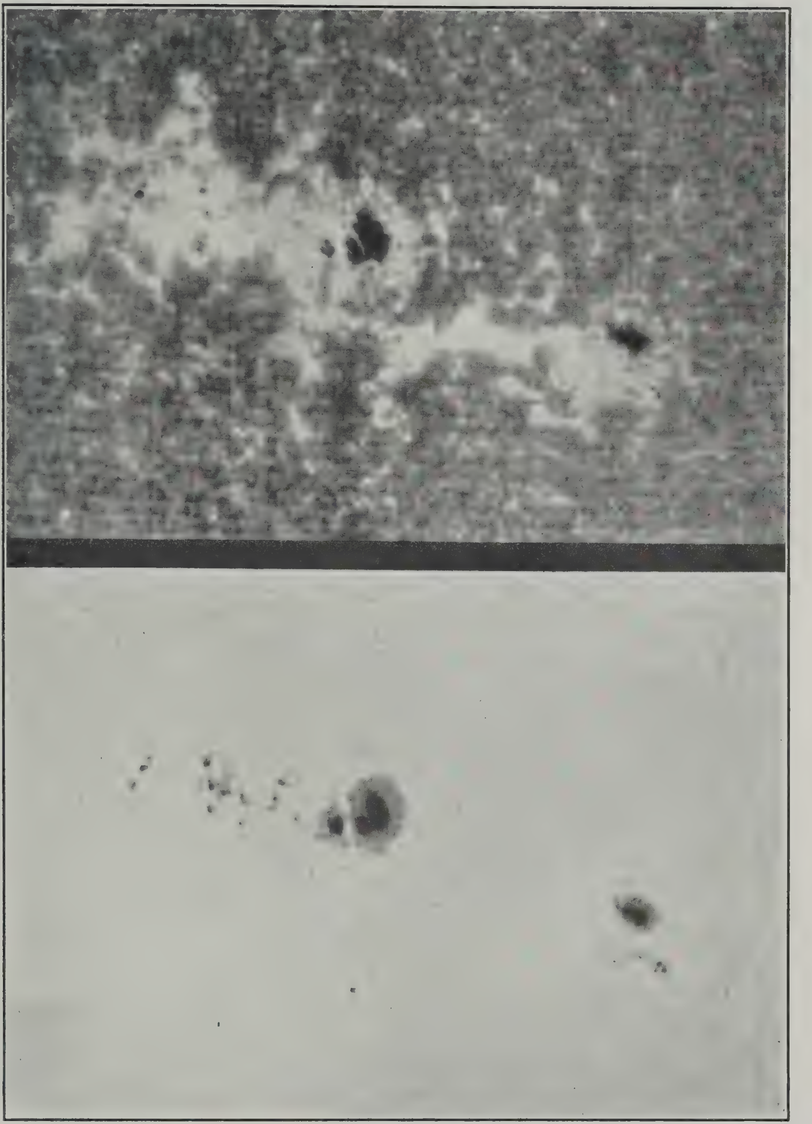
PLATE I



Photograph of the Sun, May 19, 1910, at $3^h\ 15^m$ G.M.T. Taken with Forty-Inch Refractor

Two-thirds of original size

PLATE II



SUN-SPOTS ON MAY 18, 1910

1. Spectroheliogram Taken with Calcium Line H_2 at $8^h\ 28^m$ G.M.T.
2. Direct Photograph Taken at $10^h\ 34^m$ G.M.T.

Scale: Sun's Diameter = 37 cm

grams show conspicuous changes in the arrangement of the flocculi and in the distribution of the eruptive jets. There are two chief scenes of activity: one, a small area south of spot A, the other, an area extending from B, 5° toward the east and 2° toward the south; that is, between B and the group of small spots which follows it. In this latter area no less than 12 eruptions can be counted on the negative taken at $8^{h}\,26^{m}.7$, May 18. The largest of these is about 20,000 km S.E. of the S.E. edge of spot B. It has an apparent diameter of 6000 km, but is partly covered by a long, narrow cloud, which appears to rise from a point near the heavy bridge of spot B and extend off toward the S.E. There is no trace of this eruption or the cloud on a plate taken 16 minutes later. The center of this region of activity crossed the central meridian of the sun May 19, 7^{h} G.M.T., spot A crossed at $2^{h}.5$, May 18, and spot B at about noon, May 19.

A direct photograph of the spectrum from $\lambda\,3920$ to $\lambda\,3980$, taken May 18, 9^{h}, with the slit across spot B, shows the reversed H and K lines considerably distorted, indicating active vertical currents, especially in the region east of B.

Two other spot groups appear on the plates of these two dates: one near the west edge, in latitude −16°, longitude 130°; the other, a pair of very small spots, just appearing around the east limb in latitude −20°, longitude 340°. The preceding spot of this pair is not visible on the engraving (Plate I, Fig. 2), having been lost in the process of reproduction.

The prominence plates show only ordinary conditions. On May 18 at $5^{h}\,56^{m}$ there were fifteen prominences, but none of unusual size or intensity. The largest extended from 30° to 47° in position-angle, and rose to a height of $40''$. The prominences of May 19 were somewhat fewer and smaller.

YERKES OBSERVATORY
June 9, 1910

ON SOME METHODS AND RESULTS IN DIRECT PHOTOGRAPHY WITH THE 60-INCH REFLECTING TELESCOPE OF THE MOUNT WILSON SOLAR OBSERVATORY[1]

By G. W. RITCHEY

After the figuring of the 60-inch mirror was completed in 1907, experiments were carried on in the optical shop, to determine how large a *daily* variation of temperature is permissible without seriously affecting the figure of so large and thick a mirror. The temperature in the optical shop can be controlled at will, so it was possible to conduct these experiments with any daily variation desired, from 0° to 10° F. In general, the temperature of the room was allowed to rise slowly, and approximately uniformly, for 12 hours, and then to fall the same amount during an equal period, close watch being kept of the resulting changes of figure throughout the day and night. The most marked effect was a decided disturbance of the figure of the outer zones of the surface for a distance of 3½ or 4 inches (10 cm) in from the edge, these zones turning forward, or becoming too high, as the temperature rose, and receding, and even becoming too low, as the temperature fell. The figure of the remaining zones of the surface was only very slightly affected, and the change of focal length was small. A daily variation of 1° F. caused a disturbance of the edge zones easily perceptible with the usual optical tests. It was decided, as a final result of these experiments, that a daily variation of the large mirror of 2° F., of the character described earlier in this paragraph, is the maximum variation which can be allowed without perceptible injury to the sharpness of photographic star-images.

Until July 1, 1909, the 60-inch reflector was used with its dome unprotected by the white canvas sun-shield for which provision had been made in the design and construction. The daily variation of temperature in the unprotected dome in clear weather in the latter half of June averaged 20° F. This large daily change of course affected very seriously the figure of the mirrors; in addition, the expansion and contraction of the steel skeleton tube was such that the apparent change of focal length at the Newtonian focus during the night frequently amounted to 0.04 inch (1.0 mm). This change was evidently due almost entirely to the contraction of the tube, since the plate-holder had to be moved *out* to follow the apparent change of focus, and the amount of change agreed approximately with that resulting from the temperature coefficient of steel.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 47.

Early in July the sun-shield was put in place on the dome. As described in a former article,[1] this shield consists of gores of heavy white canvas laced to a strong framework of steel pipe. The canvas is thus held at a distance of about two feet from the sheet-steel covering of the dome; provision being made for free circulation of air beneath the canvas. With this protection, the daily variation of temperature in the dome decreased to 10° F. in July, and the apparent change of focus during the night decreased to 0.02 inch (0.5 mm).

In addition to the protection of the entire dome from large temperature changes by means of the canvas sun-shield, it was originally planned to hold the large mirror during the day at the expected night temperature by means of a removable insulating jacket inclosing the lower end of the telescope tube; through the hollow walls of this jacket was to be circulated air or water cooled by a small refrigerating apparatus and controlled by a thermostat.

When we came actually to design this jacket, several serious difficulties and objections were encountered. Chief among these was the danger of local or unsymmetrical cooling of the mirror, because the forms of the adjacent metal parts are such that the jacket could not be perfectly symmetrical in form. This led to the adoption of a simpler and more economical plan which has proved so effective in preventing injurious changes of figure of the large mirror, and in reducing the apparent change of focus during the night, that it merits a somewhat detailed description.

A light removable room or chamber with insulating walls was constructed to inclose the telescope during the day. This chamber we call the "canopy." It is rectangular, 15 feet (4.6 m) high, 15 feet long (north and south), and 11 feet (3.4 m) wide, and weighs about 500 pounds (227 kilos). The top, side, and end walls each consists of four thicknesses of fine woolen blankets quilted between covers of white canvas. The floor of the canopy consists of mats 2 inches (5 cm) thick, made of cheap woven hair sewed between covers of heavy canvas. These lie on the checkered-steel floor of the dome, and fit closely against the telescope mounting.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 36; *Astrophysical Journal*, **29**, 198, 1909.

The flat top of the canopy is stretched over a strong rectangular frame of wood; the side and end walls hang vertically like heavy flat curtains, with air-tight connection down the four vertical edges. Near the top of the south wall is an octagonal opening six feet across through which the upper end of the skeleton tube of the telescope projects when the canopy is in place over the instruments; this opening is then closed air-tight by a folding wooden cover lined with wool-felt. Below this opening is a central vertical slit which enables the canopy to be opened wide at this end and drawn off from the telescope toward the north.

The canopy is suspended from above by two small wire ropes which are connected to the top wooden frame of the canopy and to the main girders of the dome. A third wire rope passing over pulleys and a windlass enables the operator quickly to draw the canopy up against the wall of the dome to a position opposite the shutter opening, where it is entirely out of the way of the telescope in all working positions, and where it amply clears all apparatus on the operating floor.

At the end of the night's observing—usually at dawn—the canopy is lowered over the telescope and quickly made air-tight. With this protection the daily variation of temperature inside the canopy in clear weather in August and September was only 3°8 F., and the apparent change of focal length during the night was frequently as small as 0.005 inch (0.127 mm), occasionally, however, being as large as 0.009 inch (0.229 mm) when the drop in the outer temperature during the night was unusually large.

The large mirror itself is still further protected by the short, massive cast-iron tube which carries it, and within which it is again inclosed air-tight during the day by the covers which are put on to protect the silver surfaces.

Two improvements should be made in the use of the canopy.

First, a very small refrigerating apparatus, controlled by a thermostat, should be installed to supply cooled air to the canopy during the day in order to prevent the small residual rise of temperature of 3°8 F.; for the effect of even this small change on the figure of the edge zones of the mirror is perceptible when the usual optical tests are used on a star. Second, the canopy should be extended to include the upper end of the skeleton tube and the small mirror which is carried there. I hope to find a simple and convenient method of doing this when the refrigerating apparatus is installed.

In designing the 100-inch Hooker telescope we have planned for a room with insulating walls to inclose the entire telescope during the day, in which the temperature can be under perfect control.

Before the canopy was in use, the large apparent change of focal length during the night called my attention sharply to the necessity of refocusing frequently during long exposures. Several methods were thoroughly tried, including the one used by Professor Keeler, in which the form of the diffraction pattern of the image of the guide-star serves as a criterion of the best focus. This method has the advantage that the guide-star is constantly under observation during the exposure, but in my experience it is not possible by this method to be sure of apparent focal changes smaller than 0.005 inch (0.127 mm) even when the atmospheric definition is good; while with moderate definition the form of the diffraction pattern is no longer sharply defined, and the method entirely fails.

The method of focusing and refocusing finally adopted is by means of the "knife-edge" used in testing optical surfaces. An extremely thin sharp edge, like that of a hollow-ground razor, is moved across the cone of light from a star, near the plane of the focus; the eye, without an eyepiece, is placed just outside the focus, so that the cone of light is received through the pupil and the entire surface of the mirror is seen illuminated. If the plane of the knife-edge is inside the plane of the focus, the illuminated mirror will be seen to darken first on the *same* side as that from which the knife-edge advances across the cone; if the knife-edge is outside the plane of focus, the mirror will be seen to darken first on the side *opposite* that from which the knife-edge advances; when the knife-edge is exactly in the plane of the focus, the mirror will be seen to darken uniformly all over.

With the 60-inch mirror a tenth-magnitude star is sufficiently bright for the purpose.

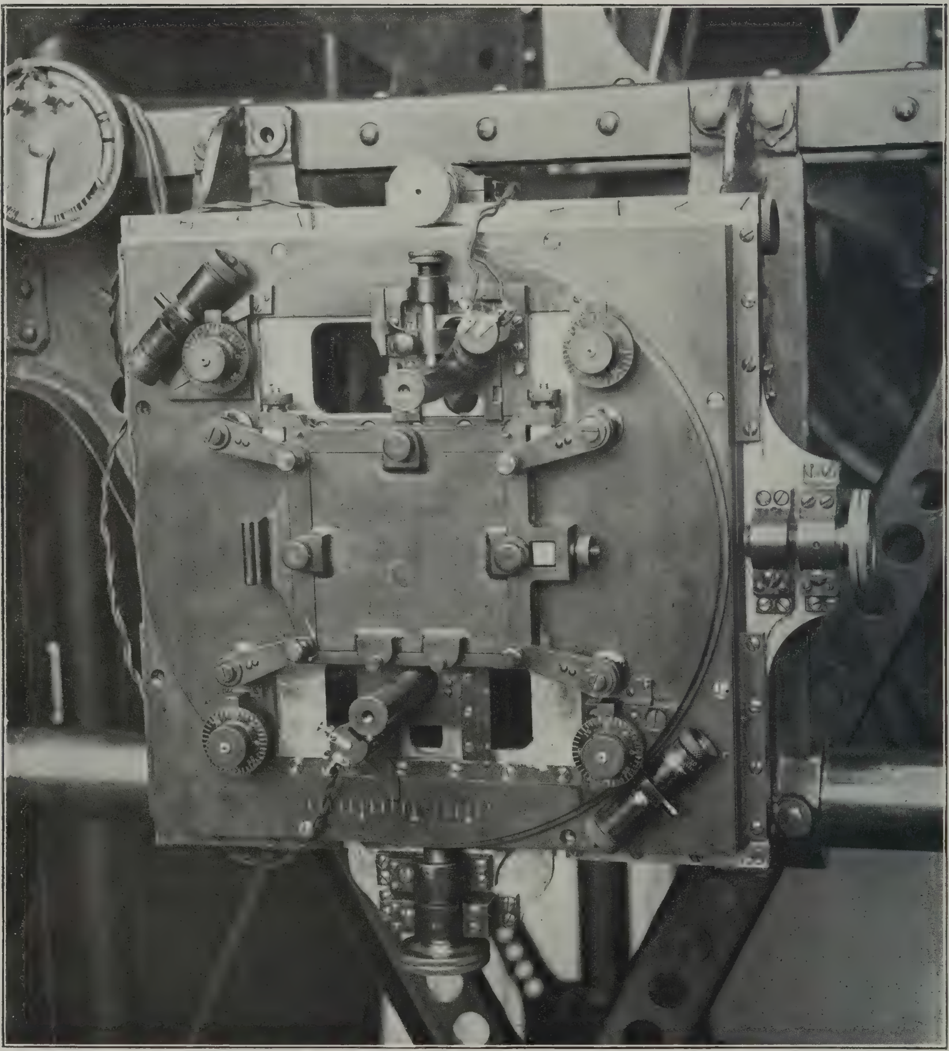
With a telescope of great angular aperture such as the 60-inch reflector, in which the ratio is as 1 to 5, this method permits a degree of accuracy of focusing which is surprising. When the atmospheric definition is at any point above 5 on a scale of 10, it is possible to locate the focal plane to within 0.001 inch (0.025 mm). With the large plate-carrier taking 6½×8½ inch (16.5×21.6 cm) plates, which was used during the summer and fall of 1909, we could not be sure that the plane of the film of the photographic plate agreed with the plane of the knife-edge within 0.001 inch. A new plate-carrier was designed in which the two planes just mentioned certainly agree always within 0.0003 inch (0.0076 mm), and which embodies all of the improvements which have been suggested by the past year's experience; this apparatus is now in use.

Before describing the new plate-carrier, I wish to call attention to the following table which shows, for slight errors in focus, the theoretical loss in magnitudes per unit area in the cross-section of the luminous cone where it intersects the sensitive film. As shown by the table this loss is so great that the surface-intensity must certainly lie below the threshhold value for large numbers of faint stars which under conditions of perfect focus would still be impressed on the photographic plate. The results are based upon the following constants of the instrument: aperture, 60 inches (152.4 cm); ratio of aperture to focal length, $\frac{1}{5}$; scale, 1 mm in focal plane=27″.2; theoretical diameter of diffraction disk of star (photographic), 0″.074.

Error in Focus	Increase in Diameter of Cone	Total Diameter of Cone	Δm=Loss in Magnitudes
inch mm	mm		
0.001=0.025	0.005=0″.136	0″.210	2.27
2=0.050	0.010=0.272	0.346	3.35
3=0.075	0.015=0.408	0.482	4.07
4=0.100	0.020=0.544	0.618	4.60
0.005=0.125	0.025=0.680	0.754	5.04

While atmospheric conditions seldom or never permit the theoretical condition corresponding to perfect focus to be realized, still the table shows the enormous importance of the utmost attainable accuracy of focusing when we are dealing with large angular apertures.

PLATE III



New Photographic Plate-Carrier of the 60-Inch Reflector

The new plate-carrier is illustrated in Plate III. It takes plates $3\frac{1}{2}$ inches (89 mm) square. This small size is chosen because it allows the guiding eyepiece to be near the center of the field. It includes a clear field about 36 minutes of arc square, and fully covers all of the central region which is free from objectionable distortion.

A second guiding eyepiece is also used, the two being on opposite sides of the center. Each gives a magnification of about 750 diameters. This allows any slight rotation of the field to be detected immediately; such rotation is corrected by rotating the bronze plate which carries the guiding eyepieces and plate-holder; two fine screws with graduated heads are provided for effecting this rotation.

The small metal plate-holder, which is shown in place on the apparatus, is so designed that it can be quickly removed and replaced as frequently as desired during long exposures, thus allowing access to the focal plane for the purpose of refocusing by means of the knife-edge. The position of the plate-holder is defined by small hardened steel surfaces so that when replaced it returns accurately to its original position with reference to the guiding eyepieces. With the small apparent change of focus which takes place since the canopy has been in use, it is found that refocusing every 25 or 30 minutes in the early part of the night, and every 40 or 45 minutes after 11:00 P.M., is usually sufficient.

With the new plate-carrier, all of the uncertainties which usually occur in making long exposures with very large telescopes are eliminated. A plate can be exposed night after night, if desired, with the assurance that no error in focus greater than one or two thousandths of an inch can occur, and that no rotation of field can take place without immediately being detected and corrected. Both of these conditions are absolutely necessary for the finest results with an instrument so powerful and sensitive as the 60-inch. With these conditions no injury or elongation of the star-images or nebular details can occur and the full effect of the prolonged exposure is secured. All of the negatives which have been secured with the new plate-carrier show perfectly round star-images. On the best of these negatives, with exposures of eleven hours, the smallest star-images are 1″03 in diameter.

As a further aid to photographic definition and resolution I have abandoned the use of rapid plates on account of their coarse grain; and am at present using Seed "23" plates almost exclusively. These are about half as rapid as Seed "27" and Cramer "Crown" and about one-fourth as rapid as Lumière "Sigma" plates. In the brands of plates named the size of grain is approximately proportional to the speed, and the photographic resolution is roughly in inverse ratio to the size of grain.

With the instrumental refinements which have been described, and with the fine-grained plates, photographs of such objects as the globular star-clusters and the spiral nebulae are revelations. The globular clusters *Messier 3*, *Messier 13*, and *Messier 15* are shown to consist of scores of thousands of stars, instead of thousands, and the angular diameters of these clusters are shown to be at least three times as great as they appear visually in the largest refractors.

Still more remarkable is the structure shown in the spiral nebulae. With fine atmospheric conditions, photographs of spirals of the class of *Messier 101*, *Messier 81*, and *Messier 51* show as much fine detail as was shown in my photographs of the Great Nebula in *Andromeda* taken with the two-foot reflector of the Yerkes Observatory.

Twelve of the larger spirals have been photographed with long exposures with the new plate-carrier. All of these contain great numbers of soft star-like condensations which I shall call *nebulous stars*. They are possibly stars in process of formation. In general they lie in streams which follow the curvature of the convolutions. Together with the smooth nebulous material in which they are apparently floating, and out of which they are apparently forming, they constitute the convolutions. The smooth nebulous material I shall call the *smooth nebulosity*, although it is not thereby meant that this is structureless. Together with the spiral dark rifts which divide it, it exhibits the most beautiful and complicated structure, especially near the center of the nebulae. The smooth nebulosity is in general much brighter near the center, becoming gradually fainter toward the extremities of the branches. In the case of the nebulous stars, however, this tendency is decidedly less marked; these, in general, are brighter and more numerous in the intermediate region between the center and the extremities of the branches. The curved streams of nebulous stars are frequently visible in the extremities of the branches far beyond the point at which the smooth nebulosity becomes too faint to be visible.

Messier 33 Trianguli contains over 2400 nebulous stars; they are present in all parts of the convolutions, from the central nucleus to the extremities; they frequently occur in groups or bunches which remind one strongly of bunches of frogs' eggs. At least twenty of these groups are present in this nebula, with from ten to sixty nebulous stars in each group.

Messier 101 Ursae Majoris contains over 1000 nebulous stars; here also they are present in all parts of the convolutions, and here also they occur in groups. No less than fifteen conspicuous groups are present, with from six to thirty nebulous stars in each. Several groups frequently occur in one branch of the spiral.

Messier 81 Ursae Majoris contains over 400 nebulous stars. This nebula bears a remarkable resemblance to the Great Nebula in *Andromeda*. The bright central region contains a very complicated system of dark rifts; as in the Great Nebula in *Andromeda* this region is entirely devoid of nebulous stars. Outside of this central region the streams of nebulous stars follow perfectly the well-defined and beautifully curved branches of the spiral; in this nebula the nebulous stars do not occur conspicuously in groups.

Messier 51 Canum Venaticorum contains over 250 nebulous stars, which in this case are present in large numbers near the center as well as in the outer parts. In this nebula the space between the two main branches of the spiral is shown to be filled with faint nebulosity of exquisite lacelike structure; the nebulous stars lie almost exclusively in the two bright main branches.

Messier 63 Canum Venaticorum contains over 200 nebulous stars; these are present in all parts of the branches, from the center to the extremities.

Messier 64 Comae Berenices is strikingly different from the other spirals photographed. The fifty or more nebulous stars in it are present only in the region immediately around the center, where they lie in spiral lines or streams. The intermediate and outer parts of the branches, in which the spiral form is very marked, are perfectly smooth and free from nebulous stars (so far as the 60-inch photographs can show). This is indicated on two good negatives, one with four hours', the other with seven hours', exposure. A large and very dark rift lies north of and very near the center; several streams of nebulous stars are conspicuous in this rift.

The foregoing description of several of the larger spirals is only preliminary and in the nature of an announcement. So much complicated structure, and so many new and suggestive features are shown, that a detailed study and description of each will be necessary later.

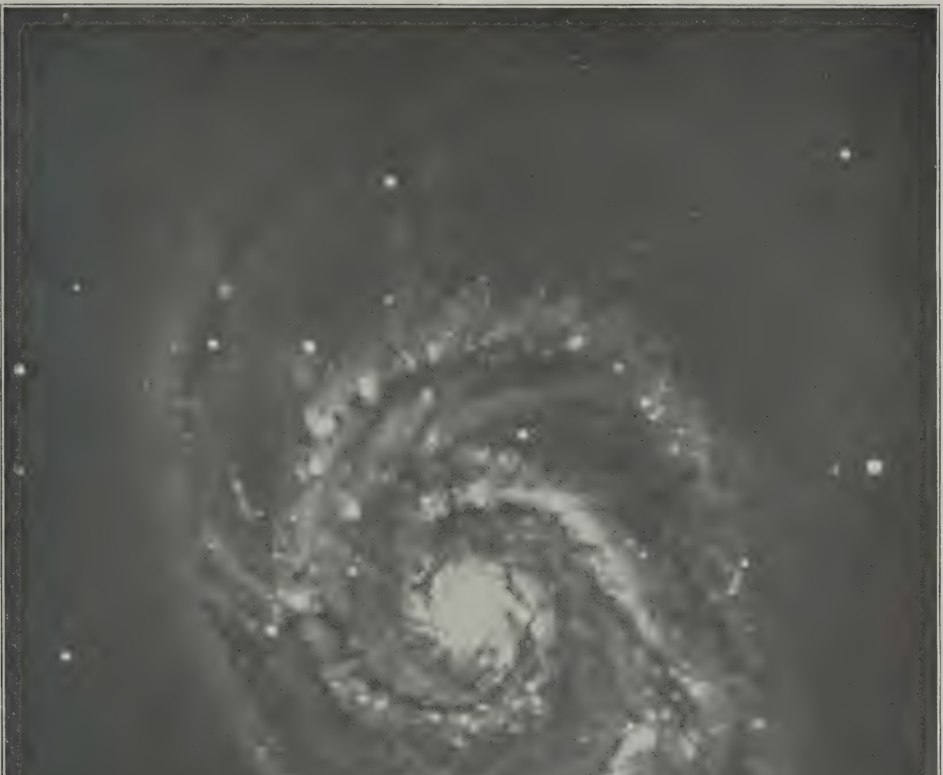
The illustration of *Messier 51 Canum Venaticorum* (Plate IV) is from a negative taken with the new plate-carrier, February 7 and 8, 1910; exposure $3^h\ 55^m$; Seed "23" plate; scale of original negative, 1 mm = $27''.2$; scale of half-tone plate, 1 mm = $4''.2$; enlargement from original negative, 6.4 diameters. This photograph was taken with rather good winter conditions; at no time during the exposure was the seeing better than 4.5 on a scale of 10.

In February 1910 two negatives of *Messier 81* were secured with the new plate-carrier. The illustration (Plate V) is from the one taken February 5; exposure $4^h\ 15^m$; Seed "23" plate; scale of half-tone plate, 1 mm = $6''.6$; enlargement from original negative, 4.1 diameters. In this case the seeing did not go above 5 (scale of 10).

The illustration of the beautiful planetary nebula *Messier 97*, the "Owl" nebula of Lord Rosse (Plate VI), is from a negative taken February 9, 1910; exposure four hours; Seed "23" plate; scale of half-tone plate, 1 mm = $3''.5$; enlargement from original negative, 7.7 diameters; seeing varied from 2.5 to 4 (scale of 10), hence the large star-images. A large amount of detail and delicate shading are shown in the nebula, which has the appearance, in general, of a slightly inclined elliptical disk superposed upon a fainter round one. An interesting description of this nebula, with a drawing, is given by Professor Barnard in *Monthly Notices*, **67**, 543, 1907. In my photograph the two dark holes are not round, but are very irregular in form and shading; the faint outer disk is irregularly round; four faint stars are shown in the nebula in addition to the central stellar nucleus. This negative is most interesting in another respect: approximately 100 very small nebulae are shown in the field 36 minutes of arc square surrounding *Messier 97*. For the field in question the nebulae are as numerous as the stars.

PLATE IV

South

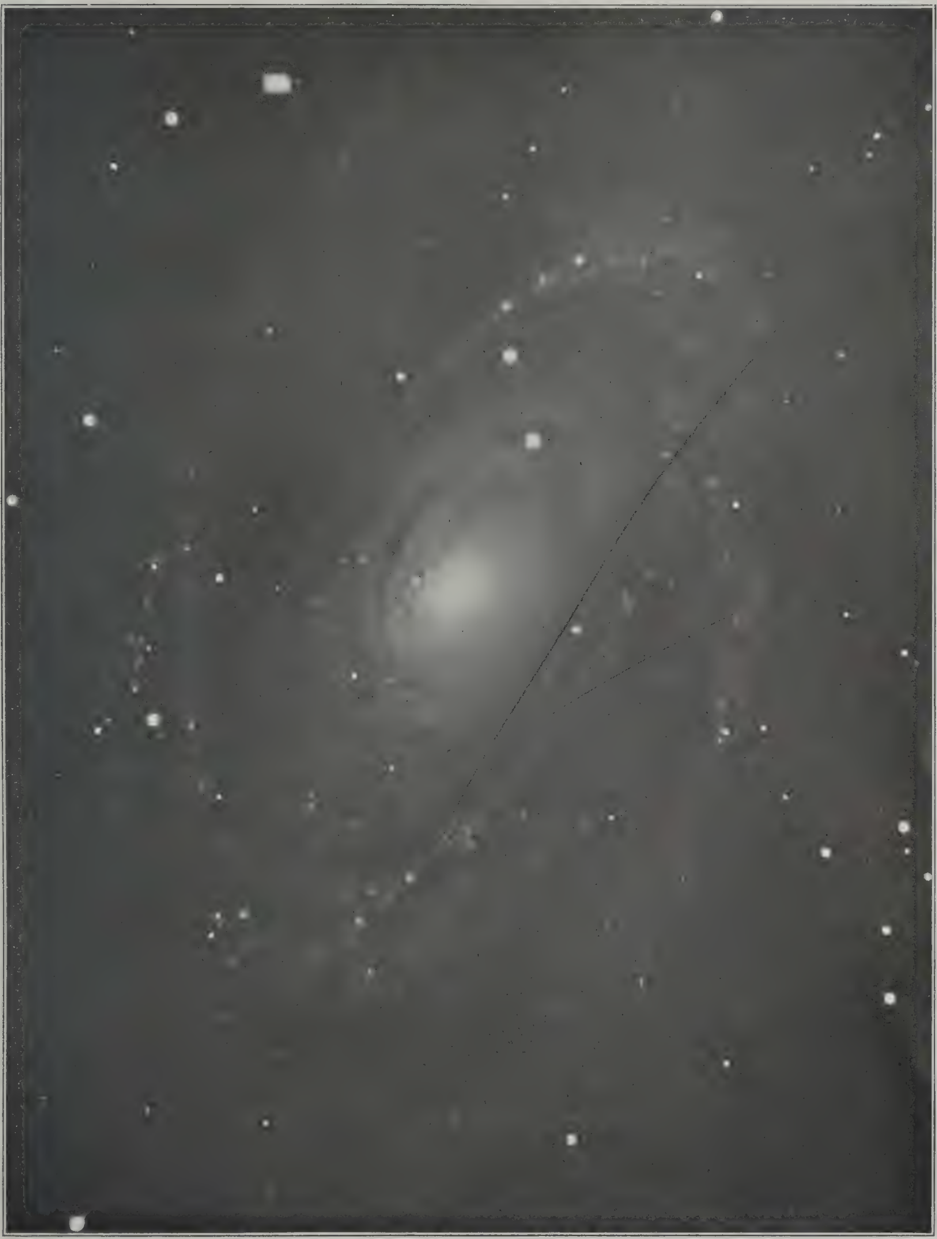


Spiral Nebula *Messier 51 Canum Venaticorum*

Photographed with 60-Inch Reflector on Seed 23 plate, February 7 and 8, 1910
Exposure $3^h\ 55^m$. Enlargement from negative 6.4 diameters. Scale: 1 mm = 4″.2

PLATE V

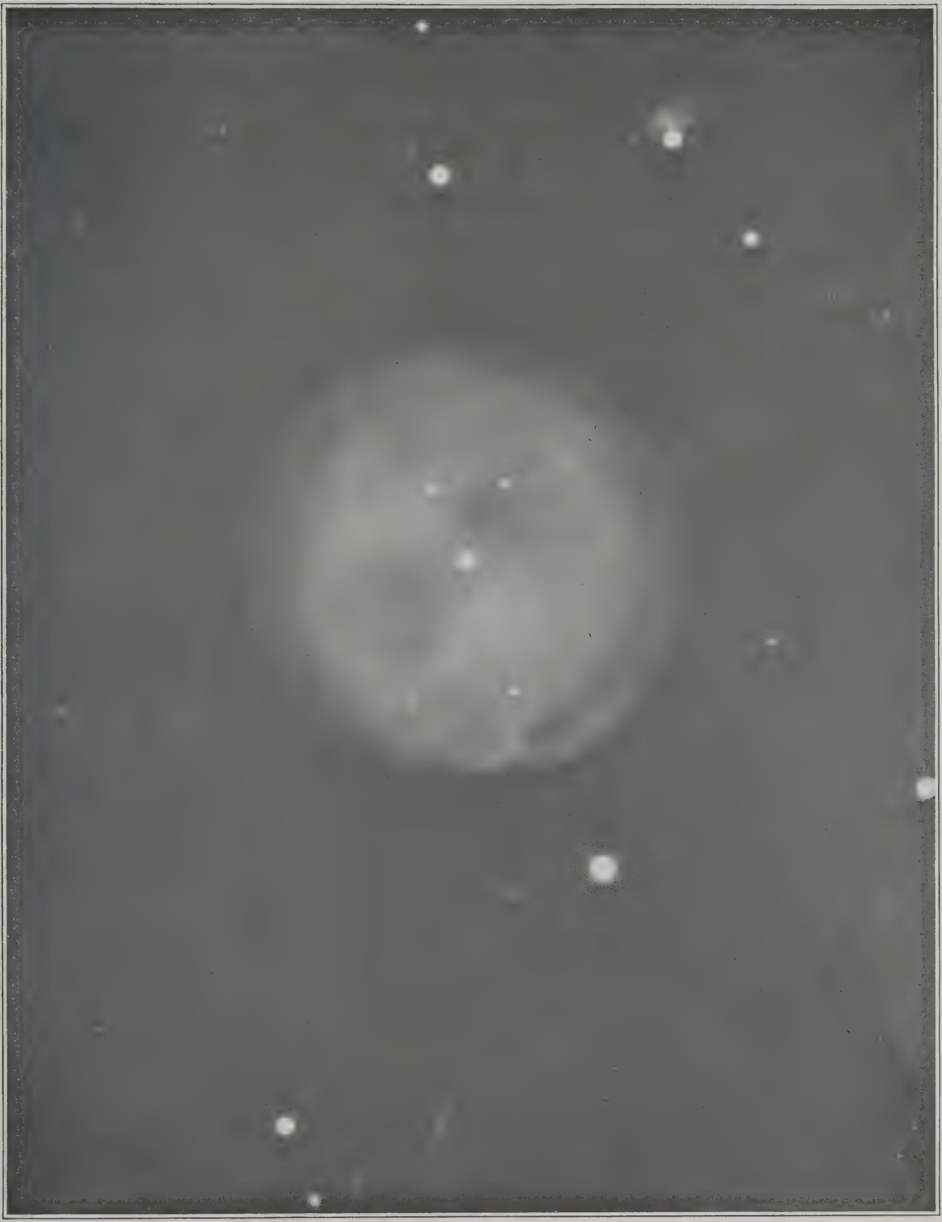
South



Spiral Nebula *Messier 81 Ursae Majoris*

Photographed with 60-Inch Reflector, on Seed 23 plate, February 5, 1910
Exposure $4^h\ 15^m$. Enlargement from negative 4.1 diameters. Scale: 1 mm = 6″.6

PLATE VI

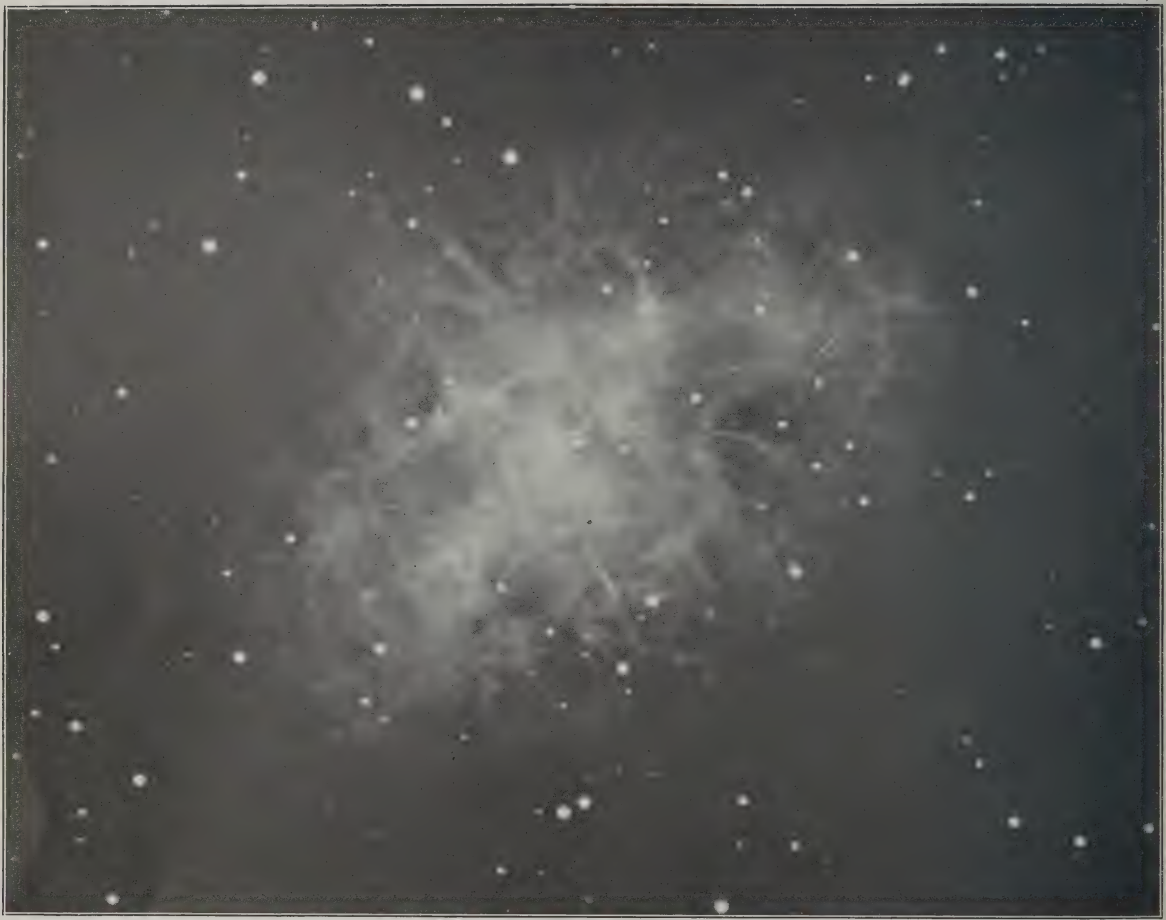


THE OWL NEBULA, *Messier 97 Ursae Majoris*

Photographed with 60-Inch Reflector, on Seed 23 plate, February 9, 1910
Exposure 4 hours. Enlargement from negative 7.7 diameters. Scale: 1 mm = 3.″5

PLATE VII

South

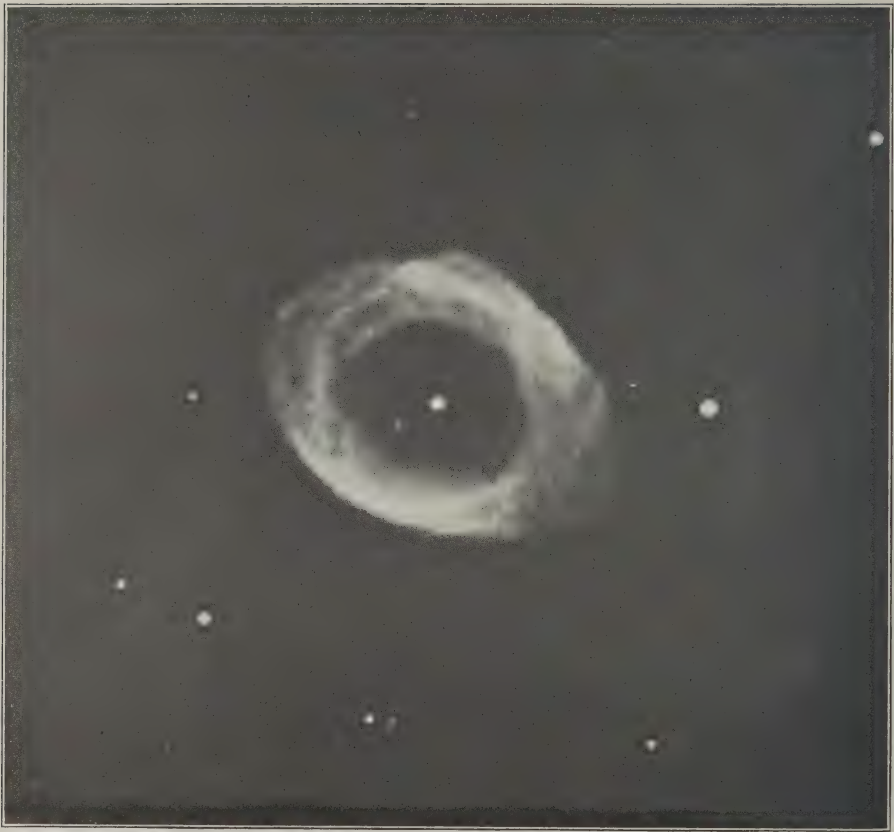


Crab Nebula in *Taurus*

Photographed with 60-Inch Reflector, on Seed 23 plate, October 13, 1909
Exposure 3 hours. Enlargement from negative 8.8 diameters. Scale: 1 mm = 3″1

PLATE VIII

South



The Ring Nebula, *Messier 57,* in *Lyra*

Photographed with 60-Inch Reflector, on Seed Process plate, September 17, 1909
Exposure 30 minutes. Enlargement from negative, 16.5 diameters. Scale: 1 mm = 1″.6

The illustration of the "Crab" nebula, *Messier 1* (Plate VII), is from a negative taken October 13, 1909; exposure three hours; Seed "23" plate; scale of half-tone plate, 1 mm = 3.″1; enlargement from original negative, 8.8 diameters. This negative was taken with the old plate-carrier, with which a slight rotation of the field, due chiefly to the change in atmospheric refraction, could not be corrected; hence the slight elongation of the star-images. Seeing varied between 6 and 7 (scale of 10), to which is due the exquisite definition of the countless antenna-like filaments surrounding the nebula; in the negative similar filaments are shown superposed over the bright body of the nebula. No stars are shown which appear to be physically related to the nebula.

The illustration of the "Ring" nebula in *Lyra, Messier 57* (Plate VII), is from a negative taken September 17, 1909; exposure, 30 minutes; Seed "Process" plate; scale of half-tone plate, 1 mm = 1.″6; enlargement from original negative, 16.5 diameters. This negative also was made with the old plate-carrier, and with seeing 5.5 (scale of 10). The "Process" plate used gives good contrast, and its fineness of grain allows very small details to be shown; hence the great enlargement possible. A large amount of structure is shown in the ring, which is seen to consist of three irregular rings intertwined; this was first noted by Professor Keeler. Even with the very slow "Process" plates 30 minutes' exposure is sufficient to show faintly the two light bands, parallel to the major axis of the ellipse, which cross the central dark space. With 60 minutes' exposure on these plates considerable structure is shown in these bands and also in the five principal protuberances outside of the ring. It may aid in appreciating the performance of the 60-inch with fine-grained plates to note that the extreme length of this nebula on the original negative is three millimeters.

MOUNT WILSON SOLAR OBSERVATORY
May 1910

THE GENERAL CIRCULATION OF THE MEAN AND HIGH-LEVEL CALCIUM VAPOR IN THE SOLAR ATMOSPHERE[1]

By CHARLES E. ST. JOHN

I. INTRODUCTION

Important as is the registering of the solar surface by monochromatic photography through the use of the spectroheliograph, of equal importance are data for the interpretation of spectroheliograms. Such photographs with the a line of hydrogen furnish in some measure the means for their own interpretation, as in the vortical movements shown and described by Hale,[2] and in the motions of the dark hydrogen flocculi such as the writer was fortunate enough to obtain on June 2 and 3, 1908.[3] But since the development of the spectroheliograph by Hale in 1889, and its first successful application, on January 12, 1892, to the photography of the solar surface and the delineation of the forms of those regions where K is reversed, a sufficient amount of definitive work bearing upon the general movement of the calcium vapor has not been done, though the total number of spectroheliograms is now many thousands and daily increasing. This lack of definitive data is especially to be regretted in the case of calcium, because of the remarkable behavior of the H and K lines over the general surface of the sun, over faculae, and over spots, and because these lines are the most favorable for use with the spectroheliograph, and the only ones that can be employed with spectroheliographs of low dispersion.

The history of the H and K lines of calcium, as far as the discovery of their characteristic appearance on the disk of the sun is concerned, is as follows:

In 1872 Young wrote concerning the reversals of H and K visually observed by him: "They were also found to be regularly reversed on the body of the sun itself, in the penumbra and the immediate neighborhood of every important spot." In 1883 Lockyer made the first attempts to photograph the spectra of sun-spots and his plates showed that "in all H and K were seen reversed over the spots, just as Young saw them at Sherman, while the blue calcium line was not reversed."[2] On Rowland's plates used for *The Preliminary Table of Solar Wave-Lengths*, taken as far back as 1886 and 1887, H and K show both single and double reversals,[3] but no notice of this was published at the time. The following observations of the single and double reversals were made independently of each other and of the preceding photographic work. In December 1891 Hale obtained some photographs of the spectrum of a spot in which the bright lines of H and K had such an extent in the surrounding regions that it was suspected that similar reversals might be found on the disk remote from spots. A series of plates was taken which confirmed this suspicion, and in a note from the Kenwood Observatory, under date of January 18, 1892, Hale says "that the H and K lines are reversed not only in the vicinity of spots but in regions irregularly distributed over the entire disk of the sun."[4] M. Deslandres reported the same results from his photographs of the spectra of spots and faculae: "Les raies H et K du calcium apparaissent souvent brillantes," and added, in a footnote: "Ces raies brillantes offrent souvent aussi un reversement au centre."[5]

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 48.

[2] *Ibid.*, No. 26; *Astrophysical Journal*, **28**, 1908.

[3] *Ibid.*, 9.

Under date of April 15, 1892, Hale reported from the Kenwood Observatory that on plates taken in December 1891 "most if not all, of these (H and K) reversals were double, i.e., a dark line ran through the center of the bright line."[6] The usual appearance of the H and K lines over the general disk of the sun is that of a broad shading—H_1 or K_1—superposed upon this a bright line—H_2 or K_2—varying greatly in width and intensity and consisting of two components separated by the narrow absorption line—H_3 or K_3—following the notation for the division of these lines introduced by Hale.[7]

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[1] *American Journal of Science*, November, 1872.

[2] *Proceedings, Royal Society*, **36**, 444, 1884.

[3] Personal letter from Professor Ames.

[4] *Astronomy and Astrophysics*, **11**, 159, 1892.

[5] *Comptes Rendus*, **114**, 277, 1892.

[6] *Astronomy and Astrophysics*, **11**, 414, 1892.

[7] *Publications of the Yerkes Observatory*, Part I, Vol. III, p. 15.

M. Deslandres called attention to the following peculiarities of the H and K lines:

> Le plus souvent, les deux composantes de la raie brillante sont dissymétriques, la composante du côté du rouge étant plus étroite que l'autre, si bien que la raie noire apparaît déplacée vers la rouge par rapport à la raie brillante.[1]

After speaking of the equality, in general, of the bright components over faculae and the occasional inflections of the lines in the neighborhood of spots, he says further:

> L'ensemble des faits précédents peut s'expliquer par un mouvement général de circulation verticale et horizontale des couches hautes et basses de la chromosphère, analogue à celui que présente notre atmosphère. Les couches basses s'élèveraient et seraient attirées vers l'équateur, comme les vents alizés, d'où un rapprochement vers la Terre; les couches élevées auraient un mouvement inverse.[2]

Returning to the subject later, M. Deslandres writes with more definitiveness:

> J'ai déjà indiqué, en 1894, ces particularités qui annoncent un mouvement d'ascension des vapeurs productrices de la raie K_2 et un mouvement de descente des vapeurs K_3. Lorsqu'on se rapproche du bord, la dissymétrie diminue et, près du bord, devient imperceptible, ce qui s'explique parce que les différences des vitesses radiales des vapeurs par rapport à la Terre diminuent elles-mêmes jusqu'à devenir nulles. En même temps, les deux composantes de K_2 s'écartent progressivement, laissant entre elles une raie K_3 qui s'élargit de la même quantité. Ce dernier fait, sur lequel je dois insister, est très net avec une grande image solaire, plus régulier et progressif que ne l'indiquent les dessins déjà publiés. Il tient évidemment à l'épaisseur constamment croissante sous laquelle se présente la chromosphère, pour le rayon Soleil-Terre, lorsque l'on va du centre au bord.[3]

Without actual measurements referred to terrestrial standards the dissymmetry of the bright components of the calcium lines can indicate with certainty only relative motions of the vapors producing K_2 and K_3, respectively, and definite conclusions cannot be drawn as to the actual motions of the vapors producing the lines.

The unsymmetrical character of the central absorption line with respect to the components of the emission line has also been reported by Jewell, who found it more easily observed in the case of K than H.[1] A number of plates were measured by him and as a result it was found,

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[1] *Comptes Rendus*, **119**, 458, 1894.

[2] *Ibid.*, 459.

[3] *Ibid.* **141**, 381, 1905.

> that the central absorption line varies in position to the extent of 0.025 of an Ångström unit between the plates upon which the central line is symmetrically placed and the plates upon which it is most unsymmetrical. . . . . An attempt was made to measure the position of the emission line in these cases. The measurements were too difficult to make to give any certain results, but there were some indications that the emission line might be slightly displaced toward the violet in the cases of dissymmetry. . . . . The obvious explanation is that in the cases of dissymmetry we have examples of motion in the line of sight, the displacement of the central absorption line being due to falling matter, the extreme difference in velocity, shown by different plates, being about seventy-five miles per minute.[2]

In a later paper he says:

> The narrow central component of the shaded lines . . . . shows a *descending* motion, over the solar surface, of the absorbing matter producing it of . . . . about a mile a second in the case of the H and K lines. . . . . These narrow components of the shaded lines are probably produced by meteoric matter falling into the solar atmosphere. The bright emission components may possibly be caused by the down-rush of this meteoric matter through the denser portions of the chromosphere such that where it meets the up-rush of the matter already referred to, the impact of the collisions or the friction caused produces an intense emission. . . . .[3]

In the paper previously referred to Jewell suggests: "In the case of H and K the central absorption line may possibly be produced by the corona." The plates examined by Jewell were not made with such measurements in view as must be carried out on these difficult lines, but were used incidentally for this purpose. The importance of having plates for this particular purpose was fully recognized and taken into account in the work reported later in this paper.

Some work was done on the H and K lines by Adams in 1905, in which, upon six plates taken on the disk, he measured the position of the absorption lines, and in some cases the position of the bright emission lines. His results for the absorption lines are in apparent disagreement with those of Jewell. Adams says:

[1] *Astrophysical Journal*, **3**, 103, 1896.

[2] *Op. cit.*

[3] *Astrophysical Journal*, **11**, 237, 1900.

> The most striking feature of these results is the general tendency toward a displacement to the violet. . . . . Taking the values already found for the wave-lengths of H and K in the arc spectrum, the average displacement amounts to 0.006 tenth-meters, which would mean a velocity of approach on the part of the calcium vapor producing the absorption lines of 0.41 kilometers a second. . . . . No certain evidence can be found in these observations of any such variation in the motion of the calcium vapor between the center and the limb, as might be expected from a general drift upward in a radial direction. A large amount of material will, however, be necessary before any conclusion can be drawn in regard to this matter. . . . . The results given by the emission lines, H_2 and K_2, also show a displacement toward the violet, although the measurement of these lines is much more difficult than that of the absorption lines. . . . . Upon most of the plates which were measured the violet and the red components are very nearly equal; in several the violet is slightly stronger, and in one distinctly weaker than the red component.[1]

The present state of the case is as follows: M. Deslandres found at the center of the disk dissymmetry with the violet component of K_2 the stronger, the inequality of the components disappearing at the limb. Without measurements he assumed a descending motion of the absorbing vapor and an ascending motion of the emission layer. Jewell found a varying dissymmetry of the components and a measured velocity of descent for the vapor producing the absorption line of about a mile a second. The measurements of the emission line were too difficult to give anything but a slight indication of a displacement toward the violet. Adams found dissymmetry to be the exception, a measured velocity of ascent for the absorbing vapor of 0.41 kilometers a second, and a slight displacement of the emission line toward the violet. The apparent contradiction in the measurements of Jewell and Adams probably depends upon the different standards used for the solar lines and the differences in wave-length assigned to the H and K lines in the arc. As to the conflicting evidence relative to dissymmetry, the explanation suggested by Adams is undoubtedly the correct one: "It seems probable, that while an effect like that found by M. Deslandres is perhaps to be expected, the local conditions of the calcium vapor at different points on the sun's surface vary so much as to mask it completely in many cases."[2]

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 6; *Astrophysical Journal*, **23**, 45, 1906.

[2] *Op. cit.*

In reviewing the work already done, three points appear to which particular care should be given in a definitive examination of the question of the general movement of the calcium vapor in the upper and middle levels of the sun's atmosphere: first, an accurate determination should be made of the wave-lengths of K_2 and K_3 at *definite positions* on the sun's surface, referring them to the same standards that were used for determining the wave-lengths of the correponding line in the arc; second, the plates should be taken with reference to the particular ends in view, and exposure times and developments should be such as to bring out as sharply as possible the bright emission line, particularly on plates taken at the center of the sun's disk; third, there should be a large number of observations extending over a sufficiently long period of time to eliminate as far as possible the masking of the general effect by temporary conditions, so that the final mean will reveal the prevailing state of the vapor.

As a preliminary to this investigation the determination of the wave-lengths of the H and K lines of calcium in the arc, the spark, and the electric furnace was made,[1] based on the secondary standards of the International Union for Co-operation in Solar Research, as determined by Fabry and Buisson.[2] It was early decided to confine the investigation to the K line of calcium in the sun because of its greater intensity and the absence of troublesome neighboring lines. The Fabry and Buisson standards being about 40 Ångströms apart in this region, only the standard λ 3935 was conveniently near to K. The wave-length of another iron line, λ 3930, was therefore determined in the arc with the same precision as the H and K lines, in order that there might be an adjacent standard on each side of the K line to which its wave-length could be referred. The results for the wave-lengths of the calcium lines in the arc were λ 3933.667 for K and λ 3968.476 for H, and for the supplementary iron line λ 3930.301.[1]

The points covered in the present paper are: the determination of the wave-lengths of the solar standards employed in terms of arc standards; the measurement of the wave-lengths of the absorption line K_3 and the emission line K_2 at different points on the solar disk as a basis for the study of general radial motion of the calcium vapor; an investigation of the frequently assumed horizontal currents along the solar surface; a study of the absolute and relative widths of the H and K lines at the limb and center; the vertical distribution of the calcium vapor in the chromosphere; the wave-length of the absorption line H_3 at the center and limb; the relative wave-lengths of H and K in the sun; and a general discussion of the data derived in the investigation.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 44; *Astrophysical Journal*, **31**, 143, 1910.

[2] *Astrophysical Journal*, **27**, 169, 1908.

2. DETERMINATION OF THE SOLAR STANDARDS

The spectrographs employed are of the Littrow form and are described as Nos. 2 and 3 in *Contributions from the Mount Wilson Solar Observatory*, No. 44. In the 18-foot spectrograph (No. 2) arranged for the Snow telescope, a 4-inch (10 cm) plane Michelson grating, having 12,500 lines to the inch (492 to the mm), and a 4-inch plane Rowland grating, having 14,438 lines to the inch (568 to the mm), were used, the latter being temporarily free from its regular service in the tower spectrograph, with which also some of the comparison plates were taken. Light from the center of the sun's disk was used for direct comparison with the iron arc, an unmagnified image of which was projected on the slit. The narrow iron spectrum fell on the plate between two narrow strips of the solar spectrum. In the case of the 18-foot spectrograph the occulting bar was free from the instrument. In that of the 30-foot spectrograph the light, easily moving occulting bar is attached to the massive head of the instrument. The plates were measured red right and red left, with at least four settings on the arc line and eight or more on the solar line for each direction. Corrections have been applied for the rotation of the earth on its axis, and the eccentricity of its orbit around the sun, using Campbell's tables.[1] The solar lines λ 3930 and λ 3935 lie in the broad shade of K_1 and therefore present some difficulties for accurate measurement. The results for the solar lines, corrected as noted above and referred to the corresponding arc lines, are given in Table I.

The increase in the wave-lengths of the solar lines over the wave-lengths in the arc is 0.005 Å for both lines.

In measuring the wave-lengths of K near the sun's limb, account must be taken of "pressure-shifts" upon the solar standards. This effect falls off rapidly with increasing distance from the limb, but in this investigation points on the disk were taken at only 1 mm from the limb. Mr. Adams kindly placed at my service his center and limb plates of this region, upon which the shifts of the solar standards were found to be as follows:

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[1] Scheiner, *Astronomical Spectroscopy* (translated by Frost), Boston, 1898, 338–344.

TABLE I

SOLAR STANDARDS

Plate	λ 3930	λ 3935	Spectrograph	Grating
396	3930.307	3935.826	18-foot	12,500
398	.306	.821	18-foot	12,500
403	.307	.826	18-foot	14,438
404 (1)	.304	.825	18-foot	14,438
(2)	.308	.825	18-foot	14,438
405 (1)	.316	.824	18-foot	14,438
(2)	.307	.824	18-foot	14,438
406	.306	.826	18-foot	14,438
B 50 (1)	.305	.822	30-foot	14,438
(2)	.306	.820	30-foot	14,438
(3)	.306	.822	30-foot	14,438
B 51 (1)	.305	.823	30-foot	14,438
(2)	.306	.821	30-foot	14,438
Means	3930.306	3935.823		

Plate	λ 3930	λ 3935
112	+0.005	+0.004
120 (1)	+0.004	+0.002
(2)	+0.004	+0.002
134	+0.003	+0.003
Means	+0.004	+0.003

The calcium vapor producing the K_3 absorption line is known to be a high-level vapor and therefore would be subject to very small pressure, possibly less than one atmosphere. The vapor producing the emission line K_2 lies at an intermediate level and may be under a pressure not much different from that of the earth's atmosphere. The results will show that these assumptions are justified. For these lines, then, the solar standards are, for the general disk, λ_A 3930.306 and λ_A 3935.823, and for the limb λ_A 3930.310 and λ_A 3935.826. Since these have been determined in terms of the arc standards used for measuring the wave-lengths of H and K in the arc, the results for K_2 and K_3 in the sun are directly comparable with the results in the arc.

3. THE WAVE-LENGTH OF THE ABSORPTION LINE K_3

The spectrograph was that used with the Snow telescope and described as spectrograph No. 2. The grating was the 8-inch Michelson, used in the third order, where the scale is approximately 1 mm=1.02 Å. The spectrograph has a first slit 45 mm long and a second slit of the same length but of variable width, so that short sections of the spectrum may be taken. The slit may be only wide enough to allow the spectrum line to pass through, or it may be increased to 12 mm. Both slits and the plate may be moved automatically or by hand. The arrangement permits one to make easily and rapidly successive exposures of the same point on the sun, for detecting changes with time, or exposures of successive points at any desired intervals. It is in a modified form the spectrograph *des vitesses* suggested by M. Deslandres and employed by him in 1892 for the photography of the spectra of spots and faculae, in which he noted for the first time the double reversal of the H and K lines.[1] From such plates taken at closely adjacent points with a long and narrow slit the forms and distribution of the calcium flocculi can be obtained in the manner M. Deslandres has shown.[2] In connection with the present investigation, the writer has had an opportunity to examine some section plates taken by Hale in 1891, when using a long slit and making the first solar survey to which this method "by sections" especially lends itself, and by which he discovered the bright reversals of the H and K lines of calcium at points on the disk of the sun. In the present case the sections used by the writer were 12 mm wide and the lines 45 mm long. They were wide enough to include the solar standards, and the lines were long enough to show the condition of the calcium vapor over a region on the sun 350,000 km in extent. A preliminary survey was made by taking points 1, 2, 3, 4, 7, 12, 17, 25, 35, and 50 mm from the limb and at the center. These were found to be unnecessarily numerous and measurements were confined to three points, namely, 1 mm and 12 mm from the east and west limbs, and the center. The change in the line-of-sight component of any radial movement of the calcium vapor takes place rapidly near the limb. With an image 172 mm in diameter, which is the mean value here used, 51 per cent of the change from limb to center

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[1] *Comptes Rendus*, **114**, 277, 1892.

[2] *Transactions of the International Union for Co-operation in Solar Research*, II, Plate XII.

TABLE II

WAVE-LENGTH OF K_3 AT 1 mm FROM THE LIMB

East			West		
Plate	Date	K_3	Plate	Date	K_3
	1909			1909	
21	Jan. 19	3933.670	42	Jan. 28	3933.663
26	" 20	.664	69	Feb. 13	.665
43	" 28	.662	70	" 13	.673
64	Feb. 8	.669	80	" 22	.670
68	" 13	.665	120	March 9	.673
71	" 13	.658	148	" 11	.675
120	March 9	.665	151	" 11	.678
145	" 10	.672	199	" 31	.674
151	" 11	.665	204	April 1	.670
199	" 31	.666	209	" 9	.673
Mean		3933.6656	Mean		3933.6714
204	April 1	.666	229	April 15	.673
209	" 9	.655	234	" 16	.664
229	" 15	.661	255	" 27	.673
234	" 16	.674	262	" 29	.671
248	" 26	.662	353	July 17	.667
255	" 27	.666	356	" 20	.672
262	" 29	.661	358	" 21	.672
361	July 22	.660	360	" 22	.662
365	" 23	.663	362	" 23	.669
366	" 24	.665	364	" 22	.667
Mean		3933.6633	Mean		3933.6690
368	July 24	.660	369	July 24	.670
371	" 26	.685	370	" 26	.659
373	" 27	.652	372	" 27	.669
376	" 28	.662	374	" 27	.662
378	" 29	.669	375	" 28	.677
379	" 30	.669	377	" 29	.659
383	" 30	.666	380	" 30	.666
387	Aug. 2	.662	383	" 30	.686
389	" 3	.664	388	Aug. 3	.678
391	" 5	.665	392	" 6	.669
Mean		3933.6654	Mean		3933.6695

occurs within 12 mm of the limb. As the total effect is small, plates taken at the center would not give very different results from those taken over the general disk unless points very near the limb were selected. Tables II, III, and IV contain the results for the three points chosen. The plates were measured red right and red left. The results are arranged in groups of ten each, with the plates in serial order, the better to bring out the regularity of the progressive change

TABLE III

Wave-Length of K_3 at 12 mm from the Limb

Plate	Date	K_3	Plate	Date	K_3
	1909			1909	
21	Jan. 19	3933.675 E	246	April 26	3933.681 W
22	" 19	.675 W	256	" 27	.678 W
26	" 20	.678 E	262	" 29	.672 E
34	" 24	.676 E	356	July 20	.676 W
38	" 28	.667 E	359	" 21	.677 W
42	" 28	.677 W	360	" 22	.667 W
43	" 28	.674 E	361	" 22	.674 E
48	" 29	.673 E	362	" 23	.676 W
50	" 29	.673 E	364	" 23	.669 W
53	" 29	.678 E	365	" 23	.672 E
Mean		3933.6746	Mean		3933.6742
59	Jan. 30	.676 E	366	July 24	.677 E
62	" 30	.678 E	368	" 24	.672 E
64	Feb. 8	.677 E	369	" 24	.671 W
68	" 13	.680 E	370	" 26	.675 W
69	" 13	.678 W	371	" 26	.676 E
70	" 13	.670 W	372	" 27	.675 W
71	" 13	.671 E	373	" 27	.671 E
77	" 22	.673 E	374	" 27	.674 W
80	" 22	.674 W	375	" 28	.671 W
187	March 30	.680 E	376	" 28	.676 E
Mean		3933.6757	Mean		3933.6738
187	March 30	.671 W	378	July 29	.674 E
210	April 9	.674 E	380	" 30	.672 W
210	" 9	.677 W	383	" 30	.675 W
214	" 12	.672 E	383	" 30	.665 E
231	" 15	.678 E	387	Aug. 2	.677 E
231	" 15	.679 W	388	" 3	.667 W
242	" 24	.680 E	389	" 3	.675 E
242	" 24	.677 W	427	Oct. 31	.680 E
244	" 25	.679 E	430	Nov. 1	.671 W
246	" 26	.677 E	434	" 1	.674 E
Mean		3933.6764	Mean		3933.6730

from the limb to the center. In the means for the groups and tables the fourth decimal has been retained as the residuals shown by the least-squares solution are confined to the fourth place except for two points. The wave-length of K_3 is given in absolute units, and referred to the two standards mentioned above. The letters E and W following the wave-lengths in Table III refer to the east and west limbs of the sun, respectively.

TABLE IV

WAVE-LENGTH OF K_3 AT CENTER OF THE DISK

Plate	Date	K_3	Plate	Date	K_3
	1909			1909	
21	Jan. 19	3933.682	366	July 24	3933.683
22	“ 19	.682	368	“ 24	.675
26	“ 20	.676	369	“ 24	.680
34	“ 24	.678	370	“ 26	.681
38	“ 28	.682	371	“ 26	.681
42	“ 28	.684	372	“ 27	.677
50	“ 29	.689	373	“ 27	.687
53	“ 29	.683	374	“ 27	.678
59	“ 30	.682	375	“ 28	.682
62	“ 30	.679	376	“ 28	.676
Mean		3933.6817	Mean		3933.6800
64	Feb. 8	.675	377	July 29	.681
68	“ 13	.677	378	“ 29	.678
70	“ 13	.679	379	“ 30	.686
123	March 9	.678	380	“ 30	.680
124	“ 9	.676	383	“ 30	.675
130	“ 9	.676	387	Aug. 2	.677
136	“ 10	.680	388	“ 3	.688
146	“ 10	.682	389	“ 3	.683
152	“ 11	.680	436	Nov. 1	.683
173	“ 12	.684	436	“ 1	.680
Mean		3933.6787	Mean		3933.6811
197	March 31	.680	444	Nov. 3	.682
197	“ 31	.682	444	“ 3	.676
206	April 1	.684	445	“ 3	.681
236	“ 16	.684	445	“ 3	.677
239	“ 24	.683	461	“ 23	.679
247	“ 26	.677	461	“ 23	.694
356	July 20	.680	462	“ 23	.681
361	“ 22	.672	462	“ 23	.684
362	“ 23	.681	471	“ 27	.678
364	“ 23	.678	471	“ 27	.676
Mean		3933.6801	Mean		3933.6808

The measurements were confined to those portions of the spectrum lines that appeared to represent the prevailing condition of the chromospheric vapor. Regions near spots and faculae were avoided, especially was this the case on the plates taken at the center and at 12 mm from the limb. On the limb plates the bright emission line K_2 was so broad and intense that it did not reveal clearly the presence of the calcium flocculi. The slits of the spectroscope being 45 mm long, it was possible on the other plates to select those portions of the line for measurement in which the bright components of K_2, bounding K_3, were of uniform width for some distance, and not of such great intensity as over the facular regions and the brilliant flocculi. The period covered by this series of observations extends from January 19 to November 27, 1909, and includes twelve rotations of the sun. The plates being taken at irregular intervals represent what must be the prevailing state of the calcium vapor producing the absorption line K_3. The results for the groups are shown in Table V.

TABLE V

MEAN RESULTS FOR THE WAVE-LENGTH OF K_3

	1 mm from the Limb	12 mm from the Limb	Center
	3933.6656 E	3933.6746	3933.6817
	.6714 W	.6757	.6787
	.6633 E	.6764	.6801
	.6690 W	.6742	.6800
	.6654 E	.6738	.6811
	.6695 W	.6730	.6808
Final means	3933.6674	3933.6746	3933.6804

Though large variations occur in the separate measurements for a given position, larger than errors of observation, the means of the groups of ten show an unmistakable tendency toward greater wave-lengths as the center of the disk is approached. The agreement among the groups is so complete that the final means must be considered as representing the prevailing condition of the vapor with a high degree of certainty. The change of the line-of-sight component is very rapid near the limb, 15 per cent of the radial motion on the sun appearing as a line-of-sight motion at 1 mm from the limb on an image 172 mm in diameter. The difference between the wave-length at the center and at 1 mm from the limb is 0.013 Å, which is 85 per cent of the change from limb to center. The total displacement between limb and center is therefore 0.015 Å, corresponding to a velocity of *descent* of 1.14 km per second for the upper level of the calcium vapor in the solar atmosphere. The falling of the high-level calcium vapor is a general phenomenon, and though its velocity of descent seems low, it is in fact far greater than hurricane velocities in the terrestrial atmosphere. The highest recorded South Sea hurricane velocity, according to Köppen,[1] is 40 m per second, so that this downrush of the calcium vapor has a mean velocity thirty times as great as the South Sea windstorms, and occasionally far greater.

The measurement of the calcium absorption line K_3 was referred to the two iron lines λ 3935 and λ 3930, which occur in the reversing layer and presumably lie at a lower level. Adams has shown that the hydrogen vapor producing the $H\alpha$ line has a considerably higher rotational velocity than the reversing layer,[2] and that the calcium vapor producing λ 4227 has a velocity intermediate between that of $H\alpha$ and the reversing layer.[3] Hale has shown a correspondingly high velocity for the hydrogen flocculi, but that the bright calcium flocculi give a somewhat lower equatorial velocity and a smaller polar retardation than the reversing layer.[4] The latter result was also obtained by Fox.[5] The calcium vapor producing the K_3 absorption line is a high-level vapor and therefore may have a relatively higher rotational velocity compared with the reversing layer. If such be the case, then the wave-length of K_3 at the east limb should be less than the mean, and that at the west limb greater than the mean, by an amount corresponding to the difference in the line-of-sight velocities of the calcium vapor and the reversing layer. In Table V there is shown a consistent difference between the results for the two limbs, those for the west limb being greater than those for the east limb. Since the probable errors of the means for the east and west limbs separately are ±0.0006 Å and ±0.0007 Å, respectively, it would appear that the difference of the means, which amounts to 0.0052 Å, is, beyond question, real. It seemed worth while to examine the values more closely from the point of view of the rotational velocity. The observations grouped themselves about two latitudes, namely, 6°6 and 38°4. From the data given in Adams' paper,[1] the radial velocities, periods, and angular velocities were calculated for these two latitudes. The differences between the wave-lengths of K_3 at the east and west limbs for latitudes 6°6 and 38°4 are 0.0042 Å and 0.0062 Å, respectively. The velocity of the calcium vapor relative to the reversing layer is sufficient at 6°6 to produce a Doppler effect of 0.0021 Å, and at 38°4 an effect of 0.0031 Å, corresponding to relative line-of-sight velocities of 0.16 km and 0.24 km, respectively. The calculated radial velocities and the resulting periods and angular velocities are given in Table VI.

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[1] Hann, *Lehrbuch der Meteorologie* (Leipzig, 1906), 281.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 24; *Astrophysical, Journal*, **27**, 213, 1908.

[3] *Contributions from the Mount Wilson Observatory*, No. 33; *Astrophysical Journal*, **29**, 110, 1909.

[4] *Contributions from the Mount Wilson Observatory*, No. 25; *Astrophysical Journal*, **27**, 219, 1908.

[5] *Science*, N.S., **25**, 606, 1907.

TABLE VI

COMPARISON OF ROTATION VELOCITIES FOR K_3, $H\alpha$, AND REVERSING LAYER

ϕ	K_3			$H\alpha$			REVERSING LAYER		
	v	ξ	Period	v	ξ	Period	v	ξ	Period
	km		days	km		days	km		days
6°6............	2.20	15°5	23.1	2.13	15°1	23.9	2.04	14°0	24.9
38°4............	1.69	15.4	23.4	1.56	14.3	25.4	1.45	13.2	27.4

From this table it is seen that the radial velocity of the calcium vapor producing the absorption line K_3 is about as much higher than that of the hydrogen producing $H\alpha$ as the radial velocity of the latter is higher than that of the reversing layer. The angular velocity of the calcium vapor is consequently greater and more nearly constant than the angular velocity of the hydrogen, and shows therefore less polar retardation. These results are in entire harmony with other observations in this paper showing the high level of the chromospheric calcium.

4. THE WAVE-LENGTH OF THE EMISSION LINE K_2 AND FURTHER CONSIDERATION OF THE WAVE-LENGTH OF THE LINE K_3

The measurements of the bright components of the K_2 line were made outside of the brilliant facular and floccular regions and are difficult since the line on the general disk is weak and its outer edges are poorly defined, especially on the red-facing border. It is present, however, at all points of the disk; and by careful timing of exposures and strong development covering a region of about 5 Å on each side of the line, that is, covering the broad shading due to K_1, a large number of plates suitable for measurement were obtained. Besides the center, the points chosen for observation were 1, 12, 35, 50, and 68 mm from the limb. As will be seen, the total change in wave-length in passing from the limb to the center is not large, and from the cosine law of the distribution of the line-of-sight components of a radial velocity it follows that to produce a variation of 10 per cent in the value of this component, it would be necessary near the center to go 35 mm toward the limb on an image of the size used. This made it feasible to take a series of exposures for points near the center on a single plate, changing the position of the sun's image on the slit a few millimeters for each exposure. In this way the effects of local disturbances in the solar atmosphere could be distributed and the probability increased that the final mean of the measurements would represent the prevailing condition of the vapor. In a more limited way this could be done with the other points selected for measurement, except for that 12 mm from the limb where the change is compara-

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 33; *Astrophysical Journal*, **29**, 110, 1909.

TABLE VII

WIDTH OF V K_2—WIDTH OF R K_2; AND WAVE-LENGTH OF K_3 1 mm FROM THE LIMB

Plate	Difference	K_3
120 W.......	+0.021	3933.673
120 E.......	+0.044	.665
199 W.......	−0.009	.674
199 E.......	+0.003	.666
204 W.......	−0.003	.670
204 E.......	+0.022	.666
209 W.......	+0.003	.673
209 E.......	+0.017	.655
229 W.......	+0.006	.673
229 E.......	+0.002	.661
234 W.......	−0.005	.664
234 E.......	+0.009	.674
248 W.......	+0.015	.673
248 E	+0.002	.662
262 W.......	+0.001	.671
262 E......	−0.006	3933.661
360 W......	+0.005	.662
378 E......	−0.004	.669
383 W......	+0.004	.686
383 E......	−0.002	.666
387 E......	−0.001	.662
389 E......	+0.019	.664
510 E......	+0.012	.664
510 W......	+0.010	.665
510 W......	+0.017	.667
510 W......	+0.013	.665
510 W......	+0.006	.672
510 E......	+0.006	.661
510 E......	+0.000	.667
510 E......	+0.014	.664
Means ...	+0.0074	3933.6672

tively rapid. The result for this point is therefore of less weight since successive exposures at this same point on the sun on a single plate would not eliminate local effects.

TABLE VIII

Wave-Lengths of K_2 and K_3 at 12 mm from the Limb

Plate	Date	K_2	K_3
	1909		
427 (1) E	October 31	3933.660	3933.678
(2) E	" 31	.657	.679
(3) E	" 31	.642	.681
(4) E	" 31	.648	.683
(5) E	" 31	.653	.680
430 (1) W	November 1	.652	.671
(2) W	" 1	.647	.671
(3) W	" 1	.640	.676
(4) W	" 1	.646	.665
(5) W	" 1	.643	.667
(6) W	" 1	.647	.670
(7) W	" 1	.653	.670
(8) W	" 1	.653	.675
434 (1) E	" 1	.660	.674
(2) E	" 1	.645	.675
437 (1) E	" 1	.654	.674
(2) E	" 1	.641	.672
(3) E	" 1	.656	.675
(4) E	" 1	.653	.675
(5) E	" 1	.657	.674
(6) E	" 1	.657	.672
(7) E	" 1	.659	.680
(8) E	" 1	.656	.671
502 (1) W	December 3	.658	.683
(2) W	" 3	.654	.674
(3) W	" 3	.648	.673
(4) W	" 3	.638	.676
508 (1) W	" 12	.640	.668
(2) W	" 13	.648	.671
(3) W	" 13	.643	.679
Means		3933.6503	3933.6744

The exposures were confined to regions on the disk outside of the faculae and flocculi, as in the case of K_3 in the preceding section, and therefore represent the general chromospheric vapor producing K_2.

In the measurement of the bright components of K_2, the method suggested by Adams[1] was followed. "It is evident," he says, "if *a* is the wave-length of one of these components, and *d* the difference of wave-length between the absorption line and the second component, that the center of the whole bright line is given by $a+d$."

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[1] *Contributions from the Mount Wilson Solar Observatory*, No 6; *Astrophysical Journal*, **23**, 45, 1906.

Measurements made in accordance with this method give data for the calculation of the wave-length not only of K_2, but of K_3 as well,

TABLE IX

WAVE-LENGTHS OF K_2 AND K_3 AT 35 mm FROM THE LIMB

Plate	Date	K_2	K_3
	1909		
465 (2) E	November 24	3933.656	3933.673
(3) E	" 24	.645	.676
467 (1) W	" 24	.656	.683
(2) W	" 24	.650	.683
(3) W	" 24	.643	.671
(4) W	" 24	.642	.676
468 (2) E	" 24	.658	.680
(3) E	" 24	.653	.683
(4) E	" 24	.656	.682
(6) E	" 24	.653	.683
(7) E	" 24	.646	.676
(8) E	" 24	.643	.683
472 (1) W	" 27	.630	.670
(2) W	" 27	.642	.669
(4) W	" 27	.633	.668
498 (1) W	December 3	.643	.676
Means		3933.6468	3933.6770

TABLE X

WAVE-LENGTHS OF K_2 AND K_3 AT 50 mm FROM THE LIMB

Plate	Date	K_2	K_3
	1909		
509 (1) W	December 13	3933.640	3933.682
(2) W	" 13	.641	.682
(3) W	" 13	.642	.681
(4) E	" 13	.657	.683
(5) E	" 13	.656	.676
(6) E	" 13	.653	.680
511 (3) W	" 23	.624	.673
(4) W	" 23	.639	.683
(6) E	" 23	.651	.681
(8) E	" 23	.651	.672
Means		3933.6454	3933.6793

and in fact part of the results already given for K_3 in Tables II–IV was derived from this material. For completeness these values are reprinted in Tables VII–XII along with the additional results for K_3 and the values of K_2. The figures in parentheses refer to successive exposures on the same plate. Table VII, however, does not give directly the values of K_2.

On the plates 1 mm from the limb the emission components are very broad and strong, and, as far as the eye can judge, symmetrical. They are too broad, however, for a determination of the wave-length of K_2 by the preceding method. But on 30 of these plates, selected for clearness and sharpness of definition and measured primarily for the purpose of obtaining the widths of K_2 and K_3, it was found that, on the whole, the V K_2 component was wider than R K_2, thus indicating a shift of K_2 relative to K_3. This result permits an indirect determination of the wave-length of K_2 for the point 1 mm from the limb. The differences in width of the components of K_2, expressed in Ångströms, and the corresponding values of the wave-length of K_3 are given in Table VII. The mean difference in width of the components of K_2 is +0.0074 Å, and indicates that here as elsewhere on the solar disk the vapor producing the emission line K_2 is rising relatively to that producing the absorption line K_3. The mean wave-length of K_3 from all of the plates 1 mm from the limb is 3933.6674. Since the shift of K_2 relatively to K_3 is one-half the difference in width of V K_2 and R K_2, it follows that the wave-length of K_2 at 1 mm from the limb is 3933.6637. The corrections to be applied to reduce these values to the limb will be given later.

TABLE XI

WAVE-LENGTHS OF K_2 AND K_3 AT 68 mm FROM THE LIMB

Plate	Date	K_2	K_3
	1909		
513 (1) E	December 23	3933.653	3933.680
(2) E	" 23	.648	.677
(3) E	" 23	.645	.678
(4) E	" 23	.637	.680
(5) W	" 23	.632	.680
(7) W	" 23	.629	.675
(8) W	" 23	.624	.684
520 (2) W	" 24	.661	.680
(3) W	" 24	.648	.678
(8) E	" 24	.639	.687
Means		3933.6416	3933.6799

It will be noticed that there is a progressive shortening of the wave-length of K_2 and a progressive lengthening of that of K_3 as the point of observation recedes from the limb toward the center, so that the difference between the wave-lengths of K_2 and K_3 increases continuously from the limb to the center. This result is clearly shown by Table XIII which contains the weighted means of all of the values for K_2 and K_3 given in Tables II–IV and VII–XII, and the respective differences in wave-lengths, for different distances from the limb.

TABLE XII

WAVE-LENGTHS OF K_2 AND K_3 AT CENTER OF SUN'S DISK

Plate	Date	K_2	K_3
436 (3)	1909 November 1	3933.654	3933.685
(5)	" 1	.655	.680
(6)	" 1	.645	.677
(7)	" 1	.662	.690
(8)	" 1	.644	.674
(9)	" 1	.653	.680
(1)	" 1	.648	.687
443 (2)	" 3	.637	.679
(3)	" 3	.654	.680
(4)	" 3	.651	.690
(5)	" 3	.645	.684
(6)	" 3	.645	.679
(7)	" 3	.638	.679
(8)	" 3	.642	.680
(9)	" 3	.645	.668
(11)	" 3	.628	.682
(13)	" 3	.642	.677
455 (4)	" 3	.646	.679
(6)	" 3	.661	.678
(7)	" 3	.645	.685
(8)	" 3	.645	.674
(9)	" 3	.640	.682
(10)	" 3	.647	.675
461 (1)	" 23	.645	.675
(2)	" 23	.645	.676
(3)	" 23	.647	.686
(4)	" 23	.627	.689
(7)	" 23	.635	.697
(8)	" 23	.635	.697
462 (2)	" 23	.631	.683
(3)	" 23	.636	.682
(5)	" 23	.639	.678
(7)	" 23	.638	.677
(9)	" 23	.642	.689
(12)	" 23	.627	.688
471 (3)	" 27	.636	.681
(4)	" 27	.611	.675
(6)	" 27	.621	.671
(7)	" 27	.625	.685
(8)	" 27	.627	.671
Means		3933.6410	3933.6811

The figures in parentheses are the weights of the corresponding mean wave-lengths. In general these are proportional to the number of exposures used, except for the point 12 mm which has been given a smaller weight for reasons already stated.

The falling of the K_3 vapor indicated by the results of the preceding section is even more definitely shown by the data in Table XIII, and at the same time there is an equally positive indication that the K_2 calcium is rising.

TABLE XIII

WAVE-LENGTHS OF K_3 AND K_2 AT DIFFERENT DISTANCES FROM LIMB

	1 mm	12 mm	35 mm	50 mm	68 mm	Center
K_3	3933.6674 (63)	.6746 (34)	.6770 (8)	.6793 (5)	.6799 (5)	.6804 (63)
K_2	3933.6637 (24)	.6503 (12)	.6468 (8)	.6454 (5)	.6416 (5)	.6410 (20)
K_3-K_2....	+0.0037	+0.0243	+0.0302	+0.0339	+0.0383	+0.0394
$O-C$ K_3 .	−0.0004	+0.0014	−0.0007	+0.0001	−0.0004	−0.0002
$O-C$ K_2 .	+0.0008	−0.0033	+0.0008	+0.0020	0.0000	0.0000

The evidence is most clearly presented by exhibiting the close agreement of the observed values of the wave-lengths of K_3 and K_2 with an equation of the form

$$\lambda_L + \Delta\lambda \cos E = \lambda$$

in which λ_L is the wave-length at the limb, $\Delta\lambda$ the change in wave-length in passing from limb to center, and E the angular elongation from the center of a point for which the observed wave-length is λ.

Least-squares reductions based upon the data in Table XIII give the following results for the wave-lengths at the limb and the changes in passing from limb to center:

	λ_L	$\Delta\lambda$
K_3............	3933.6655±0.0004	+0.0151±0.0006
K_2............	3933.6668±0.0010	−0.0258±0.0014

The appended quantities are the probable errors. The corresponding residuals for the observed values are given in the last two lines of Table XIII. The results are shown graphically by Fig. 1. The abscissae are distances from the limb, one division corresponding to 1 mm. The ordinates are wave-lengths with one division equal to 0.001 Å. The curves are drawn by points calculated from the

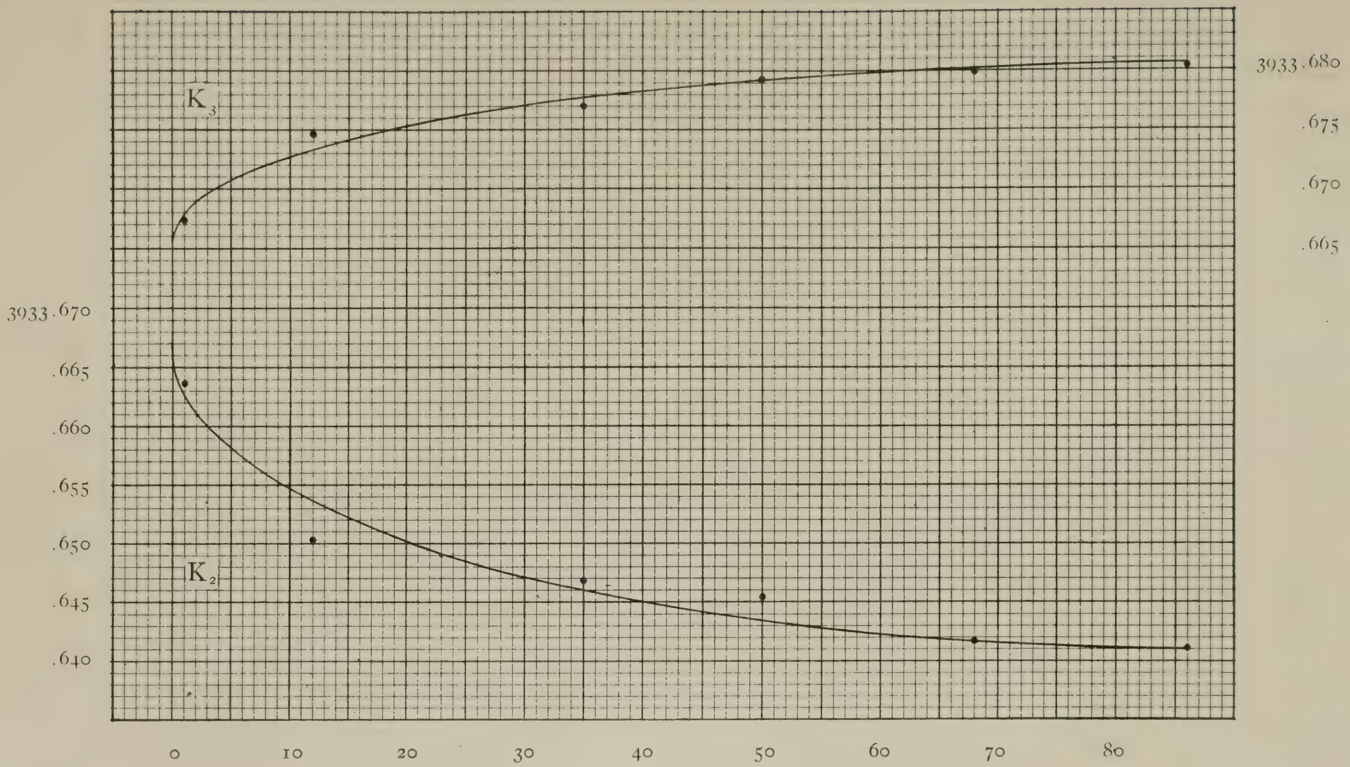

FIG. 1.—Wave-Lengths of K_3 and K_2 for Varying Distances from Limb

cosine law. That for K_3, referred to the ordinate values on the right, has the line λ 3933.6655 (wave-length of K_3 at the limb) as the axis of abscissae and corresponds to an *increase* of 0.0151 Å from limb to center. The curve for K_2, referred to the ordinates at the left, has the line λ 3933.6668 (wave-length of K_2 at the limb) for its axis and corresponds to a *decrease* of 0.0258 Å from limb to center. The dots represent the observed data as collected in Table XIII. Considering the scale of wave-lengths and the difficulties of measurement, especially in the case of K_2, the agreement of the observations with the calculated curves is extremely satisfactory except for the point 12 mm from the limb on the K_2 curve, where, as previously indicated, the effects of temporary conditions are probably not eliminated to the same extent as elsewhere. The curves may be considered as representing the prevailing condition of the vapors to which the respective lines are due, that is, a *downward* velocity for the vapor producing K_3 of 1.14 km per second, and an *upward* velocity for that producing the bright emission line K_2 of 1.97 km per second.

Retaining the fourth decimal, the wave-length of the K line derived from laboratory investigations[1] is 3933.6670±0.0002, which is in close agreement with the limb values given above for K_2 and K_3. It will be noted that the difference between the wave-lengths for K_3 and K in the arc is in the direction agreeing with the relatively low pressure conditions probably associated with the high-level K_3 vapor, and in view of the small pressure shift per atmosphere, viz., 0.002 Å,[2] would tend to strengthen the evidence given later in this paper for the assumption of a low density in the case of the vapor producing the absorption line. This fact cannot be regarded as of positive significance, however, since the difference involved is not greatly in excess of the probable errors; and moreover, the comparison is made between results produced under conditions so vastly different that causes other than difference of pressure might conceivably produce small changes in wave-length.

As far as the writer has been able to ascertain, this is the first direct proof of the agreement of the solar or cosmic wave-length of a sub-

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 44; *Astrophysical Journal*, **31**, 143, 1910.

[2] Humphreys and Mohler, *Astrophysical Journal*, **3**, 114, 1896.

stance with its wave-length in a terrestrial source, to the degree of accuracy obtainable in terrestrial comparisons. Only in the case of a substance like hydrogen or calcium, occurring at so high a level in the solar atmosphere that the pressure does not differ greatly from that of the terrestrial atmosphere, and for measurements from which line-of-sight motion may be eliminated, can such an agreement be expected. That an element can exhibit its characteristic properties under such widely differing conditions and in regions so greatly separated in space, thus emphasizing the essential unity of our system, is a fact of profound significance, and an accumulation of exact data establishing such agreement under terrestrial and cosmic conditions is very much to be desired.

5. HORIZONTAL MOVEMENTS IN THE SOLAR ATMOSPHERE

The idea of general currents in the sun's atmosphere between the equator and the pole has appeared at various times. Their existence seems in general to have been assumed from analogy to the great movements in the terrestrial atmosphere; but the conditions producing the terrestrial system of circulation, equatorial and polar regions of unequal temperatures, have never been observed in the sun. M. Deslandres, in speaking of vertical and horizontal currents in the solar atmosphere, says:

> Ce mouvement général était prévu par les théories de M. Faye, et les expériences récentes de Mgr. Rougerie. . . . . D'ailleurs, le courant de pôle à l'équateur pourrait avoir lieu dans la photosphère, et le courant inverse de retour au-dessus, mais alors avec des vitesses croissantes pour des hauteurs croissantes.[1]

M. Faye states the following conclusion drawn from his study of the origin of sun-spots and the equal velocity of spots and the photospheric zones in which they lie:

> That these bands move nearly parallel to the equator and never exhibit currents that, as in the upper regions of our atmosphere, would be constantly directed toward the pole.[2]

Oppolzer, when speaking of the upward and downward currents in the solar atmosphere and the interactions of their dynamic and thermal working, says:

[1] *Comptes Rendus*, **119**, 459, 1894.

[2] Vogel, *Populäre Astronomie*, Dritte Auflage, 317.

> Die Folge davon wird sein, dass die vertikal abwärts gerichtete Bewegung schliesslich aufhört und die oben nachdrängenden Massen gezwungen sind, ihren Weg seitwärts in horizontalen Bahnen zu nehmen. . . . . In den polaren Regionen herrschen aufsteigende Ströme, wie in unserer Erdatmosphäre am Aequator, die in einer gewissen Höhe als horizontale Ströme gegen die niederen Breiten in lang gezogenen Spiralen ziehen. . . . .[1]

To detect, if possible, any differences in the behavior of the calcium vapor between the polar and equatorial regions of the sun, a series of plates was taken at 12 mm from the polar limbs, with the results given in Table XIV. The slit of the spectrograph is fixed in a vertical plane and there is no arrangement for rotating the solar image. For the dates of these plates the polar axis of the sun's image formed an angle of 10° with the vertical.

TABLE XIV

WAVE-LENGTHS OF K_2 AND K_3 AT 12 mm FROM THE POLAR LIMBS

North				South			
Plate	Date	K_2	K_3	Plate	Date	K_2	K_3
	1909				1909		
419 (1)....	Oct. 30	3933.666	.681	420 (1)....	Oct. 30	3933.648	.684
(2)....	" 30	.658	.680	(2)....	" 30	.650	.679
(3)....	" 30	.669	.680	(3)....	" 30	.651	.675
(4)....	" 30	.663	.671	(4)....	" 30	.644	.673
(5)....	" 30	.669	.677	(5)....	" 30	.640	.678
(6)....	" 30	.659	.676	(6)....	" 30	.654	.677
(7)....	" 30	.654	.675	(7)....	" 30	.663	.679
(8)....	" 30	.651	.679	(8)....	" 30	.645	.675
431 (1)....	Nov. 1	.637	.675	426 (1)....	" 30	.633	.669
(2)....	" 1	.631	.666	(4)....	" 30	.635	.679
432 (1)....	" 1	.664	.676	(5)....	" 30	.643	.666
(2)....	" 1	.637	.678	(10)....	" 30	.645	.666
(3)....	" 1	.641	.674	435 (2)....	Nov. 1	.652	.673
(4)....	" 1	.638	.674	(3)....	" 1	.664	.681
(5)....	" 1	.654	.676	(4)....	" 1	.652	.679
(6)....	" 1	.644	.676	(5)....	" 1	.651	.677
(7)....	" 1	.652	.680	(6)....	" 1	.652	.677
(8)....	" 1	.644	.680	(7)....	" 1	.644	.679
(9)....	" 1	.643	.669	(8)....	" 1	.640	.675
(10)....	" 1	.637	.682	(9)....	" 1	.637	.680
Means		3933.6506	.6760	Means		3933.6471	.6761

The final means for Table XIV, and their probable errors, are compared below with those for points 12 mm from the equatorial limbs as given in Table XIII. The figures in parentheses indicate the weights.

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[1] Vogel, *Populäre Astronomie*, Dritte Auflage, 328.

	Equatorial Limbs	Polar Limbs
K_2…………	3933.650±0.0008 (12)	3933.649±0.0011 (16)
K_3…………	3933.675±0.0003 (34)	3933.676±0.0004 (16)

The very high-level calcium vapor producing the absorption line K_3 is particularly well adapted for an examination of the suggestion made by Deslandres that the current from the equator to the pole is high up in the solar atmosphere and has increasing velocities with increasing heights. This vapor ought, therefore, from its extreme altitude, to show the maximum effect. The velocities demanded by the Oppolzer theory are low. The shortest time he suggests for the movement from the pole to the equator is a month. Such a movement taking place in six months would produce a shift in the K line of 0.0012 Å, which is greater than the probable error for the present measurement of the K_3 line. Unless the motion is common to the high-level calcium vapor and the low-level iron vapor of the reversing layer which supplied the secondary standards used, such currents between the equator and the pole cannot greatly exceed the velocity suggested in the preceding sentence. The high-level currents suggested by Deslandres and Oppolzer are in opposite directions. The practical identity of the wave-lengths of K_2 and K_3 in the polar and equatorial regions would, however, negative both hypotheses in the case of the calcium vapor producing the H and K lines. At a point 12 mm from the limb 85 per cent of a horizontal motion would appear as a line-of-sight velocity, and in case of opposite motions in the upper regions and in the reversing layer the relative effect would be greatly increased. The absence of any observable effect for calcium vapor is conclusively shown by comparison of the measurements for the polar and equatorial limbs.

6. WIDTHS OF H AND K AT CENTER AND LIMB

The change in width of the calcium lines H and K in passing from the center to the limb is very marked. There is also a change in the *relative* width of H and K. The absolute measurement of the widths of spectrum lines presents great difficulties, but for similar lines, measured under the same conditions and in immediate sequence, considerable weight can be given to the relative widths when the number of lines is sufficiently large. Measurements were made of H_3 and K_3 on 22 plates taken at 1 mm from the limb, and upon 20 plates taken at the center. In these cases, of course, the second slit was wide enough to admit both H and K, so that the exposure time and development were the same for both lines. H_2 and K_2 were measured on 15 plates taken at 1 mm from the limb, but width measurements were not possible for H_2 and K_2 at points representing the ordinary condition of the vapor, on plates taken at the center. The results are given in Table XV, in which widths are expressed in Ångströms.

TABLE XV

RELATIVE WIDTHS OF H AND K

1 mm from Limb							Center			
Plate	H_3	K_3	$K_3 \div H_3$	H_2	K_2	$K_2 \div H_2$	Plate	H_3	K_3	$K_3 \div H_3$
120 (1)...	0.315	0.360	1.14	0.598	0.680	1.14	128 (1)...	0.118	0.138	1.17
(2)...	0.338	0.372	1.10	0.679	0.717	1.06	(2)...	0.119	0.134	1.12
199 (1)...	0.332	0.374	1.13	0.657	0.706	1.07	129 (1)...	0.126	0.154	1.20
(2)...	0.320	0.347	1.09	0.708	0.797	1.12	(2)...	0.124	0.145	1.20
204 (1)...	0.329	0.384	1.17	0.712	0.765	1.07	130 (1)...	0.113	0.141	1.24
(2)...	0.358	0.385	1.08	0.680	0.738	1.08	(2)...	0.104	0.128	1.23
209 (1)...	0.328	0.374	1.14	.....	.....	....	136 (1)...	0.119	0.142	1.20
(2)...	0.276	0.311	1.13	.....	.....	....	(2)...	0.123	0.140	1.14
229 (1)...	0.298	0.350	1.17	0.684	0.760	1.11	146 (1)...	0.137	0.163	1.19
(2)...	0.285	0.327	1.14	.....	.....	....	(2)...	0.128	0.153	1.19
234 (1)...	0.320	0.356	1.11	0.723	0.749	1.03	211 (1)...	0.139	0.163	1.18
(2)...	0.303	0.345	1.14	0.702	0.752	1.07	(2)...	0.127	0.153	1.20
248 (1)...	0.310	0.358	1.16	0.684	0.710	1.04	212 (1)...	0.123	0.149	1.21
(2)...	0.302	0.326	1.08	0.692	0.725	1.05	(2)...	0.149	0.171	1.15
262 (1)...	0.295	0.349	1.18	0.757	0.717	1.03	512 (1)...	0.145	0.165	1.14
(2)...	0.293	0.325	1.11	0.723	0.741	1.02	(2)...	0.138	0.164	1.19
510 (1)...	0.375	0.410	1.09	0.865	0.931	1.08	(3)...	0.127	0.145	1.14
(2)...	0.359	0.424	1.18	0.792	0.840	1.06	(4)...	0.116	0.132	1.14
(3)...	0.352	0.383	1.09	.....	.....	....	(5)...	0.134	0.149	1.11
(4)...	0.201	0.225	1.12	.....	.....	....	(6)...	0.148	0.170	1.15
(5)...	0.222	0.253	1.14	.....	.....	....				
(6)...	0.222	0.255	1.14	.....	.....	....				
Means ...	0.306	0.345	1.13	0.710	0.759	1.07		0.128	0.150	1.17

The writer showed in a previous paper[1] that in the case of the lines produced in the arc the relative width and intensity of K to H increased as the lines became narrower as a result of decrease in the density of the vapor. The condensed data for the arc are given below with the mean results for H and K at the center and limb of the sun. The last line refers to the relative width of H_2 and K_2 at points where the lines are locally strengthened.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 44; *Astrophysical Journal*, **31**, 143, 1910.

Source	Width		Relative Width
	H	K	K ÷ H
Arc, 13 bright lines	0.147	0.177	1.20
Arc, 12 absorption lines	0.064	0.087	1.36
	H_3	K_3	$K_3 \div H_3$
Sun's limb, 22 lines	0.306	0.345	1.13
Sun's center, 20 lines	0.128	0.150	1.17
	H_2	K_2	$K_2 \div H_2$
Sun's limb, 15 lines	0.710	0.759	1.07
Sun's center, 4 lines	.....	.....	1.09

The ratio of K_3 to H_3 increases on passing from the limb to the center, thus showing an increasing preponderance of K_3 over H_3 similar to that shown by K over H when the vapor in the arc decreases in density. An explanation is readily found by assuming that the vapor producing the absorption lines H_3 and K_3 exists in a relatively thin high-level layer in which the effective depth of vapor producing the absorption lines rapidly increases as the limb is approached. The same conditions, of course, explain the absolute increase in width of H_3 and K_3 on passing from the center to the limb, the mean width of K_3 at the limb being 2.3 times its width at the center, and the mean width of H_3 at the limb being 2.4 times its width at the center. The fact that H_3 and K_3 continue to increase in width, however near the limb is approached, would indicate that even at the limb the effective depth is not sufficient to produce black-body absorption. The widths of H_3 and K_3 at the center of the sun's disk are about twice the widths of the corresponding absorption lines for a moderate amount of vapor in the arc, and less than the widths of the corresponding bright lines for a small amount of vapor in the arc. From this low absorptive power in the sun's atmosphere it would appear that the vapor of calcium in the solar envelope producing these absorption lines must be of very moderate thickness and, owing to its high temperature and low pressure, exceedingly rare. The ratio of the width of K_2 to H_2 at the limb is 1.07. From this low ratio it follows by the same line of reasoning that the quantity of vapor producing the emission lines is greater than that producing the absorption lines. The same fact is also shown by the greater absolute width of the emission lines. The measurements of the widths of H_2 and K_2 at the center were not in general possible, but at points of local strengthening the ratio of the width of H_2 to K_2 was found to be 1.09. The increased preponderance of K_2 indicates, as in the case of K_3, a less effective layer at the center than at the limb, but the change in effectiveness between limb and center is much less than in the case of the K_3 absorbing vapor. The absolute widths, however, are greater near the limb, and by their continued increase indicate that even at the limb the vapor is not sufficiently deep and dense to produce black-body radiation. The vapor producing the emission lines is probably at a middle level and in a stratum thicker and under greater pressure than that producing the absorption lines H_3 and K_3, and probably of greater density also, the latter depending, however, upon its temperature. The question of its temperature and consequent increased radiating power will be discussed later in this paper. The intensity of H_2 and K_2 near the limb is also very much greater than over the general disk.

7. DISTRIBUTION OF THE CHROMOSPHERIC CALCIUM VAPOR

To investigate the distribution of the chromospheric calcium vapor, and determine the thickness of the absorbing and emitting layers, a series of plates was taken with a radial slit. The spectroscope was the 30-foot Littrow described as No. 3. It was used in the first order with exposures 12 to 20 times as long as required for the center of the disk. Points were chosen 30° apart around the limb and those spectra selected for measurement which seemed to represent the general chromosphere, that is, outside the vicinity of prominences. The plates were developed strongly until the spectrum of the skylight showed distinctly. The broad shadings in the atmospheric spectrum, due to H_1 and K_1, permitted this treatment of the plates and were favorable to the bringing out of the chromospheric lines. The measurements are extremely difficult to make as it is quite impossible to be sure of the extremity of bright lines which gradually fade out, and equally difficult to determine the point where the lines due to the lower layer of vapor producing the bright disk lines blend into those due to the high-level vapor. As one examines the lines he is quite easily convinced that the lower layer is far the thicker of the two. Near the photosphere the lines are still reversed, consisting of very bright broad lines, with relatively dark centers. The bright components grow narrower and weaker and blend into the more extended central portions. There is an apparent increase in the intensity of the central line just as the two components lose their identity. This was of assistance in forming a judgment as to the upper limit of the originally emitting vapor. The measurements are given in Table XVI, where the heights above the photosphere are given in millimeters with the equivalent of the means in kilometers and in seconds of arc.

TABLE XVI

HEIGHT OF THE CHROMOSPHERIC CALCIUM. MEAN DIAMETER OF SUN'S IMAGE 174 mm

Plate	H_2	K_2	H_3	K_3
	mm	mm	mm	mm
C 2..........	0.460	0.464	0.171	0.219
C 2..........	0.400	0.548	0.130	0.169
C 2..........	0.393	0.461	0.144	0.121
C 3..........	0.398	0.481	0.190	0.200
C 3..........	0.474	0.491	0.194	0.212
C 4..........	0.405	0.407	0.188	0.227
C 4..........	0.340	0.363	0.126	0.190
C 4..........	0.419	0.494	0.202	0.217
C 6..........	0.342	0.345	0.140	0.142
C 6..........	0.329	0.350	0.194	0.200
C 7..........	0.403	0.437	0.198	0.209
C 7..........	0.389	0.473	0.115	0.118
C 7..........	0.443	0.530	0.117	0.185
Means ...	0.400	0.450	0.159	0.185
Heights.......	3200 km 4″.4	3600 km 5″.0	1270 km 1″.8	1480 km 2″.0

A high degree of precision cannot be attributed to the separate measurements but from their general agreement some definite conclusions can be drawn. The height above the photosphere at which the calcium vapor still shows the K line is 600 km greater than that at which the H line disappears; that is, the outer 600 km of calcium vapor is not sufficiently dense to emit the H line to a sensible amount, though still emitting the K line with an appreciable intensity. This is in harmony with the increasing relative intensity of H to K in the arc with decreasing density of the calcium vapor, and with the increasing relative width of K_3 to H_3 in passing from the limb to the center of the sun. The K line, therefore, is the most persistent line due to the vapor of calcium.

The upper limit to which the general chromospheric calcium extends in an appreciable quantity is approximately 5000 km or 7."0 of arc. The outer 1500 km may be assigned with considerable certainty to the vapor producing the absorption line K_3. The upper limit of the layer producing the emission line K_2 is then approximately 3500 km. Its lower limit may be assumed to be that of the upper reversing layer, that is, approximately 700 km. It is of interest to compare the mean height of 53″ found by Belopolsky,[1] who gave attention only to the calcium in prominences and over faculae at the limb, with these results which aim to give what may be called the normal elevations at which the calcium can be detected, in undisturbed regions, by the instrumental equipment employed.

Assuming these thicknesses for the respective layers, it is possible to determine the change in the depth of the layers in the line of sight on approaching the limb, and to compare them with the changes in the width of the line. With a solar image 172 mm in diameter, such as was available for this investigation, and with 697,000 km as the semi-diameter of the photosphere, the following results were obtained for the thickness of the layers:

	At the Center	1 mm from the Limb	At the Limb
Reversing layer......	0–700 km	4,600 km	31,800 km
K_2.................	700 km–3500 km (2800 km)	16,400 km	62,300 km
K_3.................	3500 km–5000 km (1500 km)	7,780 km	45,800 km

The effective depth of the layer assigned to the absorbing vapor is 5.2 times as great at 1 mm from the limb as at the center, while the mean width of K_3 at 1 mm from the limb is 2.3 times its mean width at the center. The measurement of the width of K_2 at the center could not be made on the general disk. Its width at the limb relative to the width of H_2 at the limb is 1.07. The small value of the ratio points to a layer of such great thickness or density that the change in the depth of the layer on passing from the center to the limb would be much less effective than in the case of the absorbing vapor, since the relative width of H to K changes most rapidly when the density of the vapor is very low and the intensity of the lines is weak. As shown on page 63, the ratio of K_2 to H_2 at points at the center of the disk, where the line is greatly strengthened over faculae, is 1.09. The slight increase in the ratio at the center is in accord with the assumption that the lines are so strong that a considerable increase in thickness and density would not alter the ratio greatly, and hence is not in disagreement with the great increase in the thickness of the layer assigned to the emitting vapor, in passing from the center to the limb.

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[1] *Mitteilungen der Nikolai-Hauptsternwarte zu Pulkowa*, 1, 157, 1906.

The quantitative relation connecting the width of a spectrum line with the thickness of the layer is unknown. The conclusion from the laboratory results is stated by Kayser as follows:[1]

> The width of lines is increased by the thickness of the layer only in those cases in which the curve representing the distribution of intensity within the line falls away slowly on one or both sides, not, on the other hand, when the form of the curve is steep or almost perpendicular on both sides.

The distribution of the intensity within the H and K lines corresponds to the first condition, and it may be assumed for these lines that increased thickness of the radiating or absorbing layer acts in the same direction as increased density. Layers of calcium vapor in the solar atmosphere lying between 700 and 3500 km and between 3500 and 5000 km would be consistent with the observed variations in the widths of K_2 and K_3, respectively, in passing from the center to the limb.

8. THE WAVE-LENGTH OF THE ABSORPTION LINE H_3

For reasons already given, the original plan was to limit the investigation to the K line of calcium; but, on certain plates taken for comparison of the iron arc with the center of the sun and subjected to strong development to bring out the lines in the broad shadings of H_1 and K_1, the H_3 line was particularly well defined, and it was possible to measure it with a good degree of accuracy. The iron lines λ 3966 and λ 3969 were used as solar standards for H_3. The wave-lengths of these lines in the arc have been measured by the writer with reference to the Fabry and Buisson standards and their wave-lengths found to be 3966.068 and 3969.261 respectively.[1] The results for the corresponding solar lines, determined from comparison plates for the iron arc and center of the sun and corrected for the rotation of the earth and the eccentricity of its orbit, are given in Table XVII.

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[1] *Handbuch der Spectroscopie*, **2**, 296.

TABLE XVII

SOLAR STANDARDS FOR H_3

Plate	Date	λ 3966	λ 3969
	1909		
396	October 6	3936.075	3969.269
398	" 6	.071	....
403	" 15	.074	.267
404	" 15	.072	.267
406	" 15	.070	.264
Means		3966.072	3969.267
Arc		3966.068	3969.261

The displacements between center and limb are assumed to be 0.005 Å—the mean of the shifts for λ 3930 and λ 3935 given in this paper and of shifts of other iron lines in this region as found by Adams.[2] The results for the wave-length of H_3 at the center of the disk are given in Table XVIII.

TABLE XVIII

WAVE-LENGTH OF H_3 AT THE CENTER OF THE DISK

Plate	Date	H_3	Plate	Date	H_3
	1909			1909	
404 (1)	October 10	3968.489	412 (4)	October 19	3969.498
405 (2)	" 15	.495	413 (1)	" 19	.487
405 (3)	" 15	.491	413 (2)	" 19	.494
405 (4)	" 15	.487	413 (3)	" 19	.491
406 (1)	" 15	.491	414 (1)	" 19	.489
406 (2)	" 15	.492	414 (2)	" 19	.488
406 (3)	" 16	.492	414 (2)	" 19	.491
412 (1)	" 19	.496	414 (4)	" 19	.495
412 (2)	" 19	.492	416 (1)	" 19	.484
412 (3)	" 19	.469	416 (2)	" 19	.491
Mean		3968.492	Mean		3968.491

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 44; *Astrophysical Journal*, **31**, 143, 1910.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 43; *Astrophysical Journal*, **31**, 30, 1910.

A few plates were taken at 1 mm from the limb for comparing the widths of the H and K lines. Upon these also the wave-length of the H_3 line was measured with the results in Table XIX.

TABLE XIX

WAVE-LENGTH OF H_3 AT 1 mm FROM THE LIMB

EAST LIMB			WEST LIMB		
Plate	Date	H_3	Plate	Date	H_3
	1909			1909	
120	March 9	3968.479	120	March 9	3968.480
145	“ 10	.469	145	“ 10	.478
151	“ 11	.469	151	“ 11	.472
199	“ 31	.475	199	“ 31	.479
204	April 1	.469	204	April 1	.486
209	“ 9	.480	209	“ 9	.487
229	“ 15	.481	229	“ 15	.479
234	“ 16	.489	234	“ 16	.478
248	“ 26	.479	248	“ 26	.483
255	“ 27	.481	255	“ 27	.480
Mean		3968.477	Mean		6938.480

The results deduced from the measurement of H_3 show a satisfactory agreement with those of K_3, especially when it is considered that they depend upon only one-third the number of plates and consequently may be more affected by temporary conditions. The wave-length of H_3 at 1 mm from the west limb exceeds that at 1 mm from the east limb by 0.003 Å, indicating a higher rotational velocity of the calcium vapor than of the reversing layer, in harmony with the results for K_3 (p. 50). The wave-length of H_3 reduced to the limb is 3968.476. The following tabular arrangement exhibits the agreement between the measurements for the two lines.

	H_3	H_3-H_{arc}	K_3	K_3-K_{arc}
Center	3968.491	+0.015	3933.681	+0.014
1 mm from limb	3968.478	+0.002	3933.667	+0.000
At limb	3968.476	0.000	3933.665	−0.002
Arc	3968.476	......	3933.667	......

The weights of the measurements for K_3 are much greater than those for H_3 because of the larger number of the K_3 plates and their consequent wider distribution in time, thus eliminating the effect due to exceptional conditions, and because of the cleaner and better defined character of the K_3 line. The main conclusions in the paper are based therefore on the measurements of the K_3 line, confirmed as they are by the result for the H_3 line.

9. RELATIVE WAVE-LENGTHS OF H AND K IN THE SUN

Adams calls attention to a probable error in the relative wave-lengths of the H and K lines as given in the Rowland tables.[1] Various determinations of the arc value of this quantity are as follows:

H – K IN THE ARC

Rowland*	34.808 Å
Jewell*	34.809
Adams*	34.811
St. John*	34.809
Mean	34.809 Å

* *Contributions from the Mount Wilson Solar Observatory*, No. 44, pp. 1 and 6; *Astrophysical Journal*, **31**, 143 and 148, 1910.

According to these measurements, the difference between the wave-lengths of H and K in terrestrial sources is 34.809, with a very small probable error. The similar behavior of these lines in terrestrial sources would lead to the expectation of similar behavior under solar conditions. The relative wave-lengths of H_3 and K_3 at the center of the sun, found by practically the same observers, are as follows:

$H_3 - K_3$ IN THE SUN

Rowland's tables	34.800 Å
Adams	34.810
This paper	34.810

There can be little doubt, as Adams points out, that the relative wave-lengths of H and K in Rowland's tables are in error by 0.010 Å. It remains to be determined where the error is and how it is distributed between H and K. Using the wave-lengths given in Rowland's tables for the iron lines λ 3930, λ 3935, λ 3966, and λ 3969 as standards, the following results are obtained for H_3 and K_3, based upon the measurements used in calculating the wave-lengths given in this paper from the secondary standards of Fabry and Buisson. For comparison the wave-lengths of H and K given in Rowland's tables of solar wave-lengths are also shown.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 6, p. 8; *Astrophysical Journal*, **23**, 52, 1906.

	H	K	H−K
Rowland's tables	3968.625	3933.825	34.800 Å
This paper	3968.634	3933.824	34.810
Mean of arc results	....	....	34.809

There can be little doubt from the above comparison that the error in the relative wave-lengths of H and K referred to arises from an error in the measurement of the H line in the solar spectrum. Such an error, if existing, would on a priori grounds be looked for in the measurement of H rather than in the measurement of K, for reasons, emphasized already in this paper, depending upon the character of the lines and their immediate surroundings.[1]

10. DISCUSSION

The appearance of the broad H and K bands in the solar spectrum has been noted by several observers. The general survey of the solar surface outside of spots and faculae made in this investigation shows the universal presence of the absorption line K_3, with a mean width at the center of the disk of 0.150 Å, and also the practically universal presence of the bright line K_2. This emission line is easily manifest at all points of the surface except near the center of the disk, and even there by properly timed exposures and strong development it can always be obtained. It varies much more in width and intensity than the absorption line K_3. Hundreds of plates have been examined for the breaking up of the broad shading of K_1 into the system of lines

	H	K	H−K
Arc single	3968.624	3933.836	34.788 Å
Arc reversed	3968.612	3933.804	34.808
Sun	3968.742	3933.881	34.761

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[1] In this connection reference should be made to the wave-lengths of these lines found by Belopolsky, whose paper, "Untersuchung der *Ca*-Linien am Sonnenrande" (*Mitteilungen der Nikolai-Hauptsternwarte zu Pulkowo*, Band I, 164, 1906), has just come to hand. The copy in the Observatory library was lost in the fire on Mount Wilson.

noted by Jewell,[1] but no evidence of such a resolution of the lines has been found. The source of the high radiating power of the vapor producing the emission line K_2 has been attributed by Jewell to the impact of the collision, or the friction, between the falling meteoric matter to which he attributed the production of the absorption line K_3 and the ascending vapor. Hale and Ellerman suggest that it may not be entirely temperature radiation, saying:[2]

> From a strict application of Kirchhoff's law it would appear that the calcium vapor in the lower chromosphere is actually hotter than the calcium vapor which lies above and below it. It seems improbable that the law can be rigorously applied in this case, and hence it may be necessary to attribute the strong radiation of the intermediate layer to causes other than temperature alone. . . . . The evidence afforded by photographs taken at successive levels goes to show that in passing from the lower K_1 region up into the K_2 region, the calcium vapor increases greatly in radiating power. For example, eruptive phenomena, which are quite inconspicuous when photographed at the lower K_1 levels, become more and more brilliant as the second slit is moved for successive photographs nearer and nearer to the center of K.

This last is very remarkable inasmuch as material thrown up by eruptions would be expected to have a high temperature and, while passing through the K_1 level, its pressure and density would be such as to broaden the line to the K_1 width.

In this paper it has been shown that the mean upward velocity of the vapor producing K_2 is 1.97 km per second, and the mean downward velocity of the absorbing vapor producing K_3 is 1.14 km per second. It is of interest to consider the loss of mechanical energy suffered by two unit masses of vapor approaching each other with these velocities on the supposition that by collision and internal friction they are reduced to a common velocity. The resulting velocity would be 41.5×10^3 cm per second and the total loss of mechanical energy 2421×10^7 ergs or an equivalent of 578 calories. A small fraction of the liberated energy would suffice to raise the temperature of the calcium vapor sufficiently to account for the increased radiation since, as King has shown, the H and K lines of calcium are high-temperature lines. He says[3] that

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[1] *Astrophysical Journal*, **3**, 108, 1896.

[2] *Publications of the Yerkes Observatory*, Vol. III, Part I, pp. 16–18.

[3] *Contributions from the Mount Wilson Solar Observatory*, No. 32; *Astrophysical Journal*, **28**, 389, 1908.

> the production of H and K is more complex, but the furnace shows that they may be obtained if the temperature is sufficiently high . . . . without the aid of chemical action. . . . . They are to be rated as high-temperature lines by reason both of the temperature required for their production, and because their increase of strength in the furnace is closely proportional to the temperature—very much more so than in the case of λ 4227.

And again, in a later paper:[1]

> The H and K lines show a very definite behavior when compared with representative arc lines, such as the group near λ 4300. . . . . These arc lines are of fair strength at a temperature just high enough for H and K to become visible. As the temperature rises, H and K increase in intensity (though without decided widening) much faster than do the arc lines, surpassing at the higher furnace temperatures the strongest lines of the λ 4300 group. If this rate of increase is maintained at higher temperatures, the strength of H and K in the arc may easily follow.

If the few hundred degrees between furnace and arc temperatures are sufficient to account for the great change in the intensities of the H and K lines in the two sources, an increase in the temperature of the calcium vapor at the K_2 level of a few hundred degrees would easily account for the higher radiating power of the vapor producing K_2 provided a similar rate of increase in intensity with temperature holds under solar conditions; but here, as in all extra-terrestrial conditions, our only basis for reasoning is the extension of terrestrial laws.

A theoretical value for the specific heat of calcium vapor may be obtained from the following considerations. Calcium is considered to be a monatomic element for which the ratio between the specific heat at constant pressure to the specific heat at constant volume is 1.67. The external work done when a gram-molecule of any gas is raised in temperature one degree centigrade under a constant pressure of one atmosphere is about 2 calories. In the case of a monatomic gas for which the ratio is 1.67 the total heat required to raise the temperature of a gram-molecule one degree is therefore 5 calories. The molecular weight of calcium being the same as its atomic weight, namely 40, its specific heat in a gaseous state would be 0.125. Assuming this value of the specific heat of calcium vapor under solar conditions, the 578 calories of heat resulting from the loss of mechanical energy would raise the temperature of the two unit masses of calcium vapor 2300° C. Other matter, however, is also present, notably hydrogen, to share in the distribution of the energy, and the assumed changes of velocity are probably only partly realized. But with liberal deductions on these grounds, there might still be sufficient heat available to raise the temperature a few hundred degrees. The observed velocity upward may be a resultant velocity, in which case the original upward velocity would be 5.08 km per second, and the loss of mechanical energy fourfold that assumed above.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 38; *Astrophysical Journal*, **29**, 381, 1909.

The fact noted by Hale and Ellerman of the increased radiating power of the calcium vapor in passing from the K_1 to the K_2 level, even in the case of eruptive phenomena, would follow if the temperature of the upper K_2 level were raised and its radiation increased by the meeting of the opposing ascending and descending currents at that relatively high level. The K_2 emission line is too narrow to admit of its being produced by a layer of highly radiating vapor reaching down to the photosphere, where it would be subjected to great pressure. Under such conditions it would have at many points a width equal to that of K_1, a width to which it does not even approximate. Over faculae and around spots, where such conditions must obtain if anywhere, it does not ordinarily reach a width of 0.75 Å, and its edges are well defined, while Hale and Ellerman, in the case of H_1 and K_1, were able to set the second slit of the spectroheliograph 5 and 6 Å from the center of the lines in photographing the low-level calcium and still obtain the characteristic distribution of the low-level vapor showing a practically effective width of 10 to 12 Å for these lines. The radiating power of the calcium vapor appears to increase progressively with increase of height above the K_1 level and to reach its maximum at a considerable elevation, and then to decrease rapidly with further increase in height. The facts point to a cause residing in the region of increased radiation which would be supplied by the transformation of mechanical energy suggested above.

In stating that the vapor producing the K_3 absorption line is descending, it must be understood that the statement refers to the average or more frequently recurring condition. Differences in the measurement of individual lines and of different parts of the same line occur which are greater than the errors of observation and indicate wide variation of the velocities.

If the explanation of the high emissive power of the vapor producing K_2 suggested in this paper represents the true condition of the vapors, there must be a transition layer where the ascending and descending currents meet and intermingle, sometimes one prevailing and sometimes the other, though for the emitting vapor the result is a preponderating upward movement, while for the absorbing vapor the result is a downward movement on the whole. Such must be the condition of the opposing currents, since the upper level is continually reinforced from below, and in general as much vapor reaches the higher levels as descends from them. The generality of the descending motion of the absorbing vapor is at first surprising in view of the constant supply from the lower level. The more intense portion of the absorption lines, however, may be thought of as produced by the cooler vapors descending from the higher level. The upper levels of the intermediate radiating layer expand by decrease of pressure, cool, and continue to rise with a lower and ever-decreasing velocity. As long as their radiating power is comparable with that of the strata to which H_2 and K_2 are mainly due, their decreasing velocities would tend to shift toward the red the center of gravity of the H_2 and K_2 lines due to the more rapidly rising and more highly radiating vapor below. This shift toward the red of the center of the emission lines would be masked by the central absorption line. In the transition state between effectiveness as an emitting and as an absorbing body these layers would not manifest themselves strongly. After their absorption becomes effective and the movement is only slowly upward, the effect would be to strengthen or widen on the violet side and weaken or narrow on the red side the absorption lines due to the cooler and more effective descending vapor, and more or less completely overcome the shift toward the longer wave-length, or at least change the character of the K_3 lines. The average width of K_3 at the center of the sun is 0.150 Å and the average shift is 0.015 Å, so that within the borders of the absorption line resultant effects might be produced which would show as changes of intensity and displacements of the center of gravity, such as are frequently observed.

The slight astigmatism of the grating may have been an advantage in smoothing out irregularities of the lines due to variations of velocity in closely contiguous regions, so causing the preponderating velocities to be more generally manifest. The same result is favored by the fact that a length of 4 mm of the line was in the field of view of the microscope and the setting of the cross-hair was doubtless somewhat influenced by the whole portion of the line in view. It was the practice in obtaining the plates for Tables II–IV to make three exposures on the center, changing the position of the sun's image upon the slit for each exposure. The one most suitable for measurement was selected; and occasionally two were measured, and not infrequently measurements were made at two or three points of the K_3 line on the plates of the limb and center, of which the means only are given in the tables. For these reasons the measurements for K_3 represent less completely the variations in velocity from point to point than the resultant or preponderating velocity. The K_3 absorption line is then the integrated effect due to masses of calcium vapor differing in absorptive power and in velocity, in which the influence of the cooler descending masses is most pronounced. In regions where this absorption is effective, the intermingling of upward and downward moving masses of vapor must result in great turbulence over the general surface; but the data of this investigation show no evidence of the localization of definite regions where upward currents prevail.

In the case of the vapor producing the emission line K_2, the movement is one of ascent over the general disk, though its generality may be more apparent than real. It may rise through the photosphere at definite but closely adjacent points and expand laterally while ascending and so cover the surrounding region. There are indications that this may be the true condition in the wavy outer borders of the emission line and the consequent beaded form of the line, which is sometimes very apparent. Careful comparison of the spectrum of the K line taken with a long slit and a calcium spectroheliogram, made in close proximity of time and with solar images of the same diameter, reveals complete coincidence between slightly widened and intensified portions of the line and the minute calcium flocculi. This is in harmony with the working hypothesis adopted by Hale and Ellerman in which they say:[1] "It is considered that these minute flocculi are columns of calcium vapor, rising above the columns of condensed vapors of which the photospheric 'grains' are the summits."

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[1] *Publications of the Yerkes Observatory*, Vol. III, Part I, p. 15.

In the measurement and reduction of the plates the bright components of the K line have been considered as part of the same line of which approximately the central portion is cut out by the absorption line K_3. The satisfactory way in which the points fall on the curve shows that this point of view is without doubt the correct one. It is difficult to reconcile it, however, with the observations of Belopolsky, who says:[1]

> Bemerkenswert ist, dass die Componenten der *Ca*-Linien oft ganz von einander unabhängige, verschiedene Geschwindigkeiten (Verschiebungen) zeigen. Ich besitze Spectrogramme vom Jahre 1906, auf welchen die Linien H_1 und H_2 und K_1 und K_2 sich ausserhalb des Sonnenrandes kreuzen, ohne irgendwo zusammenzufliessen.

The writer understands that the subscripts 1 and 2 refer to the red and violet components, respectively, of the H and K lines. Such behavior of the bright components of H and K would be very remarkable and indeed inexplicable upon any current conception. Confirmation of such anomalous effects would be desirable. In the plates taken by the writer with a radial slit across prominences, the appearance has been quite different. The bright components form a broad line projecting a short distance beyond the limb, where in general they narrow rapidly into a less intense, single, fine line extending to the photographic height of the prominence. Less often the line shows a reversal. It is frequently wavy and distorted by the motion of the vapor in the line of sight and sometimes discontinuous, but otherwise entirely normal in appearance.

To determine whether the up-rush of the calcium occurs at all points of the solar surface or whether it is more or less localized, that is, limited to the granulations, a larger solar image is necessary, on which there will be a sufficient separation of the granulations. Such an image 425 mm in diameter will be produced by the new tower telescope of 150 feet (45.7 m) focal length now under construction, and with it the question will be taken up again. On such an image it will be possible, it is hoped, to separate the effects over the photospheric grains from the effects over the interspaces.

Owing to the fact that this investigation has been mainly devoted to the general chromospheric calcium, that is, to the regions outside of faculae and spots, the bearing of the results upon the interpretation of spectroheliograms taken with the second slit inclosing the H_2 or K_2 emission line is important in respect to the less conspicuous and less frequently considered features of the disk. Such a calcium spectroheliogram presents over the general surface a mottled appearance in which the relative dark background is very extensive compared to the small, scattered, and irregularly shaped bright regions. The question arises as to what the background is due. The bright regions have always been attributed to H_2 or K_2. An inspection of hundreds of spectra taken at the center, limb, and intermediate points of the disk, with the H and K lines and with a long slit, shows that the bright emission line is present at every point of the disk, though at times quite inconspicuous over the central regions. It varies greatly in width and intensity, with occasional marked increases of intensity and breadth over short distances. From the universality of its presence the general background of the calcium spectroheliogram may as confidently be assigned to it as are the brilliant flocculi.

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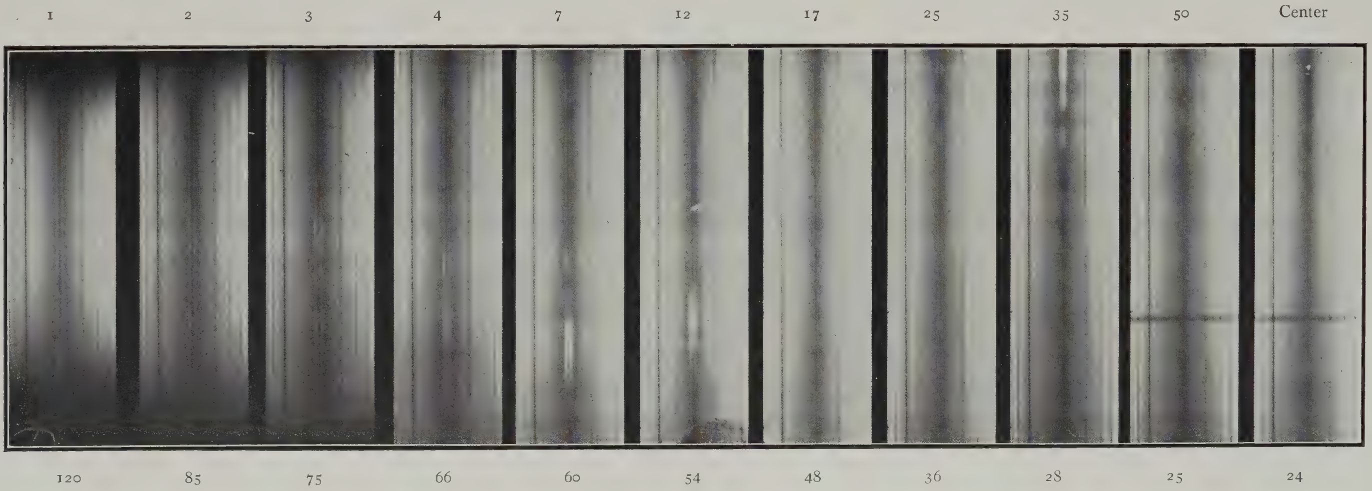
[1] *Mitteilungen der Nikolai-Hauptsternwarte zu Pulkowo*, 1, 160, 1906.

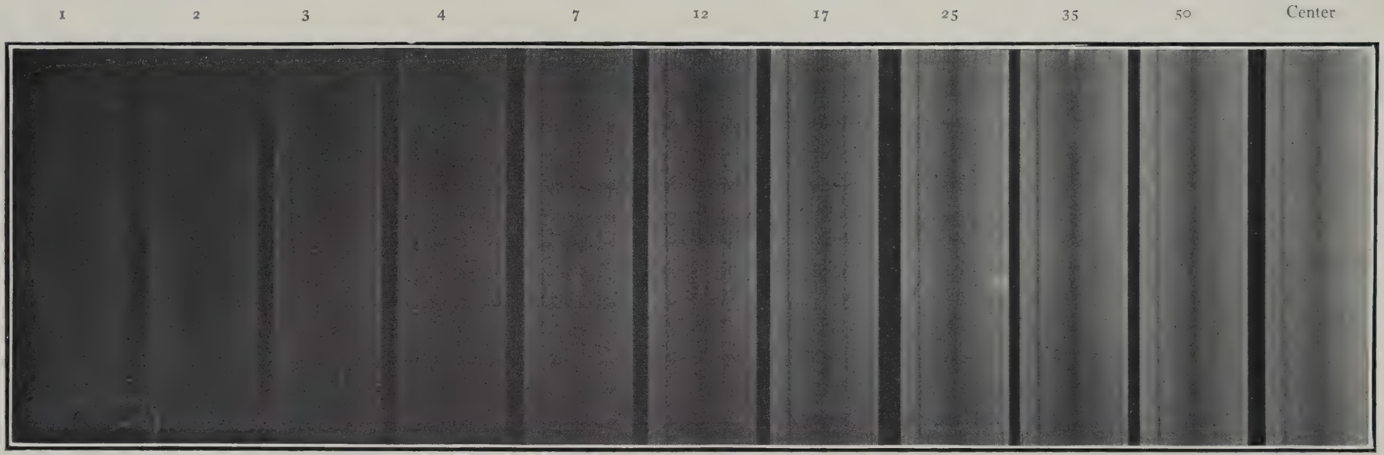
In the study of such spectroheliograms it should be borne in mind that they are taken with equal exposure times for limb and center. If it were possible to give graduated exposure times, such that the continuous spectrum of the background would be of the same intensity, the spectroheliogram would present a very different appearance. With such exposure times the bright K_2 line for a considerable distance from the limb is as broad and intense as at the center over the faculae, and a spectroheliogram corresponding to such condition would show a border of very brilliant calcium extending some distance inward from the limb, in contrast to the weak and inconspicuous peripheral regions usually shown on such. For equal exposure times the bright K line near the limb is reduced to a lower intensity than at the center, so that notwithstanding the inherent greater intensity of the emission components due to the increasing depth of the radiating layer on approaching the limb, the usual spectroheliogram shows in the peripheral region the same general appearance as over the central portions of the disk. This perhaps is an advantage from some points of view, but for a complete interpretation it is necessary to keep in mind how the actual distribution in the solar atmosphere of the calcium vapor producing the emission lines H_2 and K_2 would manifest

PLATE IX



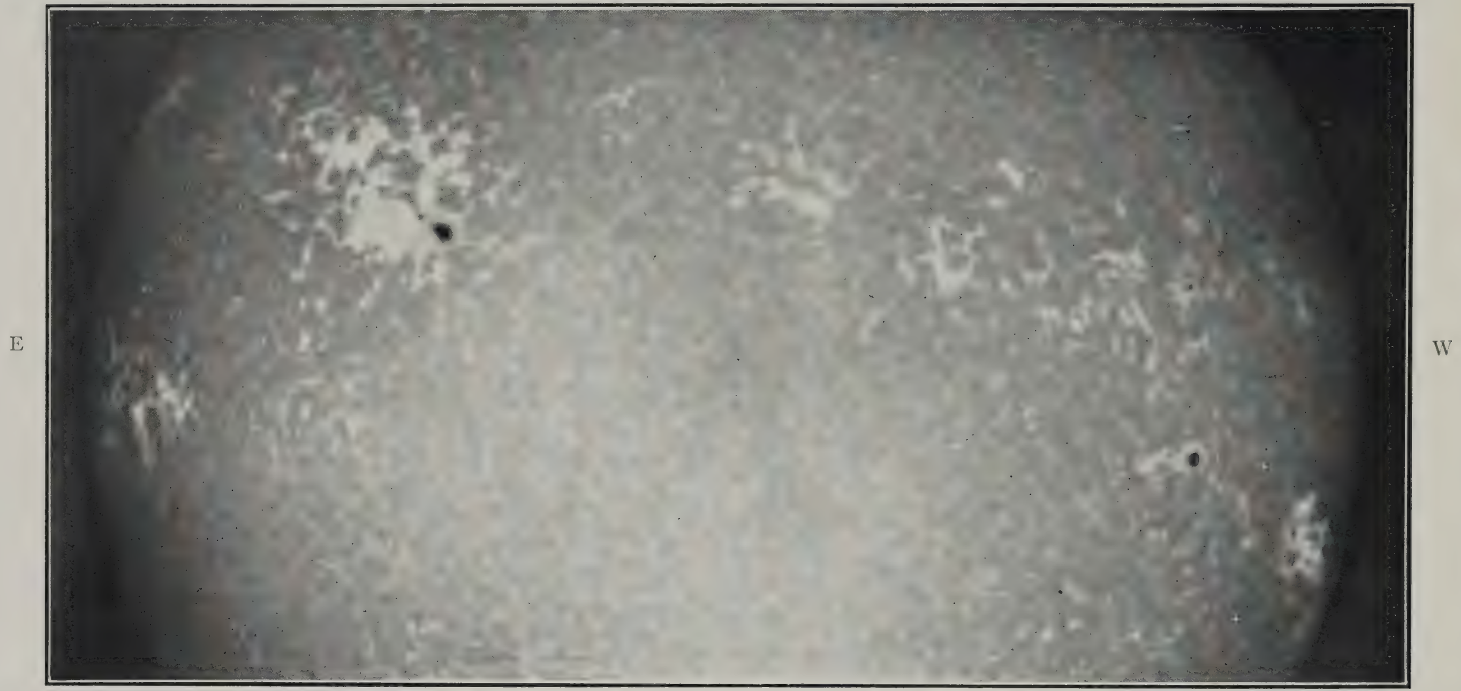
The K Line of Calcium Exposed to Obtain Uniform Intensities. 1909, November 3. Distances from East Limb in Millimeters Above Exposure Times in Seconds Below. Enlargement 1.4

PLATE X



The K Line of Calcium with Constant Exposure Time of 24 Seconds. 1909, November 3. Distance from East Limb in Millimeters Above
Enlargement 1.4

PLATE XI



H_2 Spectroheliogram of 1909, November 3. Scale of Original Negative

itself under photographic conditions allowing intensities more nearly proportional to the effective thickness of the radiating layer.

Plate IX is a reproduction of a series of exposures from the east limb to the center, timed for nearly uniform intensity. The distances from the limb in millimeters are given above, and the exposure times in seconds below. The progressive widening of the K_2 and K_3 lines in passing from the center to the limb can plainly be seen on the reproduction, and, less distinctly, the increasing dissymmetry of the emission components on passing from the limb to the center. The enlargement is 1.4. Plate X is a reproduction on the same scale of a series of equal exposures at the same points, and Plate XI one, in original size, of the corresponding section from the H_2 spectroheliogram of the same date, so that the positions at which the spectra are taken can be identified and a direct comparison made between corresponding points. The reproduction of the spectroheliogram has been made without any attempt to change the contrast at the limb, and represents more nearly the effects that follow equal exposure times than do the customary reproductions.

GENERAL SUMMARY

1. The calcium vapor producing the absorption line K_3 in the solar spectrum has a descending motion over the general surface of the sun of 1.14 km per second in the mean, as indicated by the progressive increase in the wave-length of the absorption line in passing from the limb to the center of the sun, where the shift toward longer wave-lengths amounts to 0.015 Å. This increase in wave-length corresponds with a high degree of exactness to the calculated line-of-sight component of a radial motion deduced from the observed displacements of 0.015 Å at the center. The agreement is so close that it is not probable that causes other than the downward movement are involved in the mean result. Other causes, if they exist, are occasional and diverse in their action, and their effects are eliminated from the final mean.

2. The calcium vapor to which the bright emission line K_2 is due has an ascending motion over the general surface of the sun of 1.97 km per second in the mean, as indicated by the progressive shortening of the wave-lengths of the emission line on passing from

the limb to the center of the sun, where it reaches a maximum of 0.026 Å. This decrease in the wave-length agrees with the line-of-sight projection of a radial motion of ascent deduced from the displacement of 0.026 Å at the center with such a degree of exactness that, in view of the difficulties involved in the measurements, causes of decrease other than the radial upward movement of the vapor are practically without effect upon the final mean. Such causes, if acting, must be casual.

3. The angular velocity of the high-level calcium vapor producing the absorption line K_3 is nearly constant for the latitudes of observation, being 15°5 and 15°4 per day at latitudes 6°6 and 38°4, respectively. The corresponding values deduced from Adams' results are 15°1 and 14°3 for hydrogen and 14°4 and 13°2 for the reversing layer. The high velocity of the calcium vapor producing the K_3 line points to a higher elevation of this layer of calcium vapor than of the hydrogen effective in the production of the *Ha* line.

4. The wave-lengths of K_2 and K_3 reduced to the limb are 3933.667 and 3933.665⁵ respectively. The corresponding wave-length in the arc at atmospheric pressure is 3933.667. The mean pressure in the intermediate emitting layer is therefore approximately one atmosphere, but in view of the small pressure-shift as observed by Humphreys and Mohler, the shorter wave-length of the K_3 line may be interpreted as indicating a somewhat lower pressure in the upper absorbing layer, though the smallness of the quantities involved does not permit a positive conclusion.

5. The mean wave-lengths of K_2 and K_3 at points 12 mm from the limb in equatorial regions are 3933.650 and 3933.675, respectively, and at points 12 mm from the limb in polar regions 3933.649 and 3933.676, respectively. The practical agreement for the two positions shows in the case of the intermediate and highest levels of calcium vapor the absence of currents of appreciable velocity parallel to the solar surface.

6. The widths of H_3 and K_3 at the center are 0.128 Å and 0.150 Å, respectively; their widths at 1 mm from the limb are 0.306 Å and 0.345 Å. The widths of these lines at the center compared to the corresponding widths in the arc point to an extremely small quantity of the calcium vapor in the upper levels of the solar atmosphere.

7. The ratio of the widths of K_3 to H_3 at the limb is 1.13; the ratio at the center is 1.17. Experiments with calcium in the arc show that their ratio increases with decrease in the density of the vapor and with weakening of the lines. The assumption of a thin layer of low density with consequent great increase in effective depth on approaching the limb accounts for both the absolute and relative changes in width. The ratio of the width of K_2 to H_2 at the limb is 1.07; the ratio at the center obtained from a small number of intensified lines is 1.09. The low ratio and the small change in the ratio between the limb and center both indicate a relatively dense and thick layer of the emitting vapor as compared with the absorbing layer, since the ratio is the nearer unity the stronger the lines, and the increase of the ratio is marked when lines already weak are still more decreased in intensity.

8. The average appreciable height of the chromospheric calcium shown by a radial slit is about 5000 km above the photosphere. The thickness of the upper absorbing layer is approximately 1500 km. Allowing 700 km for the reversing layer, the emitting layer would have a thickness of approximately 3000 km. The elevation at which the K line is appreciable is about 500 to 600 km above the level at which the H line ceases to show.

9. The wave-length of the H_3 line from 20 plates is 3968.491, 3968.478, and 3968.476, at the center, 1 mm from the limb, and at the limb, respectively. The shift between limb and center is 0.015 Å for the H_3 line, and in agreement with that obtained from the K_3 line.

10. The difference between the wave-lengths of H_3 and K_3 found in this investigation is 34.810 Å. The mean difference between H and K in the arc is 34.809 Å. The difference between H and K in Rowland's tables is 34.800 Å. The indicated error of 0.010 Å in the relative wave-lengths of H and K is in accord with that found by Adams.

11. When the wave-lengths given in Rowland's tables are used for the standards in reducing the measurements upon which the results given in this paper for H_3 and K_3 depend, the wave-lengths of H_3 and K_3 are λ_R 3968.634 and λ_R 3933.824, respectively; the corresponding wave-lengths from Rowland's tables are 3968.625 and 3933.825. The discrepancy occurs in the measurement of the

H line, as would be expected from the poorly defined character of the line in general.

12. A cause for the high radiating power of the emitting layer may be found in its increased temperature resulting from the transformation into heat of the mechanical energy set free by the loss of velocity in the opposing upward and downward currents.

13. The justification for the assumption of a high level, small depth, and extreme tenuity for the absorbing layer is found in the measurements with the radial slit, in the low pressure suggested by the slightly lessened wave-length of the K_3 line, in the narrowness of the absorption lines as compared with their widths in terrestrial sources, in the continued increase in absolute width on passing from the center to the limb, in the increase of the relative width of K_3 to H_3 on passing from the limb to the center, and in the high and nearly constant angular velocity. None of the facts is in disagreement with any part of the assumption and each supports one or more components of the assumption.

Mount Wilson Solar Observatory
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ON THE VELOCITY OF THE SUN'S MOTION THROUGH SPACE AS DERIVED FROM THE RADIAL VELOCITY OF *ORION* STARS

By J. C. KAPTEYN and EDWIN B. FROST

In what follows v will stand for the sun's linear velocity in its motion through space; λ will denote the angular distance of a star from the apex; ρ, the measured radial velocity; ρ_0, that velocity freed from the sun's motion. The ordinary method of determining v by means of the radial velocities of stars is founded on the supposition that the directions of the peculiar motion of the stars are distributed at random. On this assumption each star will furnish an equation of condition of the form

$$v \cos \lambda = \rho \,, \qquad (1)$$

from all of which the most probable value of v must be found.

This procedure may evidently still be retained if, instead of supposing a random distribution of the directions, we admit a preference for two opposite directions, provided that, for every part of the sky, we assume that the numbers of stars having opposite velocities are equal. Accidental deviations from this equality will not affect the final result.

Now, as the peculiar velocities, which thus play the part of accidental errors, are particularly small for the *Orion* stars, these objects must offer particular advantages for the determination of the sun's motion. It is further evident that the stars near the apex and antapex have by far the greatest weight in this determination. And further, (*a*) the results to be derived from them are much less dependent on the error in the assumed position of the apex than are those for which λ differs more widely from 0° or 180°. (*b*) By taking nearly equal numbers of stars near these two points, any constant error in the measures is completely eliminated.

To these considerations may be added the following one in favor of the *Orion* stars: (*c*) The result to be obtained for the sun's velocity will be the velocity in respect to the center of gravity of the stars used in its derivation. It seems desirable for this reason to choose stars

RADIAL VELOCITIES OF *ORION* STARS

Star	Mag.	α 1900	δ 1900	Spectrum	100 μ	ψ	100 τ	λ	Sources	ρ obs.	ρ comp.	$\rho_0 =$ O.−C.
		h m	° ′		″	°	″	°		km	km	km
γ *Pegasi*	2.87	0 8	+14 38	B 2	1.1	157	−0.7	85	Y. II	+ 7.0	− 2.0	+ 9.0
ζ *Cassiopeiae*	3.72	0 31	+53 21	B 2	3.3	102	+0.7	70	Y. II	+ 4.5	− 8.0	+12.5
π *Andromedae*	4.44	0 32	+28 32	B 3	2.3	113	+0.3	82	S.B. Allegh. I, 136	+11	− 3.2	+14.2
ϵ *Cassiopeiae*	3.44	1 47	+63 11	B 3	3.8	113	+0.8	74	Y. II	− 5.9	− 6.4	+ 0.5
2 Hev. Camelop.	4.42	3 21	+59 36	B 9	2.0	87	+1.7	84	Y. unp.	− 4	− 2.4	− 1.6
3 Hev. Camelop.	4.76	3 22	+58 32	B 9	1.4	354	+0.6	85	Y. unp.	− 4	− 2.0	− 2.0
40 Persei	5.04	3 36	+33 39	B 2	1.5	110	+0.9	107	Y. unp.	+ 3	+ 6.8	− 3.8
o Persei	3.94	3 38	+31 59	B 2	3.0	195	−2.2	109	S.B. Ap. J., **17**, 216	− 2	+ 7.6	− 9.6
30 e Tauri	5.1	3 43	+10 50	B 3	3.3	157	−0.9	127	Y. unp.	+17	+14.0	+ 3.0
ζ *Persei*	2.91	3 48	+31 35	B 1	2.0	136	+0.5	110	Y. II	+24.0	+ 8.0	+16.0
λ *Tauri*	3.4–4.2	3 55	+12 12	B 3	1.5	203	−1.3	127	S.B. Allegh. I, 136	+10	+14.0	− 4.0
μ *Tauri*	4.32	4 10	+ 8 39	B 3:	3.4	135	+0.7	133	Y. unp.	+27	+15.9	+11.1
d Persei	4.89	4 14	+46 16	B 3	4.3	151	+0.4	99	Y. unp.	+ 4	+ 3.6	+ 0.4
9 Camelop.	4.38	4 44	+66 10	B	2.4	77	+2.4	82	S.B. Y. unp.	+ 8	− 3.2	+11.2
π^4 *Orionis*	3.78	4 46	+ 5 26	B 3	0.9	159	−0.1	140	S.B. Allegh. I, No. 15	+23.3	+17.8	+ 5.5
π^5 *Orionis*	3.87	4 49	+ 2 17	B 3	0.6	26	+0.5	143	S.B. Y. unp.	+18	+18.6	− 0.6
ψ *Eridani*	4.81	4 57	− 7 19	B 8	1.8	313	−0.6	152	Y. unp.	+22	+20.6	+ 1.4
η *Aurigae*	3.28	5 0	+41 6	B 3	8.4	148	+2.8	107	Y. unp.	+ 6	+ 6.8	− 0.8
β *Orionis*	0.34	5 10	− 8 19	B 8	0.1	98	+0.1	155	S.B. Ap. J., **30**, 31	+22.6	+21.1	+ 1.5
η *Orionis*	3.44	5 19	− 2 29	B 1	0.3	168	0.0	150	S.B. Ap. J. **17**, 68	+35.5	+20.2	+15.3
β *Tauri*	1.78	5 20	+28 31	B 8	18.0	170	0.0	120	Y. unp.	+16	+11.7	+ 4.3
ψ *Orionis*	4.66	5 22	+ 3 2	B 2	1.2	149	+0.3	145	S.B. Ap. J. **28**, 266	+12.0	+19.1	− 7.1
δ *Orionis*	2.48	5 27	− 0 23	B	0.2	187	−0.1	149	S.B. Ap. J. **19**, 268	+23.1	+20.0	+ 3.1
λ *Orionis*	3.66	5 30	+ 9 52	Oe 5	3.1	182	−0.6	139	Y. unp.	+36	+17.6	+18.4
ι *Orionis*	2.87	5 31	− 5 59	Oe 5	0.3	141	+0.1	154	S.B. Ap. J. **30**, 373	+21.5	+20.9	+ 0.6
ζ *Tauri*	3.00	5 32	+21 5	B 3	3.3	166	+0.4	128	S.B. Ap. J., **22**, 115	+16.4	+14.4	+ 2.0
ζ *Orionis*	2.05	5 36	− 2 0	B	1.6	152	+0.5	151	Y. II	+18.3	+20.4	− 2.1
B.D. −1°1004	5.00	5 36	− 1 11	B 3	1.4	236	−1.3	150	S.B. Ap. J., **30**, 373	+26.1	+20.2	+ 5.9
κ *Orionis*	2.20	5 43	− 9 42	B	0.3	158	+0.1	158	Y. II	+18.7	+21.6	− 2.9
χ^2 *Orionis*	4.71	5 58	+20 8	B 2	2.1	95	+2.1	129	Y. unp.	+20	+14.7	+ 5.3

ε Canis Majoris	1.63	6 55	−28 50	B 1	0.3	326	−0.3	168	Y. II	+29.1	+22.8	+ 6.3
κ Cancri	5.14	9 2	+11 4	B 8	2.2	235	−0.4	119	S.B. Ap. J., **25**, 315	+26.3	+11.3	+15.0
a Carinae	3.56	9 8	−58 33	B 3	4.9	262	+2.2	138	S.B. Lick. B. IV, 153	+23.3	+17.3	+ 6.0
κ Velorum	2.63	9 19	−54 35	B 3	3.4	236	+2.5	138	S.B. Lick. B. IV, 155	+21.9	+17.3	+ 4.6
α Leonis	1.34	10 3	+12 27	B 8	24.6	270	−15.3	107	Potsdam VII	− 9:	+ 6.8	−15.8
α Virginis	1.21	13 20	−10 38	B 2	5.3	232	+0.3	78	S.B. Allegh. I, No. 10	+ 1.6	− 4.8	+ 6.4
η Ursae Majoris	1.91	13 44	+49 49	B 3	11.6	259	+1.6	50	Potsdam VII	−26:	−15.0	−11.0
β Librae	2.74	15 12	− 9 1	B 8	10.1	256	−5.5	56	Potsdam VII	−10:	−13.0	+ 3.0
τ Herculis	3.91	16 17	+46 33	B 5	2.9	2	−2.6	25	Y. II	−12.7	−21.1	+ 8.4
ζ Draconis	3.22	17 8	+65 50	B 5	2.2	324	+0.6	36	Y. II	−14.4	−18.8	+ 4.4
68 u Herculis	4.6–5.4	17 14	+33 12	B 3	2.6	259	+1.0	10	S.B. Allegh. I, No. 11	−21.2	−23.0	+ 1.8
ι Herculis	3.79	17 37	+46 4	B 3	0.5	46	−0.4	16	Y. II	−15.4	−22.4	+ 7.0
67 Ophiuchi	3.92	17 56	+ 2 56	B 5	1.7	138	+1.2	28	Y. II	− 1.8	−20.6	+18.8
102 Herculis	4.32	18 5	+20 48	B 2	2.2	159	+0.5	10	Y. II	− 8.8	−23.0	+14.2
μ Sagittarii	4.01	18 8	−21 5	B 8	0.6	255	−0.6	52	S.B. Ap. J., **26**, 157	− 7	−14.4	+ 7.4
β Lyrae	3.4–4.5	18 46	+33 15	B 2*p*	0.8	150	−0.8	10	S.B. Ap. J., **6**, 328	−14.8	−23.0	+ 8.2
2 Cygni	4.86	19 20	+29 26	B 2	2.0	60	+1.3	18	Y. unp.	−18	−22.2	+ 4.2
8 Cygni	4.85	19 28	+34 14	B 3	1.0	22	+0.9	19	Y. unp.	−22	−22.0	0.0
ι Aquilae	4.28	19 32	− 1 31	B 5	0.9	122	+0.4	39	Y. unp.	−27	−18.1	− 8.9
B.D. + 25°.4165	4.82	20 11	+25 17	B 3	0.4	236	−0.3	30	Y. unp.	−10	−20.2	+10.2
α Pavonis	2.12	20 18	−57 3	B 3	9.2	180	−4.4	93	S.B. Lick B. IV, 154	+ 2.0	+ 1.2	+ 0.8
ω′ Cygni	4.89	20 27	+48 37	B 3	1.3	60	+0.2	33	Y. unp.	−32	−19.5	−12.5
ε Delphini	3.98	20 28	+10 58	B 5	2.7	157	−1.4	40	Y. II	−26.2	−17.8	− 8.4
β Cephei	3.32	21 27	+70 7	B 1	1.4	67	0.0	49	S.B. Ap. J., **24**, 259	− 5	−15.3	+10.3
π¹ Cygni	4.78	21 39	+50 44	B 3	0.9	238	−0.4	45	Y. unp.	− 7	−16.5	+ 9.5
π² Cygni	4.26	21 43	+48 51	B 3	0.9	96	−0.3	45	Y. unp.	−20	−16.5	− 3.5
2 Lacertae	4.66	22 17	+46 2	B 5	1.7	156	−1.5	52	S.B. Allegh. I, No. 13	− 9.0	−14.4	+ 5.4
4 Lacertae	4.64	22 20	+48 58	B 9	3.1	212	−2.8	51	Y. unp.	−22	−14.7	− 7.3
10 Lacertae	4.91	22 35	+38 32	Oe 5	1.8	131	−0.8	56	Y. unp.	− 9	−13.0	+ 4.0
1 Cassiopeiae	4.93	23 2	+58 53	B	0.6	129	−0.3	57	Y. unp.	− 8	−12.7	+ 4.7
σ Cassiopeiae	4.93	23 54	+55 12	B 2	0.8	173	−0.7	65	Y. unp.	− 4	− 9.9	+ 5.9

at considerable distance from the solar system, which shall represent a much greater part of the whole of the stellar system than if we choose stars within a relatively small sphere surrounding the sun. Attention has been already called to the fact that the stars near to our system give an exceptionally low value for the sun's motion.[1] That the *Orion* stars are as a rule at great distances from the sun will appear below, even from the scanty material discussed in the present paper.

Considerations like these led one of us to devote some particular attention to the determination of the radial velocities of the *Orion* stars within moderate distance from the apex and antapex. The measures obtained, together with those collected from other sources, are given in the following list. We have included (1) the stars for which the measures as yet obtained do not show variability of motion; it is likely that a longer series of observations will show that a few of these are really binaries; (2) all the binaries for which the velocity of the center of mass of the system has been determined.

The magnitudes were taken from *Harvard Annals*, **50**; the spectra from *Harvard Annals*, **28**[2] and **56**, or if contained in neither of these, from Vol. **50**.

The tenth column contains the source from which the radial velocities ρ were taken: *Y. II* refers to the 20 *Orion* stars observed by Frost and Adams in the *Publications of the Yerkes Observatory*, **2**. The stars which have since been found to be binary have been excluded. The velocities have been slightly corrected by the introduction of improved values for the wave-lengths of the silicon lines. *Y. unp.* refers to unpublished observations at the same observatory. Every result is based on at least two plates, but in nearly every case on three or more spectrograms. *Ap. J.* stands for *Astrophysical Journal*. The other abbreviations will need no explanation.

The λ were computed by assuming for the position of the apex

$$a=269\overset{\circ}{.}7 \qquad \delta=+30\overset{\circ}{.}8 \qquad (1875.0).$$

Nearly all of the centennial proper motions, 100 μ, and their angle of position, ψ, had been computed before the publication of Boss's

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[1] J. C. Kapteyn, "Star Streaming," *Report of the British Association for the Advancement of Science*, 1905.

[2] The notations given in Vol. **28**, Part I, were transformed to the more usual ones by means of the table of Vol. **28**, Part II, 145.

Preliminary General Catalogue. For them were used the data given in Newcomb's *Fundamental Catalogue*, or, if not contained in that work, either those of Auwers-Bradley, according to *Groningen Publications*, No. 9, or those of Groombridge's *Catalogue*, according to the reduction by Dyson and Thackeray. For the five stars π *Andromedae*, λ *Tauri*, *B.D.* $-1^\circ 1004$, β *Lyrae*, *B.D.* $+25^\circ 4165$, the proper motions of Boss have served. $100\,\tau$ represents the centennial proper motions at right angles to the great circle star—apex. The values ρ comp. were found by introducing our final value (6) for v in equation (1). The last column gives the residuals. It is evident that these residuals represent the *peculiar* radial velocities ρ_0.

The solution of our equations of condition leads to the value

$$v=-23.16\pm 1.06 \text{ km per sec., from 61 stars.} \tag{2}$$

If we compare this result with that recently brought out by Hough and Halm (*Monthly Notices*, **70**, 100, 1909), viz.:

$$v=-20.85\pm 0.95, \text{ from 496 stars,} \tag{3}$$

the advantage of the present choice of stars in regard to accidental error is well shown. With one-eighth the number of stars, we get a probable error only slightly larger. That our value for v is somewhat in excess of (3) is not surprising, for in the case of Hough and Halm a great many stars relatively near to the solar system have been included, which, as mentioned above, seem really to lead to a somewhat smaller value of the sun's velocity, i.e., they seem to participate to some extent in the motion of the sun through space.[1]

Meanwhile our numbers bring out a somewhat unexpected fact, namely that the velocity of the sun relative to the stars near the apex is found to be very different from that relative to the stars near the antapex. To show this more clearly, a separate solution was made for the stars for which $\lambda < 90^\circ$ and for which $\lambda > 90^\circ$. We thus find:

$$\left.\begin{array}{ll} \text{Near apex} & v=-18.38\pm 1.40 \text{ km, from 32 stars} \\ \text{Near antapex} & \underline{v=-28.38\pm 1.36 \text{ km}}\text{, from 29 stars} \\ \text{Simple mean} & v=-23.38 \text{ km, from 61 stars} \end{array}\right\} \tag{4}$$

The difference is very considerable and cannot well be attributed to accidental error alone. It is confirmed, moreover, by the stars contained in the paper by Hough and Halm. To show this, we simply take the average radial velocity of all the stars within two hours of right ascension and 30° in declination from the apex and within an equal area around the antapex. We find

---

[1] It will be understood that we fully recognize the necessity of establishing this result on more extensive data than are now generally available.

$$\left.\begin{array}{l}\text{average radial velocity near apex} \quad = 16.6 \text{ km from } 31 \text{ stars} \\ \text{average radial velocity near antapex} = 24.7 \text{ km from } 71 \text{ stars}\end{array}\right\} \quad (5)$$

the difference of which is in the same sense and of the same order of magnitude. How is it to be explained? A constant error, depending on instrumental and personal influences and on errors in the assumed wave-lengths of the lines both in the star spectrum and in the comparison spectrum such that the positive velocities would result too great, might of course explain the difference.

The average velocity of all the Hough-Halm stars is +2.2 km and might possibly be interpreted as such an error.[1] This would explain more than half of the difference of the values (5). The rest might be attributed to accidental error. Assuming the same error for the *Orion* stars, there would remain in the values (4) a difference of 5.6 ± 1.3 km. It is hard to believe that even this residual amount is wholly attributable to accidental error, so that we should have to assume for the *Orion* stars a somewhat greater constant error, which then would have to be attributed to the diffuse nature of the lines. As the lines are symmetrical, however, their diffuseness ought not, in the long run, to introduce a constant error.

The amount of the error to be assumed is thus so much higher than seems well admissible that we must look for other possible explanations. The most plausible would seem to be that either the stars near the apex, or those near the antapex, or both, belong in unequal numbers to the two great star-streams. If we call Stream I the stream moving *toward* the true vertex $\alpha = 94°$, $\delta = +10°$, and Stream II that moving toward $\alpha = 274°$, $\delta = -10°$; then the assumption that the stars of our list near the antapex belong preferentially to Stream I, or that the stars near the apex belong for the greater part to Stream II, may explain the difference. Such differences in the mixture of the stars have been brought out by Eddington and more recently and more clearly by Hough and Halm. Our result, however, is *not* explained by the formula of the last-named astronomers, for that gives an identical mixture for any two points of the sphere diametrically opposite.

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[1] Küstner (*Astrophysical Journal*, **27**, 324, 1908) finds a systematic error in his measures of 1.1 km in the sense here required. Frost and Adams, *Publications of the Yerkes Observatory*, **2**, 18–37, find their systematic error to be practically zero.

As long as we have no further data, it will be safest to assume for the sun's velocity, as derived from the radial velocity of the *Orion* stars, the simple mean of the two values (4), in which mean any constant error of measurement is completely eliminated. We may thus adopt

$$v = -23.3 \text{ km per sec.} \tag{6}$$

as the definitive result of the present paper. It is with this value that the computed ρ of the above table have been calculated. We refrain from giving any probable error, because evidently such a quantity, derived in the usual way, must be misleading.

From the present materials we may derive two further results:

1. The average amount $\bar{\rho}_0$ of the radial velocities freed from the sun's motion, all taken positively. Adopting the value (6) for the whole sky, we get

$$\bar{\rho} = 6.7 \text{ km per sec.}$$

Adopting the separate values (4) for the two hemispheres,

$$\bar{\rho}_0 = 5.9 \text{ km per sec.}$$

We cannot be far wrong, therefore, if we adopt definitively

$$\bar{\rho}_0 = 6.3 \text{ km per sec.} \tag{7}$$

In Kapteyn's recent paper in this *Journal* (**31**, 247, 1910) the value $\bar{\rho}_0 = 6.5$ km was adopted. The difference is due to the exclusion of some stars recently discovered to be really binaries, to the introduction of a couple of new stars, and to further improvements in the values of the measured velocities.

2. A rough estimate may be made of the average parallax of the stars discussed in this paper.

The average value of 100 τ, disregarding signs, for the stars of our list, is

$$1\overset{''}{.}23.$$

The velocities τ, being independent of the sun's motion and representing projections on a line, are perfectly comparable to the quantities $\bar{\rho}_0$. It follows that from a distance corresponding to the average parallax π of our stars, a velocity of 6.3 km per sec. (7) shows itself, as seen without foreshortening from the sun, as an angular motion of $1\overset{''}{.}23$ per century.

This implies the value:[1]

$$\pi = 0\overset{''}{.}00924$$

for the average parallax of the 61 stars in our list. The average magnitude of these stars is 3.72.[2] Hence we infer, by the aid of the formula of *Groningen Publications*, No. 8, p. 24, that for the *Orion* stars of the magnitude 5.0 the parallax will be:

$$\pi_{(5.0)} = 0\overset{''}{.}0064. \qquad (8)$$

In a paper by Kapteyn, following this article, the same parallax is derived from the parallactic motion of 440 *Orion* stars, with the result:

$$\pi_{(5.0)} = 0\overset{''}{.}0068. \qquad (9)$$

The agreement of these values, obtained by wholly independent methods, is astonishingly close, in fact far better than we had a right to expect.

GRONINGEN. WILLIAMS BAY
May 1910

NOTE.—Since the above was in type there has come to our notice the interesting paper by Paul Stroobant (*Bullétin de la Classe des Sciences, Académie Royale de Belgique*, 1910, **1**, 39–51) entitled "Nouvelle détermination de la vitesse du système solaire dans l'espace." Adopting Newcomb's co-ordinates of the apex, and employing 64 stars within 41°5 of the apex or antapex, M. Stroobant derives 19.4 km per second as the value of the solar velocity. He also treats the 49 stars near the apex separately from the 15 near the antapex, finding values respectively of 18.75 and 21.55 km per second. From the separate consideration of *Orion* stars, he obtains 15.9 km per second as the solar velocity from 9 stars near the apex, and 26.9 km from 13 stars near the antapex, with a mean of 22.5 km. It will be seen that his results, although based upon only one-third as many stars as ours, are in close agreement as to the difference of 10 km in the solar velocity from stars near the apex and from stars near the antapex; and the mean value is in as good accord with ours as could be expected.

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[1] According to the formula: $\pi = 0.0473 \dfrac{\text{average value of } 100\,\tau}{\text{average value of } \rho_0}$. More rigorously we should have $\pi = 0.0473 \times \text{average value of} \left(\dfrac{100\,\tau}{\rho_0}\right)$. In this confessedly rough estimate we took the first formula, which dispenses with any elaborate system of weighting.

[2] For the variables the greatest light was adopted.

ON THE AVERAGE PARALLAX OF THE STARS OF THE FOURTH TYPE AS COMPARED WITH THAT OF STARS OF OTHER TYPES

By J. C. KAPTEYN

1. The question as to the absorption of light in space naturally led me to look for classes of stars the distance of which is exceptionally large. The fact that up to the present time no star of the fourth or fifth type is known to have a sensible proper motion makes it not improbable that these classes must have very small parallaxes.

Meanwhile, owing to the faintness of nearly all of the stars of both classes, the data for proper motion are so scanty that in reality they prove very little.

For many of the stars of the *fourth* type we have older observations: for all of them which are of magnitude 9.0 or brighter the *Astronomische Gesellschaft Zone Catalogue* contains positions, which, in the majority of cases, were obtained more than thirty years ago. Better data might be obtained, therefore, by reobserving all the stars of this type of which suitable older observations, particularly in the *A.G. Zone Catalogue*, are available.

Professor E. F. van de Sande Bakhuysen, with whom I discussed the matter, offered at once to have these observations made at the Leyden Observatory. It is a great pleasure to me to express my deep feeling of gratitude for this act of kindness.

The stars of both the fourth and of the fifth types which promised useful results had been already put on the program of the observatory, when I learned that the observation of the fourth-type stars had actually been undertaken at the Observatory of Copenhagen. I addressed myself at once to Professor Strömgren, the director of the observatory. Both he and Mr. Nörlund, who made the observations, kindly offered to hasten the observations and reductions and to place the results for proper motion, based on all the available material, at my disposal as soon as finished—an offer which of course was gratefully accepted. It is only through this truly courteous and powerful help that the result given in this paper could be obtained with so little delay. In fact, the possibility of any reliable conclusion about the average parallax of the fourth-type stars is entirely due to the labors of Mr. Nörlund.

2. *Stars of the fourth type.*—Nörlund's list, as it was put in my hands, contains 140 stars with data for proper motion in both co-ordinates. From these I have excluded (*a*) the stars not in the *A.G.C.*, at least if there exist no considerably earlier observations; (*b*) stars in the *A.G.C.*, if the difference of epoch Copenhagen − *A.G.C.* is less than 24 years, and if no other early observations are available.

There thus remain 120 stars for which the proper motion must be entitled to some confidence. As Nörlund will no doubt soon publish the complete list, I will, with his permission, communicate only the averages for the parallactic motion, as given below. In the computation of this motion I adopted for the position of the apex the co-ordinates

$$\alpha = 17^{h}\ 58^{m}.8, \qquad \delta = +30^{\circ}.8 \qquad (1875.0). \tag{1}$$

If

v represents the proper motion away from the apex;
λ, the angular distance from star to apex;
h, the linear velocity of the sun;
ρ, the star's distance;

then we have the well-known formula (see, for instance, *Astronomische Nachrichten*, No. 3487, **146**, 99, 1898):

$$\text{average value of } \frac{h}{\rho} = \frac{\Sigma v \sin\lambda}{\Sigma \sin^2\lambda} = \text{mean secular parallax.} \tag{2}$$

The average value of the proper motion in right ascension, in arc of the great circle, was found to be +0″.0038.

If we consider this quantity as the effect of the magnitude-equation on the proper motions in right ascension, we get the values headed II in the following summary.

If we neglect the correction, we get the values I. The magnitudes have been taken from the *Bonner Durchmusterung*, but have been reduced to the Harvard scale by applying Seeliger's corrections (*Bayrische Akademie der Wissenschaften*, 2d class, Vol. **19**, Part III, 575). For the variable stars, I took the magnitude at maximum.

The agreement between the separate values has not been improved by the introduction of the correction for magnitude-equation. As was to be expected, the final results I and II differ very little. The difference would still have been much smaller had the distribution of the stars been somewhat more uniform. I have finally adopted the mean of the two values, with a probable error derived from the agreement of the values II.

TABLE I

FOURTH-TYPE STARS (N)

α	Mag. Harvard Scale	$\Sigma v \sin \lambda$		$\Sigma \sin^2 \lambda$	Number of Stars	$\frac{h}{\rho}$	
		I	II			I	II
$0^{h}.0$ to $3^{h}.0$....	8.58	−0″.046	−0″.082	11.10	12	−0″.0041	−0″.0074
3.0 to 6.0....	8.46	+ .206	+ .178	21.33	28	+ .0097	+ .0083
6.0 to 9.0....	8.16	+ .130	+ .158	13.20	25	+ .0098	+ .0120
9.0 to 12.0....	7.57	+ .044	+ .063	8.84	7	+ .0075	+ .0108
12.0 to 15.0....	7.77	+ .062	+ .082	5.18	6	+ .0120	+ .0158
15.0 to 18.0....	7.95	− .037	− .031	1.68	4	− .0221	− .0185
18.0 to 21.0....	8.03	− .099	− .180	8.59	24	− .0115	− .0210
21.0 to 0.0....	8.42	+ .022	− .022	9.72	14	+ .0023	− .0023
Totals.......	8.21	+0.282	+0.166	76.64	120	+0.0037	+0.0022

Adopted.. +0″.0030±0″.0029 (p.e.) (3)

TABLE II

Orion STARS (B)

α	Mag. Harvard	$\Sigma v \sin \lambda$	$\Sigma \sin^2 \lambda$	Number of Stars	$\frac{h}{\rho}$	
$0^{h}.0$ to $3^{h}.0$....	4.92	+1″.010	41.19	48	+0″.0246	
3.0 to 6.0....	4.63	+1.661	66.64	112	+ .0250	
6.0 to 9.0....	4.42	+0.268	14.08	42	+ .0190	
9.0 to 12.0....	4.59	+0.789	20.38	28	+ .0387	
12.0 to 15.0....	3.54	+1.706	28.15	31	+ .0606	
15.0 to 18.0....	4.25	+1.507	36.13	55	+ .0417	
18.0 to 21.0....	5.09	+0.675	26.54	71	+ .0254	
21.0 to 0.0....	4.98	+0.966	39.02	53	+ .0248	
Totals........	4.63	+8.582	272.13	440	+0.0315	±0.0020 (p.e.)*

* In deriving this p.e., I first reduced the separate results to the same magnitude.

3. *Orion stars (B) and stars of the third type (M)*.—For the sake of comparison, I have derived in the same way the average parallactic motion of the *Orion* stars and of those of the third type, contained in the catalogues of Miss Maury (*Harvard Annals*, **28**, Part I) and of Miss Cannon (*Harvard Annals*, **28**, Part II, and **56**, Part IV)

for which good proper motions are available[1] in Auwers-Bradley (according to *Groningen Publications*, No. 9), Newcomb's *Fundamental Catalogue*, Groombridge (Dyson-Thackeray), Auwers' *Southern Fundamental Catalogue*, and Hedrick (*Washington Observations*, **8**). As the distribution over the several hours is tolerable, no attempt was made to find a correction of the proper motions in right ascensions for magnitude error.

TABLE III

THIRD-TYPE STARS (M)

α	Mag. Harvard	$\Sigma v \sin \lambda$	$\Sigma \sin^2 \lambda$	Number of Stars	$\frac{h}{\rho}$	
$0^{h}0$ to $3^{h}0$....	3.99	+0".973	6.71	8	+0".0705	
3.0 to 6.0....	4.60	+0.180	9.19	17	+ .0196	
6.0 to 9.0....	4.90	+0.274	5.82	9	+ .0471	
9.0 to 12.0....	4.48	+0.193	7.35	9	+ .0263	
12.0 to 15.0....	4.53	+1.548	13.18	16	+ .1175	
15.0 to 18.0....	4.59	+0.305	4.79	10	+ .0637	
18.0 to 21.0....	4.86	+0.130	5.93	13	+ .0219	
21.0 to 0.0....	4.75	+0.434	15.35	19	+ .0283	
Totals........	4.62	+3 537	68.32	101	+0.0518	±0".0092 (p.e.)

4. *Stars of the first and second type and general summary.*—Finally, we have, from *Astronomische Nachrichten*, No. 3487, taking general averages:

	Mean Harvard Mag.	Number of Stars	$\frac{h}{\rho}$
Type I (A and a few B)........	5.26	1088	+0".0381
Type II (F, G, K).............	5.23	1036	+0.0882

In the following general summary, I have for the sake of convenience reduced the results for all the types, except those of the fourth, to what would have been found had we used stars of the average magnitude 5.00. The necessary reductions were easily obtained by means of the formula of *Groningen Publications*, No. 8, p. 24, which permits the computation of such small corrections with perfect safety. For the reduction of the secular parallaxes $\frac{h}{\rho}$ to ordinary annual parallaxes, I adopted for the sun's velocity 4.20 units per year.

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[1] Boss's *Preliminary Catalogue General of 6188 Stars* had not yet been published when our computations were made.

TABLE IV

General Summary

Spectrum	Mag. Harvard	$\frac{h}{\rho}$	P.E.	$\bar{\pi}$	P.E.	Number
B..........	5.00	+0″.0284	±0″.0018	0″.0068	±0″.0004	440
A*.........	5.00	+ .0411	(.0022)	.0098	(.0005)	1088
F, G, K.....	5.00	+ .0942	(.0040)	.0224	(.0010)	1036
M..........	5.00	+ .0465	.0082	.0111	.0020	101
N..........	8.21	+ .0030	.0029	.0007	.0007	120

* A small number of stars of type B are included.

The probable errors or the types A and F, G, and K, are mere estimates. In a former paper[1] we found, by a perfectly independent method, the value +0″.0064 for the average parallax of the *Orion* stars of magnitude 5.00. The agreement is extremely satisfactory.

A somewhat similar table to the above was given some years ago by Pannekoek (*Proceedings of the Academy of Science*, Amsterdam, **9**, 140). The results agree fairly well, considering the small number of stars used by Pannekoek. Fourth-type stars are of course not contained in Pannekoek's paper.

The most striking fact brought out by the present investigation is the extremely small value of the parallax found for these fourth-type stars. In fact, the final value for the parallactic motion does not exceed its probable error, so that it is hardly too much to say that as yet the motion of the sun through space is not at all indicated by the stars of this class of spectrum. The distance of a great part of these stars is undoubtedly such that, if we adopt the value for the selective absorption of space found in this *Journal* for November 1909 (**30**, 284), the quantity

photographic − visual magnitude

must amount for them to at least half a magnitude. It may be much more considerable.

No interpretation of the spectra of the fourth type will be quite satisfactory, therefore, if it neglects the consideration of the effect of the selective absorption of light in its passage through space.

Groningen
May 1910

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[1] Kapteyn and Frost, *Astrophysical Journal*, **32**, 83, 1910.

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SOLAR DISTURBANCES AND TERRESTRIAL TEMPERATURES

By W. J. HUMPHREYS

INTRODUCTION

The effort of this paper is to bring harmoniously together, in the sense of cause and effect, a number of solar and terrestrial phenomena, some of which are well known, while others, perhaps, are comparatively new and unfamiliar.

Since the sun is the source of practically all the radiant energy we receive, therefore any change in its surface or envelope that affects either the quantity or the quality of its radiation must, through the resulting modification of the energy received, also affect certain terrestrial phenomena, some of which are of vital importance.

Changes in the number and extent of its spots, flocculi, prominences, and coronal streamers, all constitute very obvious and probably closely connected disturbances of the sun; and many efforts have been made to determine what relation, if any, exists between these solar disturbances and such terrestrial phenomena as auroral displays, magnetic storms, temperature changes, rainfall, plant-growth, and, in fact, everything else that directly or indirectly can be connected with the sun.

That the magnitude and frequency of auroral displays and magnetic disturbances follow closely the corresponding changes in sunspots appears to be well established. Concerning similar relations, however, of other phenomena to spot-frequency, the conclusions of investigators are less uniformly in accord, and perhaps only two at most of these, namely, temperature and, possibly, rainfall, have been measured with sufficient accuracy and recorded long enough and at enough places to justify a minute examination of their relations to solar activity.

A careful study of the variation of rainfall in Europe with sun-spot numbers has been made by Hellmann,[1] who concludes that there is a slight increase in precipitation during sun-spot minima. This agrees with, and doubtless depends upon, the corresponding temperature changes; and as these latter are the most accurately measurable they alone, therefore, of meteorological phenomena, will be considered in what follows.

SUN-SPOTS AND EARTH TEMPERATURES

Of the various studies of the relation of terrestrial temperatures to sun-spot numbers, one of the latest and best is by Abbot and Fowle.[2]

The stations whose temperatures were used in this investigation were selected in accordance with the conclusions of a careful theoretical study of the problem to be solved, and the results obtained may, therefore, be accepted as the best available. From this paper it appears that:

1. Spot maxima are accompanied by temperature minima and spot minima by temperature maxima.
2. The average range, from spot maxima to spot minima, in the mean annual temperatures, at the selected stations, is about 1° C.
3. The decrease in radiation from the spot area itself can account for only about $\frac{1}{25}$ of the observed temperature change.

The above small range of 1° C. in the average yearly temperature, it will readily be admitted, is full of theoretical interest, but that it is also of great practical importance may not be so generally realized. However, this too will at once be granted when it is recalled that the growth of many plants, other things being equal, is, through a range of several degrees, roughly proportional to $N\ (T-T_0)$, in which N is the number of days of growth, T the average temperature during this time, and T_0 the critical temperature for the plant in question, or that temperature below which it will not grow.

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[1] *Veröff. des Kön. Preuss. Meteorol. Instituts*, Berlin, 1909.

[2] *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **2**, 177–201, 1908.

An excellent illustration of this general law, and one emphasizing the great importance to plant-growth of even small changes in the average temperature, has been given by Walter,[1] who shows that a change of only 0°.7 C. may alter, and in Mauritius has actually been observed to alter, by as much as an entire year, the time required for the development of sugar cane.

THE SOLAR ATMOSPHERE AS AFFECTED BY SUN-SPOTS

Surrounding sun-spots, and doubtless because of their accompanying or preceding eruptions, are extensive areas of flocculi, or great clouds of metallic vapors. Besides, the streamers and other phenomena of the corona, so conspicuous during a total eclipse of the sun, are much more pronounced and extensive during periods of spot maxima than at times of spot minima. It is also known, from observations made during total eclipses of the sun, that part at least of the coronal light is polarized, and therefore we infer that some of it is reflected or scattered light.

Sun-spot maxima, then, appear to be accompanied by maxima in the amounts of "dust" (any particles that perceptibly reflect or scatter light) and clouds in the solar atmosphere, which, because of diffuse scattering, and, possibly, selective absorption, may be expected to reduce the amount of radiation of short wave-length that is finally emitted.

SOLAR RADIATION AS AFFECTED BY THE SUN'S ATMOSPHERE

In this connection it is important to note the fact[2] that the solar intensity decreases from center to rim of the sun, that is to say, that the intensity decreases as the portion of the line of sight within the "dust" layer increases; and, especially, that this falling off of the intensity is much *greater* for the *shorter* than it is for the longer wave-lengths. The effective radiating layer of the sun, therefore, becomes cooler and, presumably, reaches higher levels as the rim is approached —results that would follow the phenomenon of diffuse scattering. In addition to this Abbot and Fowle[3] have shown that the distribution of radiation in the solar spectrum can vary materially from time to time, especially in the region of short wave-length, even when there is but little if any change in the total intensity.

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[1] *On the Influence of Forests on Rainfall and the Probable Effect of "Déboisement" on Agriculture in Mauritius* (1908), 44–45.

[2] Hale and Adams, *Astrophysical Journal*, **25**, 300, 1907; Abbot and Fowle, *op. cit.*, Part 3, chap. i (1908); Adams, *Astrophysical Journal*, **31**, 30, 1910.

[3] *Astrophysical Journal*, **29**, 281, 1909.

On examining Wolfer's[1] curves of sun-spot numbers it is seen that the days on which Abbot and Fowle found the amount of violet radiation to be relatively small were days of many spots, while the one on which they found it relatively large was a day of exceedingly few spots.

These observations, made on only four days in all, are not sufficiently numerous to justify, of themselves, any conclusions as to the relation of this change in the intensity of the violet to the size and number of sun-spots. Nevertheless, they are sufficient to show that changes in the violet do take place; and it seems highly probable that a decrease in the violet, through scattering, would be caused by the more extensive corona that prevails during the time of many spots.

It would appear reasonable, too, to expect some change in the total radiation output of the sun to accompany a change in the conditions of its surface, but the connection of any such change with the sun-spot period is not yet established, nor do we know enough about the causes of solar disturbances to give us confidence in theoretical deductions in regard to the total output of solar energy at times of spot maxima and minima.

Any change in the solar constant, other things being equal, would, of course, lead to corresponding changes in the surface temperatures of the earth; though, owing to the fact that the radiation of the earth is proportional, approximately, to the fourth power of its absolute temperature, and to the further fact that most of its surface is covered with water, the proportional change of the temperature would be much less than that of the total radiation, and, in point of time, would lag behind it.

A change in the relative intensity of the violet seems definitely established, both for different places on the sun and for the sun as a whole at different times, and it seems quite probable that this intensity must be least at the time of maximum sun-spot disturbance, or when the solar "dust" is most extensive.

It therefore is worth while to trace the effect on the temperature of the earth of a change in the amount of violet and ultra-violet radiation, while the total energy output remains the same.

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[1] *Astronomische Mittheilung*, No. 98, 1907; No. 100, 1909.

PRODUCTION OF OZONE BY ULTRA-VIOLET LIGHT

During the past few years a number of persons have examined and reported the production of ozone by the action of ultra-violet light on dry oxygen. One of the latest to investigate this subject carefully is Van Aubel,[1] who concludes that there can be no question about the ozonizing action of radiation of short wave-length. It is also known that this action is best when the gas is dry and very cold—the conditions that obtain in the upper portions of the earth's atmosphere.

One would, therefore, expect to find an appreciable amount of ozone in the upper atmosphere, where the conditions are the same as those under which it is best produced in the laboratory, but, owing to its unstable nature and its great power of oxidation, very little near the earth; and this is confirmed by direct observations. Only traces of ozone are found in the lower portions of the air, and when there is much moisture and the temperature is high the amount is vanishingly small. On the other hand, by examining the absorption bands in the solar spectrum, Ångström[2] has detected the presence of what seems to be a very considerable amount of ozone, which of course must be in the upper air, since it is not in the lower.

Now, ozone, as Ladenburg and Lehmann[3] have shown, has a number of absorption bands scattered over a wide range of the spectrum, from well within the ultra-violet to far out in the ultra-red. There is a strong band of wave-length $0.3\ \mu$ and less, a moderate one in the neighborhood of $0.6\ \mu$, another strong one at $4.7\ \mu$, and, besides several of lesser intensity at different places in the ultra-red, a very strong band extending from about $8.5\ \mu$ to approximately $10.5\ \mu$, and a small one at $11.4\ \mu$. The most intense portion of the solar spectrum falls about half-way between the first two of these bands, the one at $0.3\ \mu$ and the other at $0.6\ \mu$, while the great absorption band, from $8.5\ \mu$ to $10.5\ \mu$, and the small ones at $7.6\ \mu$ and $11.4\ \mu$ belong to the region of strong earth-radiation. Presumably there are other ozone bands of still greater wave-length, but the region to which they would belong does not appear to have been examined in this connection.

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[1] *Comptes Rendus*, **150**, 96, 1910.

[2] *Arkiv för Matematik, Astronomi och Fysik*, **1**, 395, 1904.

[3] *Annalen der Physik*, **21**, 305, 1906.

From a consideration of the positions and intensities of the absorption bands of ozone, though they are known to only 12.5 μ, together

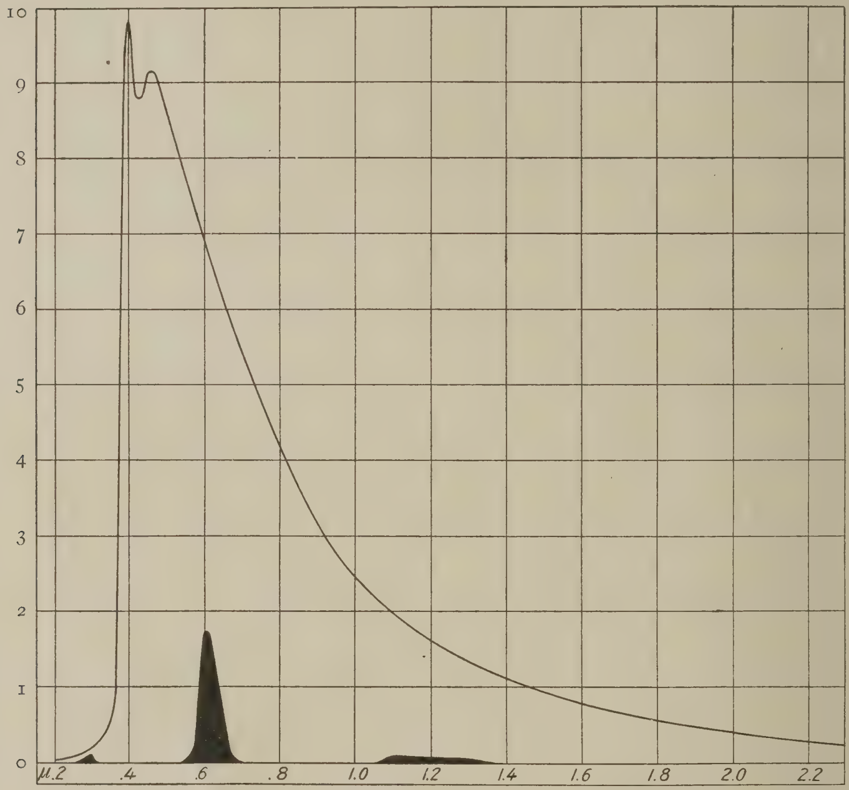

Fig. 1.—Ozone Absorption of Solar Radiation

with the distribution of energy in the solar and terrestrial spectra (assuming the earth to radiate as a black body), it is evident (1) that the coefficient of absorption, due to ozone, of solar radiation, is not the same as that of earth radiation, and (2) that the latter, probably, is greater, perhaps several fold greater, than the former.

These relations are graphically represented in Figs. 1 and 2, in which wave-length is plotted against intensity. Fig. 1 shows (*a*) the approximate spectral distribution of the solar energy outside the earth's atmosphere,[1] and (*b*) the portion of this energy, indicated by the dark areas, that would be absorbed by a certain quantity of ozone. Fig. 2 gives (*a*) the corresponding amount (only, because of loss by reflection, about two-thirds of the solar) and distribution of terrestrial radiation,[2] and (*b*) the fraction of this, in the region examined, that would be absorbed by the above assumed quantity of ozone.

These particular values of absorption were obtained by Ladenburg and Lehmann (*op. cit.*) with a tube of ozone one meter long, apparently at room temperatures and under a pressure of 200 to 300 mm. Unfortunately they did not extend their examination beyond 12.5 μ, but still this is far enough to show that ozone absorbs earth radiation to a much greater extent than solar, or in other words, that it lets solar energy in more readily than it lets earth energy out.

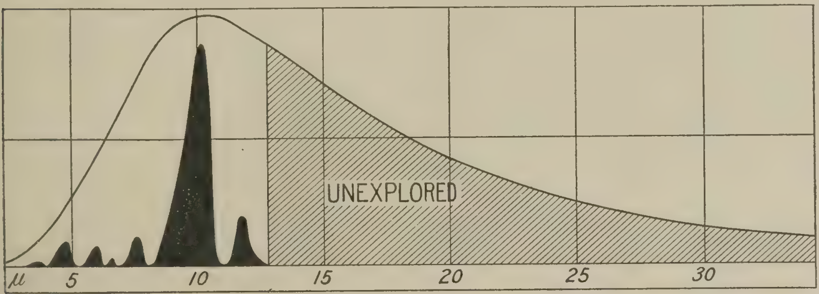

FIG. 2.—Ozone Absorption of Earth Radiation

Fig. 2, of course, does not accurately represent the average spectral distribution of the earth's planetary radiation, nor is this definitely known. However, since water vapor—always present in the atmosphere—absorbs and radiates at ordinary temperatures very much as a black body, it probably does approach the actual distribution near enough for the present purpose, that is, to give some idea of the greater absorptive power ozone has for terrestrial than for solar radiation.

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[1] This curve is taken, with a slight modification, from Plate XVI, Vol. II, *Annals of the Astrophysical Observatory of the Smithsonian Institution* (Abbot and Fowle). The ultra-violet is extended farther here than in the original curve, though without other justification than the fact that solar energy is known to extend beyond 0.3 μ.

[2] Really the radiation of a black body at the average temperature of the earth.

EFFECT OF A SELECTIVELY ABSORBING LAYER ON THE TEMPERATURE OF THE EARTH

The next step is to determine what effect a selectively absorbing layer, particularly an upper atmosphere rich in ozone, would have on the temperature of the earth.

The solution of this problem is complicated by the alternations between day and night; by the constant changes, during the day, in the sun's inclination; by reflection, especially from clouds, and doubtless by many other things that more or less modify the final result. However, since the coefficient of absorption is independent, so far as we know, of intensity of radiation, it is possible, by the use of average values, to obtain an approximate solution.

The average intensity of the normal component of the solar radiation over the entire earth is the total radiation flux in a solar beam, at the earth's distance from the sun, whose cross-section is equal to the area of a terrestrial great circle, divided by the area of the earth's surface.

This average intensity then is $\frac{I_0 \pi R^2}{4\pi R^2} = \frac{1}{4} I_0$, in which R is the effective radius of the earth as an intercepter of solar energy, and I_0 the intensity of the solar radiation at the outer limit of the atmosphere, or the amount of solar radiant energy received per second at that place per unit area at right angles to the path of radiation.

A portion of this radiation is reflected by clouds and other things and of course produces no change in the temperature of the earth. If we adopt 37 per cent, the value determined by Abbot and Fowle,[1] as the total albedo of the earth, or the reflected fraction of all incident radiation, and represent by I'_0 the average effective intensity of the normal component of the solar radiation over the entire earth, then

$$I'_0 = 0.16 I_0, \text{ nearly.}$$

Let aI'_0 = the amount of solar energy, both direct and reflected, absorbed by the outer atmosphere, per second per unit area. Approximately, and for convenience we will assume exactly, half this is radiated to space and half to the earth, including in this term the lower atmosphere.

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[1] *Annals of the Astrophysical Observatory of the Smithsonian Institution*, 2, 163, 1908.

The earth, which we shall follow step by step as it would warm up from an initially cold condition, now receives $(1-a)\ I'_0$ solar energy per unit time and unit area, and $\frac{a}{2}\ I'_0$ energy per same time and area from the absorbing outer layer, which, presumably, is roughly coincident with the isothermal layer and rich in ozone. These two together amount to $I'_0\left(1-\frac{a}{2}\right)$.

After a time the earth will send back an equally intense radiation of its own, but one in which the energy is very differently distributed as to wave-length. Therefore, when equilibrium with this radiation is established, $I'_0\left(1-\frac{a}{2}\right)$ is the intensity of the outgoing earth radiation also.

Let $bI'_0\left(1-\frac{a}{2}\right)$ be the part of this long-wave earth radiation absorbed by the outer layer. One-half of this, or $\frac{b}{2}I'_0\left(1-\frac{a}{2}\right)$, will be radiated back to the earth, there absorbed, and once more sent out. The next amount returned by the shell is $\left(\frac{b}{2}\right)^2 I'_0\left(1-\frac{a}{2}\right)$, and so on indefinitely.

The total intensity of radiation, therefore, reaching the earth, when completely warmed, is

$$I'_0\left\{1-\frac{a}{2}\right\}\left\{1+\frac{b}{2}+\left(\frac{b}{2}\right)^2+\ .\ .\ .\ .\ .\ +\left(\frac{b}{2}\right)^\infty\right\}.$$

Now b is either unity, the maximum value possible, or less, probably much less, and positive. Therefore the terms inclosed by the second pair of braces form a rapidly converging series, whose maximum value, when $b=1$, is 2, and whose minimum value, when $b=0$, is 1, but whose actual value probably is not much greater than unity. Calling this sum S, we have $I'_0\left(1-\frac{a}{2}\right)S$ as the average intensity of the entire radiant energy used in heating the earth.

Since the albedo of the earth is roughly one-third, we can say that the sum of the incident and reflected energy passing through the outer layer of the atmosphere is approximately $\frac{4}{3}$ that originally incident, so that $\frac{1}{4}aI'_0$ represents the absorption per unit time and area of reflected radiation. If there was no absorption of solar radiation in the outer layer then, of the additional $\frac{3}{4}aI'_0$ that would go on per second to each unit area of the earth below, one-third, or $\frac{1}{4}aI'_0$, would be reflected. Therefore, if the outer absorbing layer did not exist, the average intensity of the energy used in heating the earth would be $I'_0\left(1-\frac{a}{2}\right)$, while that which actually does exist is $I'_0\left(1-\frac{a}{2}\right)S$.

Let $b=0.20$, a value, as shown by Fig. 2, possibly of the correct order; then $S=1.11$, or the radiant energy actually received by the earth is about 11 per cent greater than it would be if there was no absorptive layer.

The average temperature of the earth is 14°4 C.[1] or 287°4 C. absolute. Therefore, since the percentage change in temperature is approximately one-fourth the percentage change in radiation, it follows, if the assumed value of b is correct, that the surface of the earth averages from 7° C. to 8° C., or say 13° F. warmer than it would be without the presence of the absorptive shell.

The amount of this difference in temperature depends, as we have seen, on the value of b, and it in turn upon the quantity of ozone or other absorptive material in the upper atmosphere. But ozone, as above explained, is produced by the action of ultra-violet radiation on cold dry oxygen. The amount of ozone in the outer atmosphere must, therefore, vary with the amount of ultra-violet radiation sent out by the sun, and this, it appears, is a variable quantity.

When this ultra-violet solar radiation is at a minimum, presumably during a sun-spot maximum, the amount of ozone in the upper layers of the atmosphere will be a minimum, unless maintained by some other process—see below—and the temperature of the earth will be correspondingly low. On the other hand, with a maximum of ultra-violet radiation, presumably during a sun-spot minimum, when the solar atmosphere is clearest, there will be a maximum amount of ozone, so far as this method of producing it is concerned, and a comparatively high temperature at the surface of the earth, even if the total radiation remains unchanged, which is improbable.

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[1] Hann, *Lehrbuch der Meteorologie*, 115.

Unfortunately the amount of change in ultra-violet radiation during spot cycles, and its relation thereto, if any, have not yet been determined, and hence it appears desirable at this point to sum up the foregoing and sharply distinguish between facts and assumptions.

The facts are:

1. The brightness of the sun drops off from center to limb.

2. This drop is greater the shorter the wave-length, and is due, almost certainly, to diffuse scattering.

3. The total amount of violet and ultra-violet in the solar radiation changes from time to time.

4. This change is greater than the change in the solar constant.

5. During sun-spot maxima the solar atmosphere is more extensive—dustier than it is during spot minima.

6. The temperature of the earth is greatest during sun-spot minima, and least during spot maxima.

7. Rainfall appears to follow the temperature curve, and therefore to be greatest during spot minima.

8. Ultra-violet light acting on cold dry oxygen converts some of it into ozone.

9. There appears to be a considerable amount of ozone in the atmosphere, which must be at great elevations since it is not found, except in traces, and could not long exist in the lower atmosphere.

10. Ozone is selectively absorptive both of solar and of terrestrial radiation, but more absorptive of the latter than of the former.

11. An upper atmosphere, rich in ozone, would influence the temperature of the earth—the temperature becoming higher, other things being equal, the greater the quantity of ozone.

The assumptions are:

a) That the solar constant is the same during sun-spot maxima that it is during spot minima.

This assumption probably is not correct, but the amount of the change, and even its sign, are unknown, and therefore it is assumed to have no cyclic change; the assumption being made for the purpose of seeing whether or not the known temperature changes can be accounted for, in any part, without invoking corresponding differences in the total intensity of solar radiation.

b) That the solar radiation is relatively weak in the shorter wavelengths during spot maxima. This appears all but necessarily true.

The conclusion is:

With the solar constant and other things, except, of necessity, the spectral distribution of the energy, unchanged, any proportionate decrease in the ultra-violet radiation, such as we believe takes place at the time of sun-spot maxima, leads to a diminished production of ozone in the upper atmosphere, an increase in its diathermacy, and a cooling off of the earth and the lower atmosphere.

PROBABLE AURORAL EFFECTS

So far only that production of ozone in the atmosphere has been considered which is due to the action of ultra-violet solar radiation.

It is known, however, that ozone is abundantly produced by the action of silent electric discharges, like those seen in a Geissler tube, on cool dry oxygen. Almost certainly, therefore, auroral displays, except those that exist above the oxygen level, and all other electrical discharges in the atmosphere must be accompanied by a greater or less production of ozone.

Auroral streamers invariably parallel, or nearly so, the earth's magnetic lines of force, and therefore, if of equal electric discharge throughout their paths, the density of this discharge must rapidly increase as either magnetic pole is approached, or, in general, with increase of latitude. Owing to the concentration of the lines of force and their nearer approach to the vertical as the distance from a magnetic pole is decreased, together with the latitude increase of the winds, there may be many electric discharges of the Foucault nature, in the atmosphere of high latitudes, that do not extend to equatorial regions. However this may be, it remains that auroral displays are seldom seen in the tropics, while they are of frequent—almost nightly—occurrence in high latitudes.

We would, therefore, expect that portion of the ozone which is due to auroral discharges to be most concentrated in the higher latitudes: (1), because of the increased density here of the discharges; and (2) because that while often probably above the oxygen level in equatorial regions (where of course no ozone can be formed) they come down, by following the lines of magnetic force, well within the oxygen, where the production of ozone is possible, as the magnetic poles are approached.

Another disturbing element in this distribution is the rather slow equator-to-pole drift of the upper atmosphere, indicated by sounding balloons, and by the spread to high latitudes of the dust thrown out by Krakatoa, which must tend to deplete the tropical regions of their supply of ozone and to concentrate it in those on either side.

These two actions then, the equator-to-pole circulation of the upper atmosphere, and the paralleling of magnetic lines of force by auroral discharges, might very well lead to a greater amount of ozone in the upper air of temperate and polar regions than in that of the tropics. A consequence of this would be a greater diathermacy of the upper atmosphere in equatorial regions than elsewhere, and therefore a comparatively low temperature and great altitude, at this place, of the isothermal layer; a condition that actually obtains, for the under surface of the isothermal layer is fully 5 kilometers higher and 15° C. colder over the equator than in middle latitudes.

Now a sun-spot maximum is accompanied by increased auroral activities, and, presumably, by a minimum of ultra-violet radiation, while a spot minimum on the contrary gives few auroral displays, but a maximum, probably, of radiation of short wave-length. Therefore the amount of ozone in the atmosphere, since it results from the action both of auroral discharges and of ultra-violet radiation that seem to increase and decrease oppositely, though not necessarily equally, tends to remain more or less fixed; and consequently it appears quite probable that the variations in the average temperatures of the earth through a spot cycle are less than they otherwise would be —that the increase of auroras with spot numbers tends to maintain equal average temperature conditions of the earth, by more or less completely offsetting the effects due to the simultaneous decrease in the ultra-violet radiation.

Finally, since changes in the quality of the solar radiation must be more effective within the tropics than elsewhere, while auroras are active almost entirely at higher latitudes, it follows that the temperature changes in the two regions must be unequal, possibly at times even in opposite directions. An increase of temperature in the first, at the time of a spot minimum, may be accompanied by a smaller increase, or at places even a decrease, in the second; while a decrease in the first, to be expected at the time of a spot maximum, may, because of auroras, be accompanied by a less decrease, conceivably even an increase, in the second.

However, since radiation is constant while auroras are fleeting and fitful, it seems probable that changes in the spectral distribution of the former rather than in the frequency of the latter may be the principal cause, through the resulting variations in the amount of ozone in the atmosphere, of the observed periodic swings in the average of terrestrial surface temperatures.

CONCLUSIONS

1. An increase in sun-spots appears certainly to be accompanied by a decrease in terrestrial temperatures, at least in many places, fully twenty fold that which can be accounted for by the decrease in radiation from the spot areas alone.

2. It seems nearly certain that sun-spot maxima, whatever the value at such times of the solar constant, must lead to a decrease in the ultra-violet radiation that reaches the earth, and a corresponding decrease in the production, by this method, of ozone in the upper atmosphere.

3. The increase in the auroral discharges that accompany spot maxima tend to increase the amount of ozone, especially in the higher latitudes.

4. The change in the temperature of the earth, and all its train of consequences, from spot maximum to spot minimum, is not necessarily dependent upon a change in the solar constant. It may depend largely, if not wholly, upon a change in the absorptive property of the atmosphere, caused, we believe, by a variation in the amount of ozone produced by ultra-violet radiation and by auroral discharges.

SUGGESTIONS

In addition to a careful determination of the solar constant and terrestrial temperatures during one or more spot cycles, it would be well to measure, at the same time, the accompanying changes in the ultra-violet portion of the radiation, and also to follow, over the same cycles, the temperature and height of the isothermal layer, and to note, if possible, the amount of ozone in the upper atmosphere.

The information here called for is difficult, though not impossible, to obtain; but much of it—it may be all—is essential, though perhaps not sufficient, to the solution of the complex problem concerning the relation of solar activities to terrestrial temperatures—a problem of great interest, both from the strictly scientific and from the purely utilitarian standpoints.

Mount Weather Observatory
Bluemont, Va.
April 1910

FORMULAE FOR THE SPECTRAL SERIES FOR THE ALKALI METALS AND HELIUM

By R. T. BIRGE

The recent extension of the principal series of the sodium spectrum to the forty-seventh member by R. W. Wood,[1] of the second subordinate series of sodium to the eleventh member by H. Zickendraht,[2] and of the principal series of potassium, rubidium, and caesium to the twenty-third, twenty-fourth, and twentieth members, respectively, by P. V. Bevan,[3] gives us a means of testing very accurately the various formulae proposed to represent spectral series. The author has found that the Kayser and Runge formula,[4] even with four undetermined coefficients, is quite inadequate. On the other hand, the Ritz[5] formula, with three undetermined coefficients and a universal constant, is quite adequate for substances with low atomic weights, but becomes less so, as we ascend from potassium to caesium, failing utterly in the case of the last-named substance.

In order to obtain the best possible results, all wave-lengths were first reduced to the new Fabry and Perot scale. This scale, founded on Michelson's[6] measurement of the red cadmium line, and remeasured by Benoit, Fabry, and Perot,[7] is undoubtedly more accurate than the Rowland values. The list of secondary standards, as determined by Fabry and Buisson,[8] extending from $\lambda=6495$ to $\lambda=2374$, gives us the material for a complete table of corrections, to be applied to the formerly accepted values. These corrections, when plotted, lie on a complicated curve, and cannot be wholly compensated for by adjustment of constants in the spectral formulae, especially when the actual wave-lengths are known very accurately, as in the case of helium.

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[1] *Astrophysical Journal*, **29**, 97, 1909.

[2] *Annalen der Physik*, **31**, 233, 1910.

[3] *Phil. Mag.*, **19**, 195, 1910.

[4] Kayser's *Handbuch der Spectroscopie*, **2**, 514.

[5] *Annalen der Physik*, **12**, 264, 1903.

[6] Michelson and Benoit, *Mémoires du bureau internationales des poids et mesures*, **11**, 1, 1895.

[7] *Comptes Rendus*, **144**, 1082, 1907.

[8] *Ibid.*, **144**, 1155, 1907.

Therefore the following table of corrections, expressed in Å, has been prepared. From $\lambda=2300$ to 4600, the comparison is between Fabry and Buisson's iron lines (terrestrial),[1] and Kayser's iron lines (terrestrial),[2] while between $\lambda=4600$ and 6500, the comparison is between Fabry and Perot's solar lines,[3] and Rowland's solar lines.

CORRECTIONS FOR REDUCTION TO FABRY AND PEROT SCALE

λ in Å	Correction Å	λ in Å	Correction Å	λ in Å	Correction Å	λ in Å	Correction Å
2400	−0.08	3300	−0.12	4100	−0.15	5300	−0.16
2500	−0.08	3400	−0.13	4200	−0.16	5400	−0.18
2600	−0.08	3450	−0.14	4300	−0.16	5500	−0.20
2700	−0.09	3500	−0.15	4400	−0.17	5600	−0.21
2800	−0.10	3600	−0.15	4600	−0.17	5800	−0.21
2900	−0.11	3700	−0.15	4800	−0.17	6000	−0.21
3000	−0.11	3800	−0.15	5000	−0.17	6200	−0.20
3100	−0.11	3900	−0.14	5100	−0.16	6400	−0.21
3200	−0.11	4000	−0.15	5200	−0.15	6600	−0.22

The above corrections reduce wave-lengths at 20° C., 760 mm on the old scale, to wave-lengths at 15° C., 760 mm on the Fabry and Perot scale. To correct to *vacuo*, the Kayser and Runge[4] values of the index of refraction of air, at 15°, 760 mm, obtained by interpolation from their original curve, were used.

This reduction to the Fabry and Perot scale necessitates the calculation of a new universal constant (N_0) for use in the Ritz formula. The recent remeasurement of hydrogen δ and ϵ by Evershed,[5] together with the Rowland determinations of hydrogen α, β, and γ, gives us five accurately determined lines, each of which gives a value of N_0 directly $\left[N_0=\frac{4\times 10^8}{h}\right.$, where h is the constant of the hydrogen formula $\left.\lambda=h\frac{m^2}{m^2-4}\right]$. The values of these five hydrogen lines in air, and *in vacuo*, with the h derived from each, and the difference between observed and calculated values, using the average h, are given below.

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[1] *Comptes Rendus*, **144**, 1155, 1907.

[2] *Annalen der Physik*, 3, 195, 1900.

[3] Fabry and Perot, *Astrophysical Journal*, **15**, 261, 1902.

[4] H. Kayser, *op. cit.*, Bd. **1**, 719.

[5] *Astrophysical Journal*, **28**, 162, 1908.

m	Letter	$\lambda_{(\mathrm{air})}$	$\lambda_{(vacuo)}$	h	$\lambda_{(\mathrm{obs.})}-\lambda_{(\mathrm{cal.})}$
3.	α	6563.045	6564.635	3647.019	−0.002 Å
4.	β	4861.527	4862.712	.035	+0.018
5.	γ	4340.634	4341.693	.022	+0.001
6.	δ	4101.900	4102.901	.023	+0.002
7.	ϵ	3970.212	3971.185	.007	−0.015
Average.				3647.021	

Then $N_o = \frac{4 \times 10^8}{h} = 109{,}678.6.$

The former Ritz value is 109,675.0. This new value gives, for hydrogen, exactly the same agreement between observed and calculated values as does the old.

In determining a formula to express the principal sodium series, it was immediately evident that the Kayser and Runge formula was unsatisfactory. In using their formula, Kayser and Runge worked with the wave-lengths in air, and Bevan,[1] in testing the complete sodium series by this formula, also used the air-values. Since the correction to be applied, in reducing to *vacuo*, varies from 1.63 Å to 0.73 Å, it might be thought that the error introduced by a failure so to reduce would be considerable. However, this was not found to be the case, and the conclusions reached by Bevan apply equally well, if the wave-lengths *in vacuo* are used, the differences between observed and calculated values reaching a maximum of 2.7 Å for the four-coefficient formula.

In calculating spectral series with any formula, certain lines are chosen for the determination of constants, as a least-square solution involves much labor. It is found, however, that the results vary considerably, according to the lines used, and the author, after much needless work, was led to the following general conclusions.

The lines for which n is large (near the head of the series) are fixed principally by the value of the convergence frequency (the first undetermined coefficient in all formulae). Small changes in the other constants do not affect these higher terms. The lower lines in the series, however, are greatly affected by changes in these other constants. Therefore, in a formula with m undetermined coefficients, use one line near the head of the series, and the first $m-1$ lines of the series, to determine the m coefficients. This rule will ordinarily give results most nearly approximating a least-square solution. In determining what line near the head of the series can best be used, the differences between observed wave-lengths of successive lines are calculated, then the differences of these differences—these latter being plotted and a smooth curve drawn through them. Any line near the head of the series which corresponds to a point lying *on* this curve will give a good average result. Moreover, the divergence of the points from this curve gives the general experimental errors, which no formula will correct, and gives therefore the minimum difference between observed and calculated values, to be obtained by the use of the correct formula, assuming that such a one exists. The Ritz formula, for all the elements tried, save only caesium, gave differences in exact coincidence with this minimum difference.

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[1] *Proc. Roy. Soc.*, A, **83**, 421, 1910.

The formula, as derived mathematically by Ritz, is an infinite series of unknown form. For the expansion Ritz used the semi-convergent development

$$\nu = A - \frac{N_0}{\left(n+a+\frac{b}{n^2}+\frac{c}{n^4}+\ldots\right)^2},$$

actually employing only the first three terms of the denominator. N_0 is the universal constant, A the convergence frequency of the series, and ν the frequency $\left(\frac{10^8}{\lambda}\right)$ of term n ($n=2, 3$, etc.). Ritz has given this formula in two different forms:

$$1.\quad \nu = A - \frac{N_0}{\left(n+a+\frac{b}{n^2}\right)^2}$$

$$2.\quad \nu = A - \frac{N_0}{\left(n+\alpha+\frac{\beta}{N_0}[A-\nu]\right)^2}.$$

This latter form can be expanded, by successive approximations, into

$$\nu = A - \frac{N_0}{\left(n+\alpha+\frac{\beta}{n^2}-\frac{2\alpha\beta}{n^3}+\frac{3\alpha^2\beta}{n^4}-\frac{4\alpha^3\beta-2\beta^2}{n^5}+\ldots\right)^2}.$$

It differs, however, from [1] only by infinitesimals of higher order. The first form has been used in the calculations, the results of which are given in the following tables. These tables also embody the differences between observed and calculated wave-lengths, recalculated with the published constants and *vacuo* frequencies of Ritz. The results vary considerably in some cases from those given in his papers, especially as regards the lower members of the series, and the discrepancy is only partly accounted for by the fact that the published constants are not given to a sufficient number of places.

The following tables give the results for the principal series (less refrangible member of the pairs) of the alkali metals and for the single-line principal series of helium.

TABLES

Column 1. Number of line in the series.
2. Wave-length in air— Ångström units.
3. Wave-length *in vacuo* (Fabry and Perot scale).
4. Estimated maximum experimental error.
5. Difference between observed and calculated wave-lengths, as published by Ritz.
6. The above difference, as recalculated by the author with the same constants.
7. The difference, computed with the new constants of the author, and with the Fabry and Perot wave-lengths of column 3.
8. The observer of the line.
* denotes the lines used in determination of constants.

TABLE I

LITHIUM

New constants $N_0=109{,}678.6$ (universal)
$A=43{,}482.115$
$a=-0.047423$
$b=+0.025801$

1 n	2 $\lambda_{(\text{air})}$	3 $\lambda_{(vacuo)}$	4 E	5 R (old)	6 R_2	7 $R'_{(\text{new})}$	8 Observer
2.......	6708.2	6709.83	0.2	±0.00*	+0.76	±0.00*	Kayser and Runge
3.......	3232.77	3233.59	0.03	±0.00*	+0.05	±0.00*	“ “
4.......	2741.39	2742.10	0.03	+0.07	+0.06	−0.07	“ “
5.......	2562.60	2563.27	0.03	±0.00*	−0.09	−0.01	“ “
6.......	2475.13	2475.78	0.1	+ .20	−0.11	−0.23	“ “
7.......	2425.55	2426.19	0.1	+0.01	−0.08	−0.19	“ “
8.......	2394.54	2395.17	0.2	+0.03	−0.03	−0.15	“ “
9.......	2373.9	2374.52	—	+0.10	+0.14	±0.00*	Liveing and Dewar
10.......	2359.4	2360.02	—	+0.22	+0.27	+0.15	“ “

TABLE II

SODIUM

New constants $A=41{,}450.083$
$a=+0.144335$
$b=-0.1130286$

1 n	2 λ (air)	3 λ (*vacuo*)	4 E	5 R (old)	6 R_2	7 R′(new)	8 Observer
2	[5896.16]	5897.563	0.00	±0.00*	+0.12	±0.00*	
3	3303.07	3303.895	0.03	+0.03	+0.04	±0.00*	Kayser and Runge
4	2852.91	2853.63	0.05	−0.10	−0.18	−0.04	“ “
5	2680.46	2681.17	0.10	−0.08	−0.07	+0.09	“ “
6	2593.98	2594.67	0.10	−0.13	−0.11	+0.09	“ “
7	2543.82	2544.49		−0.22	−0.22	−0.05	Wood
8	2512.15	2512.90		−0.15	−0.15	+0.07	“
9	2490.70	2491.36				−0.04	“
10	75.60	76.26				+0.03	“
11	64.53	65.18				+0.10	“
12	56.02	56.67				+0.05	“
13	49.46	50.11				+0.04	“
14	44.24	44.89				−0.03	“
15	40.06	40.71				+0.01	“
16	36.70	37.35				+0.06	“
17	33.85	34.50				+0.04	“
18	31.43	32.08				−0.01	“
19	29.42	30.07				−0.02	“
20	27.72	28.37				−0.01	“
21	26.28	26.93				+0.02	“
22	25.00	25.65				+0.03	“
23	23.88	24.53				+0.01	“
24	22.90	23.55				±0.00	“
25	22.04	22.69				±0.00*	“
26	21.29	21.93				+0.02	“
27	20.60	21.25				+0.01	“
28	20.02	20.67				+0.04	“
29	19.50	20.15				+0.07	“
30	19.09	19.65				+0.06	“
31	18.44	19.09				−0.05	“
32	18.09	18.74				±0.00	“
33	17.71	18.36				−0.01	“
34	17.38	18.03				+0.01	“
35	17.10	17.75				+0.02	“
36	16.80	17.45				±0.00	“
37	16.56	17.21				+0.03	“
38	16.33	16.98				+0.04	“
39	16.11	16.76				+0.03	“
40	15.89	16.54				+0.02	“
41	15.70	16.35				+0.03	“
42	15.52	16.17				+0.02	“
43	15.37	16.02				+0.03	“
44	15.21	15.86				+0.04	“
45	15.06	15.71				+0.03	“
46	14.94	15.59				+0.05	“
47	14.78	15.43				+0.01	“
48	14.64	15.29				−0.01	“
49	14.50	15.15				−0.04	“

Experimental data seem to indicate, in the case of the alkali metals especially, a relation between the principal series and the second subordinate (sharp) series, and both Rydberg[1] and Ritz have incorporated this connection in their formulae, Ritz writing his

TABLE III

POTASSIUM

New constants $A=35,006.51$
$a=+0.286917$
$b=-0.22083$

1 n	2 $\lambda_{(\mathrm{air})}$	3 $\lambda_{(vacuo)}$	4 E	5 $R_{(\mathrm{old})}$	6 R_2	7 $R'_{(\mathrm{new})}$	8 Observer
2.......	7699.30	7701.17	0.2	±0.00*		±0.00*	Average of all observations
3.......	4047.36	4048.36	0.03	±0.00*		+0.01*	Kayser and Runge
4.......	3447.49	3448.34	0.03	+0.13		+0.04	" "
5.......	3217.76	3218.58	0.03	±0.00*		−0.02	" "
6.......	3102.37	3103.15	0.10	+0.10		+0.08	" "
7.......	3034.94	3035.71	0.10	−0.20		−0.26	" "
8.......	2992.33	2993.09	0.15	−0.19		−0.19	" "
9.......	63.36	64.11	0.20	−0.13		−0.25	" "
10.......	42.8	43.54	1.00	−0.27		−0.27	" "
11.......	28.0	28.74				+0.07	Bevan
12.......	16.6	17.33				+0.16	"
13.......	07.6	08.33				+0.09	"
14.......	00.4	01.13				−0.02	"
15.......	2894.6	2895.33				−0.10	"
16.......	89.7	90.43				−0.32	"
17.......	85.9	86.63				−0.25	"
18.......	82.9	83.63				±0.00*	"
19.......	80.3	81.02				+0.15	"
20.......	77.9	78.62				+0.09	"
21.......	75.8	76.52				+0.01	"
22.......	74.1	74.82				+0.07	"
23.......	72.5	73.22				±0.00	"
24.......	71.1	71.82				−0.04	"
25.......	70.0	70.72				+0.05	"

$$\pm\nu=N_0\left\{\frac{1}{\left(n+a+\frac{b}{n^2}\right)^2}-\frac{1}{\left(m+a'+\frac{b'}{m^2}\right)^2}\right\}$$

$m=1.5$ $n=2, 3,$ etc. for principal series.
$n=2$ $m=1.5, 2.5,$ etc. for second subordinate series.

It is evident that the first term of each series is the same, except that ν is negative in one case, and positive in the other. In Ritz's formula the sign of ν has no significance, so that the two lines are really identical.

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[1] *K. Svenska Vetensk. Akad. Hand.*, 23, No. 11, 1890.

By comparison with the sodium formula already evaluated,

$$A \equiv \frac{N_0}{\left(1.5+a'+\frac{b'}{(1.5)^2}\right)^2}=41450.083$$

and, for the second subordinate series, using the derived values of N_0, a, and b,

$$\pm\nu=109{,}678.6\left\{\frac{1}{\left(2+.144335-\frac{.1130286}{2^2}\right)^2}-\frac{1}{\left(m+a'+\frac{b'}{m^2}\right)^2}\right\}.$$

Thus, of the three normally arbitrary conditions arising from three undetermined coefficients, only one remains arbitrary in the case of the second subordinate series. For the principal series' constants furnish us the value of A, and one relation between a' and b', namely,

TABLE IV

RUBIDIUM

New constants $A=33{,}689.07$
$a=+0.345595$
$b=-0.265769$

1 *n*	3 λ(*vacuo*)	4 E	5 R (old)	7 R′ (new)	8 Observer
2	7952.38	?	±0.00*	±0.00*	H. Lehmann
3	4216.75	0.03	±0.00*	±0.00*	Kayser and Runge
4	3592.62	0.05	±0.00*	+0.20	“ “
5	3351.87	0.05	−0.11	+0.16	“ “
6	3230.08			−0.01	Ramage
7	3159.00			−0.22	Bevan
8	3113.79			−0.23	“
9	3082.68			−0.65	“
10	61.08			−0.41	“
11	44.97			−0.40	“
12	32.87			−0.26	“
13	23.37			−0.23	“
14	15.86			−0.18	“
15	09.86			−0.08	“
16	04.66			−0.27	“
17	00.76			−0.03	“
18	2997.06			−0.26	“
19	94.16			−0.21	“
20	91.76			−0.09	“
21	89.56			−0.13	“
22	87.66			−0.14	“
23	86.16			±0.00	“
24	84.86			+0.14	“
25	83.85			+0.41	“
26	82.85			+0.53	“

TABLE V

CAESIUM

New constants $A=31,394.23$
$a=+0.411966$
$b=-0.33269$

1 *n*	2 λ (air)	4 E	5 R (old)	7 R′ (new)	8 Observer
2	8949.92	?	±0.00*	−0.02*	H. Lehmann
3	4593.34	0.05	±0.00*	±0.00*	Kayser and Runge
4	3888.83	0.1	±0.00*	−0.32	“ “
5	3617.08	0.3	−0.72	−1.12	“ “
6	3477.25			−4.00	“ “
7	3398.40			−2.80	Ramage
8	48.72			−1.37	“
9	13.5				Bevan
10	3288.9			−1.73	“
11	70.6			−1.72	“
12	57.1				“
13	45.9				“
14	37.6				“
15	30.9			−1.19	“
16	26.0				“
17	21.6				“
18	17.4			−0.27	“
19	14.3			±0.00*	“
20	11.8				“
21	09.6				“
22	08.0			+1.18	“

TABLE VI

HELIUM

New constants $A=32,033.20$
$a=+0.011131$
$b=-0.004767$

1 *n*	2 λ (air)	3 λ (*vacuo*)	4 E	5 R (old)	6 R_2	7 R′ (new)	8 Observer
2	20,400		?	−80.		−20.	F. Paschen
3	5015.732	5016.966	0.02	±0.000*	+0.010	±0.00*	Runge and Paschen
4	3964.875	3965.848	0.02	±0.000*		±0.00*	“ “
5	3613.785	3614.662	0.02	±0.000*	+0.003	+0.01	“ “
6	3447.734	3448.575	0.02	−0.001		+0.025	“ “
7	3354.667	3355.499	0.02	−0.038		+0.01	“ “
8	3296.900	3297.726	0.02	−0.032	−0.028	+0.03	“ “
9	3258.336	3259.152	0.02	−0.116		−0.04	“ “
10	3231.327	3232.139	0.02	−0.13	−0.11	−0.04	“ “
11	3211.626	3212.437	0.02	−0.12	−0.11	−0.04	“ “
12	3196.81	3197.62	0.02	−0.09	−0.10	−0.03	“ “
14	3176.6(?)			[+0.08]			“ “

that relation which will give a correct calculated value for the line $m=1.5$ (or $n=2$), which is common to both series. The infra-red line $\lambda=11{,}381.1$ ($m=2.5$) has been measured by Lewis with great accuracy. Therefore this line must be taken as the other determining one. The resulting coefficients are:

$$A=24{,}493.93$$
$$a'=+0.151022$$
$$b'=-0.054833.$$

In the following table, column *a* gives the differences between observed and calculated values, using these constants.

TABLE VII

SODIUM, SECOND SUBORDINATE SERIES

1 m	2 λ (air)	3 λ (*vacuo*)	4 E	*a*	*b*	8 Observer
1.5....	−[5896.16]	−5897.563		±0.00*	[−20.]	
2.5....	11,381.1	11,384.2	0.2	−0.16*	−0.04*	P. Lewis
3.5....	6154.62	6156.12	0.1	+0.67	+0.02*	Kayser and Runge
4.5....	5149.19	5150.47	0.1	+0.56	+0.19	“ “
5.5....	4748.36	4749.52	0.15	+0.45	+0.28	“ “
6.5....	4541.70	4542.80	0.20	+0.08	−0.03	Zickendraht
7.5....	4419.9	4420.96		+0.02	±0.00	“
8.5....	4341.4	4342.46		+0.20	+0.15	“
9.5....	4287.7	4288.75		−0.17	−0.17	“
10.5....	4249.5	4250.54		+0.11	±0.00*	“
11.5....	4220.5	4221.53		−0.41	−0.43	“
12.5....	4198.5	4199.52		−0.65	−0.75	“

Zickendraht estimates his experimental error as 0.05 Å at 4500, increasing to 0.3 Å at 4200. According to this, the differences are much greater than the experimental errors. But the wave-lengths, as determined by different observers, were measured under different conditions of excitation, and seem to vary considerably as a result. Thus for $m=6.5$, Kayser gives $\lambda=4542.75$ (error=0.2 Å) while Zickendraht finds $\lambda=4541.70$ (error=0.05 Å). Taking these facts into consideration, the formula seems to be fairly satisfactory.

Deriving a formula for this series, independent of the principal series, so that $m=1.5$ is not included, we get these new values for the constants

$$A=24{,}493.41$$
$$a=+0.150440$$
$$b=-0.050864.$$

The differences between observed and calculated values, using these new values, are given in column *b*, of Table VII. They are seen to be a slight improvement, but if $m=1.5$ had been included, there would have been absolutely no improvement. This is shown by the fact that the convergence frequency (A) calculated independently of the principal series ($A=24{,}493.41$) differs from the A furnished by the principal series ($A=24{,}493.93$) by only 5 units in the sixth place. Therefore the main relation between the principal series and second subordinate series (namely, that the difference between the convergence frequencies of these two series gives the frequency of the term common to the two series) is accurately true.

Now, as to the results for the principal series: The calculated differences, using the new constants, in the case of lithium, sodium, and potassium, lie practically within the estimated maximum errors given by Kayser and Runge. The experimental errors estimated by Wood and Bevan (.02 Å for sodium, 0.2 Å for potassium, and 0.3 to 0.4 Å for rubidium) consider only the accidental variations in measurement, and do not take into account constant errors of any kind, such as would be caused by varying conditions of temperature and pressure, finite width of the lines (when caused by *assymmetrical* broadening), etc. A consideration of these statements, together with the fact that the calculated differences vary quite irregularly over the positive and negative region, is sufficient to show that the calculated differences must have been caused by experimental errors, and that, therefore, the Ritz formula is satisfactory.

For helium the estimated maximum error is 0.02 Å, and here again the calculated differences have a maximum of twice this amount but it is instructive to note that, in all cases, the smaller the estimated error, the smaller are the maximum calculated differences, the two preserving an almost constant ratio. These conclusions do not hold at all in the case of caesium, where the differences have a maximum of 4 Å, with an estimated error of 0.4 Å. This discrepancy may be considered also from the following point of view.

The Ritz formula has theoretically the form of an infinite series. How many terms of the series need actually be used depends directly upon the size of the coefficients of the several terms. But these coefficients are found to increase with increasing atomic weight, so that higher terms of the theoretical series become less negligible, as the atomic weight increases. This increase in coefficients is shown in the following table:

TABLE VIII

Substance	Atomic Weight	a	b
Hydrogen	1	0	0
Helium	4	0.0111	0.0047
Lithium	7	0.047	0.026
Sodium	23	0.144	0.113
Potassium	39	0.287	0.221
Rubidium	85	0.345	0.266
Caesium	133	0.412	0.333

Moreover, in the case of *Na*, *K*, *Rb*, and *Cs*, the coefficients bear a definite relation to the atomic volumes, namely, that both a and b are directly proportional to the atomic volume. Hicks,[1] using the formula

$$\nu = A - \frac{N_0}{\left(m+a+\frac{b}{m}\right)^2},$$

found similar relations. These relations are shown below.

TABLE IX

Substance	a	b	b/a	Atomic Volume	$a/a.v.$	$b/a.v.$
Na	0.14433	−0.11302	0.784	23.606	613×10^{-5}	480×10^{-5}
K	0.28692	−0.22083	0.770	44.617	642 "	495 "
Rb	0.34559	−0.26577	0.768	56.05	627 "	473 "
Cs	0.41196	−0.33269	0.807	70.584	585 "	472 "

These results show far more than a chance coincidence, and indicate very clearly, as Hicks points out, that the coefficients have some relation to the physical properties of the atom. These relationships are an argument for continuing the use of the Ritz formula, increasing the number of coefficients, if necessary, rather than seeking for an entirely new type of formula, as several authors have done, since Ritz first published his results.

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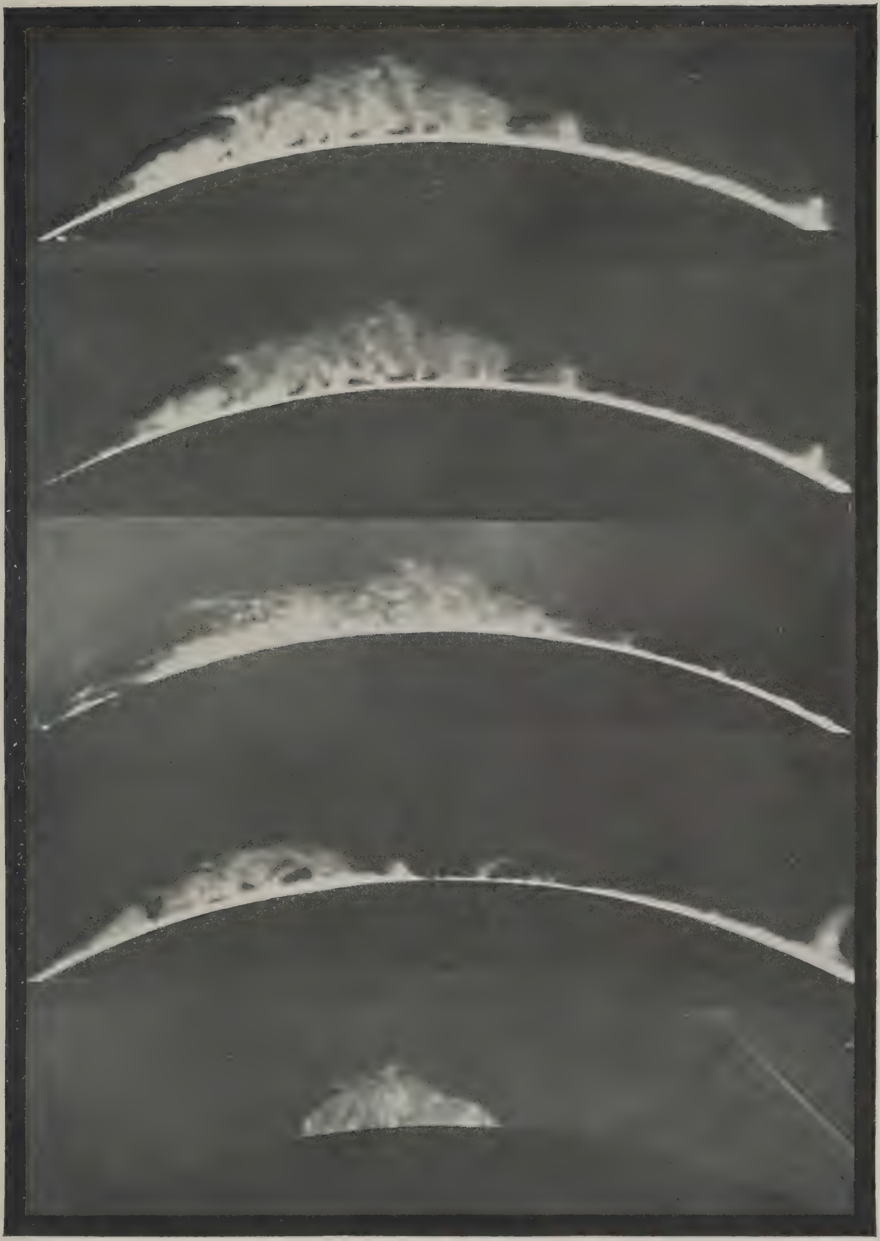
[1] "A Critical Study of Spectral Series," *Proc. Roy. Soc.*, **83**, 226, 1910.

Conclusion.—We may conclude, then, that the Ritz formula, with three undetermined coefficients, will represent practically within experimental errors the principal series of helium, and of all the alkali metals, except caesium. Sodium is represented with especial accuracy, contrary to the conclusions of Bevan (*loc. cit.*). For sodium, also, the chief relation between the principal series and the second subordinate series is quite accurately fulfilled.

Department of Physics
University of Wisconsin
June 1910

PLATE XII



1 1910
March 17
G.M.T.
$5^{h}\ 26^{m}$

2 March 17
$5^{h}\ 30^{m}$

3 March 18
$4^{h}\ 47^{m}$

4 April 13
$7^{h}\ 47^{m}$

5 April 28
$4^{h}\ 22^{m}$

Spectroheliograms of Solar Prominences
Scale: Sun's Diameter = 225 mm

TWO SOLAR PROMINENCES

By FREDERICK SLOCUM

During March and April of the present year the surface of the sun presented an appearance characteristic of the approach to a minimum sun-spot period. Sun-spots were very few and small. Faculae and flocculi were weak and scattered. Solar prominences, on the other hand, were both numerous and active. Two prominences were of especial interest: one, on March 25, on account of its remarkable activity and brief existence; the other, observed from March 4 until April 28, on account of its great size, slow changes, and long life.

I. PROMINENCE OF MARCH 4—APRIL 28

The following table gives the chief facts in regard to the location of the prominence on the sun's limb and its dimensions on the various dates of observation. Plate XII shows reproductions of several of the photographs, all of which were made with the Rumford spectroheliograph, using the H_2 line of calcium.

Date	Number of Exposures	Limiting Position-Angles	Lateral Extent	Apparent Height	
1910			km		km
March 4.........	2	226°–249°	280,000	95″.6	69,000
March 16.........	3	52°– 79°	328,000	84.9	61,000
March 17.........	6	48°– 83°	426,000	95.5	69,000
March 18.........	9	37°– 84°	572,000	106.1	77,000
March 31.........	2	224°–236°	146,000	29.6	21,000
April 1.........	1	222°–230°	97,000	16.1	12,000
April 12.........	2	63°– 71°	97,000	26.0	19,000
April 13.........	7	50°– 87°	450,000	64.3	47,000
April 14.........	3	59°– 80°	255,000	29.5	21,000
April 15.........	1	80°– 90°	122,000	24.1	18,000
April 27.........	1	225°–240°	182,000	67.1	49,000
April 28.........	5	218°–233°	182,000	83.1	61,000

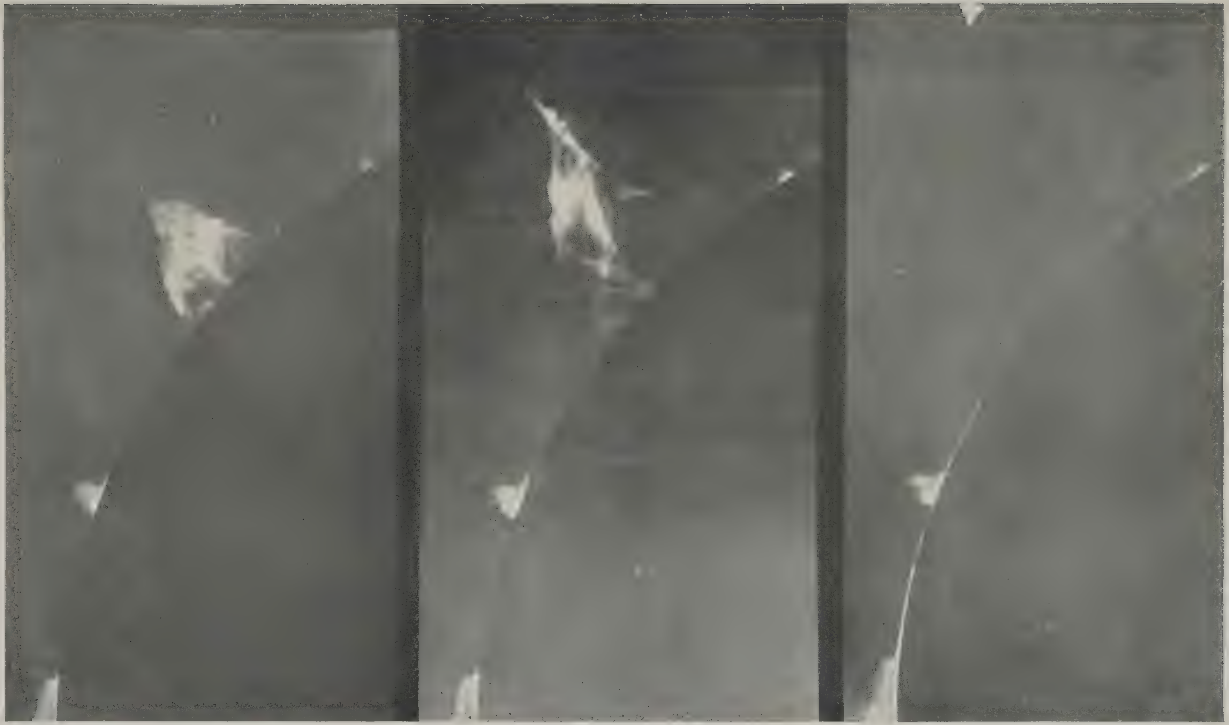
When first seen on March 4 the prominence resembled a great compact mass with fine irregular markings on its surface, somewhat similar to those shown in the photograph of April 13, Fig. 4. It was then just passing around the western edge of the sun. On March 16 it reappeared, rising over the eastern edge, greater in extent and transformed in appearance. By the next day it had reached its greatest apparent elevation, and resembled a grove of banyan trees with short, rugged trunks and intertwining branches. The general aspect remained the same throughout the day, but the minor details were constantly changing. Some such changes are shown in Figs. 1 and 2, Plate XII, which were taken on this date at an interval of 4 minutes. The most conspicuous difference is the bright jet which appeared near the right-hand edge of the main mass of the prominence at some time between $5^h\ 26^m$ and $5^h\ 30^m$. This particular feature may be an independent eruption projected against the prominence. Other differences may be noted in the size and shape of the tree trunks and in the position of the branches.

On March 18 the rotation of the sun had brought the prominence around so that the trunks and lower branches were projected against the disk and were therefore invisible. The tops of the branches remained visible and from these great streamers went out to the right and to the left, curving down to the photosphere, reaching over an arc of 47° of the sun's limb, or 572,000 km.

At this time the prominence extended from latitude $-20°$ to about $+25°$, and its longitude was approximately 70°. This region was carefully examined on the calcium flocculi plates of the following days, but no trace of the prominence could be seen projected on the disk. Near the south end of the prominence was a scattered group of calcium flocculi, and about 20° following the north end was a small sun-spot closely surrounded by flocculi. Between these two there was nothing but the ordinary small granular flocculi which cover the whole disk of the sun.

No plates were obtained on March 29 or 30, but on March 31 and April 1 the top of the prominence was seen passing over the western edge. Twelve days later it reappeared once more on the eastern limb. Photographs were obtained on April 12, 13, 14, and 15. The maximum apparent elevation was reached on the afternoon of April 13. The appearance of the prominence at $7^h\ 47^m$ G.M.T. is shown in Fig. 4, Plate XII. Visual observations a few minutes later showed that the size and configuration of the prominence in the light of $H\alpha$, $H\beta$, and D_3 were essentially the same as shown on the photograph taken in the light of calcium H.

PLATE XIII



1
G.M.T. $4^{h}\,14^{m}.7$

2
$4^{h}\,57^{m}.9$

3
$5^{h}\,45^{m}.0$

SPECTROHELIOGRAMS OF SOLAR PROMINENCES, MARCH 25, 1910
Scale: Sun's Diameter = 182 mm

During the following week the spectroheliograms of the disk were again carefully examined for traces of the prominence, but, as before, without success.

For the fifth time the prominence made its appearance on April 27 and 28 on the western edge of the sun. Its appearance had greatly changed. Its lateral extent was much less and its general appearance more like a compact mass, Fig. 5, Plate XII. Prominence plates were obtained on May 11, 12, 13, and 14, but no trace of a prominence could be seen where the great prominence had been. After an existence, probably continuous, of at least 55 days, it had at last vanished. Throughout the observations the southern extremity of the prominence remained in the vicinity of latitude −20°, while the northern limit varied greatly; starting near the equator on March 4, it moved up to +25° on March 18 and then gradually retreated to −10° on April 28.

II. PROMINENCE OF MARCH 25, 1910

This prominence was first noticed on a plate taken at $7^{h}\ 46^{m}.7$, G.M.T., March 24. It was conical in shape, the base extending from position-angle 230° to 235°, and the apex rising to an apparent height of 46,500 km. The first plate of March 25, at $2^{h}\ 54^{m}.9$, showed that the form had become similar to a rugged oak tree 75,500 km high, the trunk at $229^{\circ}.7$, and the branches extending from 225° to 235°. From this time on, the development of the prominence was very rapid. At $4^{h}\ 14^{m}.7$ it had risen to a height of 120,000 km, Plate XIII, Fig. 1. This increase in elevation was produced in two ways: first, by the tapering aloft of the tree; and secondly, by the ascent of the tree as a whole, including the trunk and roots. No less than eleven rootlike filaments connected the trunk and spreading branches with the chromosphere. During the next forty minutes the prominence lost all resemblance to a tree. At $4^{h}\ 56^{m}.1$, Plate XIII, Fig. 2, two brilliant condensations at an altitude of 115,000 km were connected with the chromosphere by several filaments, and other fainter streamers reached up to a point 240,100 km high. The axis of the prominence, which had been normal to the surface of the sun, now appeared inclined 25° south of the vertical.

Between $4^{h}\ 56^{m}.1$ and $4^{h}\ 57^{m}.9$ occurred the maximum observed motion, 11,600 km in 1.8 minutes, giving a rate of 107 km per second. At 5^{h} $10^{m}.6$ the two condensations had apparently blended and all the filaments united into one, giving a fair representation of a giant flamingo standing on one leg. At 5^{h} $43^{m}.0$ all that remained was a long thin filament rising, with a very slight curve toward the south, to a height of 281,000 km, then making a sharp turn and extending 97,000 km horizontally toward the south, and finally terminating in a cloud 74,000 km in diameter. The total length of the prominence as shown on this plate is 450,000 km, its apparent elevation 290,500 km. Fig. 3, Plate XIII, shows the prominence two minutes later. The point of the arrow marks the location of the faint terminal cloud, which shows clearly on the original negative. Unfortunately this and the following plates do not show quite the whole of the prominence, the exposure being cut off too near the sun's limb. At 5^{h} $55^{m}.4$ the filament had vanished, the detached cloud alone remaining, floating toward the south at a height of 319,500 km or 7′ 22″, and at a rate of 50 km per sec. On the next plate, at 7^{h} $43^{m}.3$, no trace of the prominence can be seen.

Date	G.M.T		No.	Plate Number	Height	
	h	m			″	km
1910 March 24...	7	46.7	1	3548	64	46,500
March 25...	2	54.9	2	3551	104	75,500
	4	14.7	3	3553	166	120,000
	4	16.7	4		171	123,900
	4	56.1	5	3554	332	240,100
	4	57.9	6		348	251,700
	5	10.6	7	3555	351	253,700
	5	43.0	8	3556	402	290,500*
	5	45.0	9		402	290,500
	5	55.4	10	3557	442	319,500*
	5	57.7	11		442	319,500
	7	43.3	12	3558	No	trace
	7	45.6	13			

* Exposure cut off below the highest point of the prominence.

The dissolution of eruptive prominences by vanishing upward in this way, rather than by subsidence, has frequently been observed and seems to be a characteristic of this type of prominence.

The source of the prominence was in latitude −16°, longitude

144°. This region was carefully examined on the calcium plates of the disk taken on March 22, 23, 24, 25, but no trace of anything of especial interest was detected. The nearest disturbance on the disk was a large area of straggling flocculi from 5° to 15° north and from 5° to 25° east of the place of the prominence. The table on p. 128 gives a summary of the observations.

Yerkes Observatory
June 3, 1910

REMARKS ON WILSING AND SCHEINER'S MEMOIR ON THE TEMPERATURE OF 109 STARS

By J. WILSING

The review of the above memoir,[1] by Mr. Abbot, published in this *Journal* for April (**31**, 274, 1910) gives to the authors a welcome opportunity for making here a further statement as to the purpose and results of their investigation, particularly as the continuation of the measurements has furnished the material for settling one of the questions touched upon by Mr. Abbot.

The realization of Kirchhoff's black-body radiation renders it possible, in connection with the theoretical investigations by Wien and by Planck as to the form of Kirchhoff's function under certain conditions, to determine the temperature of a radiating body by measuring the energy-curve of the spectrum. These premises are thoroughly discussed in our Memoir, p. 20. In the first place, it is permissible to employ the radiation formula only in the case of pure temperature radiation. In the case of the temperature of celestial bodies, the question can in general be answered only a posteriori according to the agreement between measurement and theory. The latest investigations on the solar spectrum by Messrs. George E. Hale, Walter S. Adams, and others, in connection with laboratory experiments, permit us to regard the solar radiation as essentially a temperature radiation. Aside from the fulfilment of this condition, however, the transition from the values of the energy measured in the spectrum to the temperatures according to Kirchhoff's law requires the knowledge of the absorptive power of the radiating body. The object of the investigation could, therefore, be in the present case only the computation of the effective temperatures of the stars, i.e., we were to test whether the temperature of an absolutely black body can be so determined that the distribution of energy in its spectrum agrees with that in the solar spectrum within the errors of observation. Of course any value thus computed of the effective stellar temperature is to be regarded only as a mean value, since, as Mr. Abbot remarks and as was emphasized by the authors themselves (p. 43), the radiations of layers of the photosphere of different temperatures mingle, and the absorption, also, has an effect upon the distribution of the energy. In spite of this, in the opinion of the authors, the values of the temperature of the photosphere computed from the observed mean energy-curve may be compared with the value which could be computed from the temperatures of the separate radiating layers.

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[1] J. Wilsing und J. Scheiner, "Temperatur Bestimmung von 109 helleren Sternen aus spectralphotometrischen Beobachtungen," *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, No. 56, **19**.

The comparison of the energy-curves of stellar spectra and of the black body (an electrically heated furnace of Heraeus, the temperature of which could be measured by a platinum, platinum-rhodium thermo-element) was made with the aid of an incandescent electric lamp with a carbon filament. From the ratio of intensity of the spectrum of furnace and of lamp, found by measurements with the spectral photometer, the energy in the spectrum of the lamp, E'_λ, was computed by means of the Planck formula for the radiation of the furnace

$$E_{\circ\lambda}=C\lambda^{-5}\left(e^{\frac{c}{\lambda T'}}-1\right)^{-1}$$

in which T' denotes the absolute temperature of the furnace, λ the wave-length, c the constant of the law of radiation, C a second constant depending upon the particular conditions of the measurement. If $\log n_\lambda$ is the logarithm of the ratio of intensity between the spectrum of the star and of the lamp, photometrically determined, we obtain, computing the energy E_λ in the stellar spectrum and the temperature T of the star, equations of condition of the form

$$\log E_\lambda=\log n_\lambda+\log E'_\lambda+\Delta_\lambda=k-5\log\lambda-\frac{c\log e}{\lambda T}-\log\left(1-e^{-\frac{c}{\lambda T}}\right).$$

Here Δ_λ denotes a correction covering the errors in the values assumed for the energy of the radiation of the lamp, for the selective absorption in the objective, for the coefficient of transmission of the atmosphere, and k is a constant. The last number on the right-hand side of the above equation is small, and can always be determined with an approximate value $T_\circ$, so that we have for the determination of the unknowns Δ_λ, x, y, or T, linear equations of the form

$$\log n_\lambda+\log E'_\lambda+5\log\lambda+\log\left(1-e^{-\frac{c}{\lambda T_\circ}}\right)=x-\Delta_\lambda+\frac{\lambda_\circ}{\lambda}y.\qquad y=-\frac{c\log e}{\lambda_\circ T}.$$

The measurements were made at five places in the stellar spectrum as free as possible from lines, so that there are five equations of condition for the determination of the quantities x and y for each star, provided we may regard the corrections Δ_λ as insignificant. It appeared, however, that on this assumption the residuals from the adjustment of the measured values of the energy by the method of least squares had constant values for each color, independent of the temperature (see p. 43).

log n (observed) — log n (computed)

Temperature	3000°—5000°	5000°—7000°	7000°—9000°	9000°—11000°	Mean
No. of Stars	25	12	13	10	
Weight	2	1	1	1	
λ					
0.448 μ......	+0.033	+0.039	+0.045	+0.037	+0.037
0.480	+0.005	+0.002	−0.005	+0.006	+0.003
0.513	−0.075	−0.068	−0.059	−0.064	−0.068
0.584	+0.025	+0.013	−0.014	−0.009	+0.008
0.638	+0.011	+0.022	+0.038	+0.036	+0.024

In order to disclose the cause of these deviations, measures were made in the spectrum of sunlight reflected from a chalk surface[1] and in the spectrum of the Argand lamp. The relations of these measurements, as well as of the energy-curves of the last two spectra of the incandescent lamp employed (which, similarly to those of the

[1] The following values were found for the logarithms of the coefficients of reflection Δ_λ (multiplied by a constant a) of the radiation reflected by the chalk (J. Wilsing and J. Scheiner, "Comparative Spectral Photometric Observations on the Moon and on Stones, etc.," *Publikationen des Astrophysikalischen Observatoriums*, No. 61, **20**, 16):

log $a\Delta_\lambda$

λ	W.	S.	$\frac{W.+S.}{2}$
0.451 μ.........	9.992	9.992	9.992
0.472	0.020	0.010	0.015
0.494	0.001	9.991	9.996
0.514	9.997	0.017	0.007
0.535	9.993	9.985	9.989
0.556	9.999	0.005	0.002
0.577	9.983	0.014	9.999
0.593	0.007	9.992	0.000
0.615	9.990	0.014	0.002
0.642	0.022	9.985	0.004

The distribution of the energy in the spectrum of light diffusely reflected from the chalk is, therefore, according to these measurements, not different in the visual portion from that in the spectrum of the source itself.

Argand flame, should agree closely with the energy-curve of a black body) give the same residuals as were found in the case of stellar spectra. Thus on p. 48 the following summary will be found:

log E (observed) − log E (computed)

	Argand Flame	Incandescent Lamp, 1	Incandescent Lamp, 2	Sun	Star
λ	Temp. = 2050°	1970°	1850°	5200°	3000°–11000°
0.448 μ......	+0.020	+0.042	+0.030	+0.005	+0.033
0.480	+0.010	+0.012	+0.018	+0.028	+0.003
0.513	−0.046	−0.086	−0.074	−0.047	−0.067
0.584	−0.012	−0.003	−0.006	+0.005	+0.012
0.638	+0.027	+0.036	+0.032	+0.011	+0.019

It follows from this agreement that the distribution of energy in the spectra of all sources of light when referred to any desired fundamental spectrum, such as that of the sun, or of the Argand lamp, is completely represented by the Planck formula and that the numerical values of the differences

$$y_* - y_\odot$$

which are independent of the quantities Δ_λ, or $\frac{1}{T_*} - \frac{1}{T_\odot}$, are determined without qualification. But the assumption that

$$\Delta_\lambda = 0,$$

made in the adjustment of the observations, is not permissible, and in fact the determination of the values of the energy of the radiation of the lamp E' by means of the black body turns out to be erroneous by amounts notably in excess in some cases of the amounts to be expected from the agreement of the measures among themselves. As Mr. Abbot properly remarks, our determination of these corrections Δ_λ were not made from independent sources of measures and the residuals above given are therefore to be regarded only as the most probable values of the quantity $-\Delta_\lambda$. But according to the explanations given above, the uncertainty from this deficiency affects only the determination of the absolute temperature of the fundamental star from the comparison with the radiation of the black body. Thus, if, for instance, the temperature assumed for the sun required the correction $\Delta T_{\odot}$, the corresponding correction of the stellar temperature ΔT_{*} would be obtained directly from the equation

$$\Delta T_{*}=\left(\frac{T_{*}}{T_{\odot}}\right)^{2}\Delta T_{\odot}\,.$$

Here the numbers found from the measurements were to be introduced for T_{*} and $T_{\odot}$. From the remarks at the close of this note, which are based upon new and independent measurements, it follows that the constant amount of error in the determination of the temperature, which may be ascribed to the differences between the most probable and the true values of the corrections Δ_{λ}, cannot for the hottest stars much exceed 1000°.

For comparison with his bolometric measurements on the solar spectrum, Mr. Abbot discussed our spectral-photometric observations on four different assumptions. Of these only the second is permissible. The values of log E assumed by the authors for the distribution of energy in the solar spectrum are given in our paper entitled "Vergleichende spectralphotometrische Beobachtungen am Monde . . . . ," p. 29. The following table contains our definitive values for log E and those computed by Mr. Abbot. The differences are so slight that in what follows Mr. Abbot's figures have been retained. The third column gives the energy of radiation when the intensity at λ 0.448 μ is placed at 1000. The fifth column gives the results of the bolometric measurements by Messrs. Abbot and Fowle.

λ	0.448 μ	0.480 μ	0.513 μ	0.584 μ	0.638 μ
log E	9.928	0.014	0.033	0.018	0.006
log E (Abbot)	9.932	0.018	0.037	0.022	0.009
E (c=14,600)	1000	1219	1274	1230	1194
E (c=14,200)	1000	1167	1172	1052	975
Abbot and Fowle	1000	1040	1000	893	800

In the attempt to explain the considerable difference between the bolometric and the spectral-photometric measurements, Mr. Abbot has overlooked the effect due to the uncertainty in the assumed numerical value of the radiation constant c. We pointed out on p. 41 of our Memoir that, according to determinations of temperature made by Messrs. L. Holborn and S. Valentiner[1] with the nitrogen thermometer, the value $c=14{,}600$ used here in the reductions is probably too large, and should be replaced by $c=14{,}200$. We then obtained from the spectral-photometric measurements $T=5500°$ for the solar temperature (see p. 41). The investigations recently completed at the Reichsanstalt in Berlin by S. Valentiner[2] seem to make certain the necessity for this correction. It also follows from these investigations that all the values of the tensions of the thermo-elements, platinum, platinum-rhodium, as previously extrapolated for all high temperatures, were too low. Hence a correction of $\Delta T_{\circ}=+7°$ (corresponding to a mean temperature of 1500° absolute) should be applied to the data of the thermo-element by which was marked the temperature of the furnace. After applying the corrections to the values of the energy of the radiation of the lamp for $c=14{,}200$, and correcting the mean temperature of the black body (1500° absolute) by 7°, we obtain for the energy-curve of the sun the values in the next to the last row of Table I. If the energy in the solar spectrum at λ 0.638 μ is taken at 100, we get the following figures:

log E

	λ 0.448	λ 0.480	λ 0.573	λ 0.584	λ 0.638
A. and F.	125	130	125	112	100
W. and S.	103	120	120	108	100
Difference . . .	+22	+10	+5	+4	0

Only at λ 0.448 μ do the differences appreciably exceed the amount corresponding to the precision of the spectral-photometric measures, particularly since the differences still involve the uncertainty in the determination of the coefficient of transmission of the atmosphere. Further, photographic determinations of the coefficient of reflection of chalk, with which the writer is occupied at present, seem to yield a small decrease with the wave-length, which would still further reduce the difference between the measurements by the different observers by several per cent. Finally, it should be noted that between λ 0.40 μ and λ 0.45 μ, and λ 0.50 μ and λ 0.54 μ, according to the data of

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[1] "Eine Vergleichung der optischen Temperaturskale mit dem Stickstoffthermometer bis 1600°," *Annalen der Physik*, **22**, 1, 1907.

[2] S. Valentiner, *op. cit.*, p. 309.

Messrs. Abbot and Fowle,[1] depressions of a solar nature exist which may have affected the two sources of observations in a different manner.

For the wave-length $\lambda_{\text{max.}}$ of the maximum of the energy, Mr. Abbot finds from the bolometric measurements $\lambda=0.460\ \mu$ and from the spectral photometric measurements $\lambda=0.571\ \mu$. The latter value is based on the assumption that $c=14{,}600$. From the formula for the ratio of the solar radiation to the radiation of the black body at wave-length λ

$$\frac{E_\odot}{E_\circ}=e^{-\frac{c}{\lambda}\left(\frac{1}{T_\odot}-\frac{1}{T_\circ}\right)},$$

we obtain as a correction $\Delta T_\odot$ of the solar temperature

$$\Delta T_\odot=\left(\frac{T_\odot}{T_\circ}\right)^2\Delta T_\circ,$$

where $\Delta T_\circ$ is the change of the temperature of the black body. Therefore for $\Delta T_\circ=+7°$, and $\frac{T_\odot}{T_\circ}=3.7$,

$$\Delta T_\odot=+100°.$$

Hence, the solar temperature becomes $T_\odot=5600°$, and the wave-length of the maximum of intensity, according to Wien's law, is $\lambda_{\text{max.}}=0.523\ \mu$, whereby the difference between the bolometric and the spectral-photometric measurements is reduced almost by half. A corresponding diminution occurs in the difference between the value $T_\odot=5962°$, derived by Messrs. Abbot and Fowle from their pyrometric measures, and the temperature 5600° (formerly 5100°), computed from the spectral-photometric measurements with the corrected constant of radiation. From Mr. Abbot's remark, "It seems misleading to compute temperatures from a spectral range of only $0.2\ \mu$," it would seem that he has not considered the form of the equations of condition from which the temperatures are determined. If λ_1 and λ_2 are the wave-lengths of the places in the spectrum at which measurements of energy have been made, we have

$$\log\frac{E_1}{E_2}=-5\log\frac{\lambda_1}{\lambda_2}-\frac{c\log e}{T}\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right).$$

We obtain for the factor $\frac{1}{\lambda_1}-\frac{1}{\lambda_2}$ the value 0.7 (when we place $\lambda_1=$

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[1] *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **2**, 108.

0.45 μ, and $\lambda_2=0.64\ \mu$). This factor becomes unity for measurements in the infra-red portion of the spectrum for $\lambda_1=0.7\ \mu$ and $\lambda_2=2.5\ \mu$, where the energy of the solar spectrum, according to the measures of Messrs. Abbot and Fowle, is but 1 per cent of its maximum value. The accuracy of the determination of temperature from the less refrangible portion of the spectrum is, therefore, with equal accuracy in the determination of the logarithmic ratio of intensity, only greater by one-half, in spite of the ninefold greater extent of spectrum than that obtained from the optical region of the spectrum covering only 0.2 μ. For deciding the question as to how far the distribution of energy in the stellar spectrum may be represented by the radiation formula holding good for the black body, it is indeed necessary to compare as large an extent as possible of both spectra. It is, therefore, very much to be regretted that measurements in the region of larger wave-lengths have only been made in the case of the sun.

After the completion of the series of observations published by the authors, the simple prism in the spectral-photometer was replaced by a Rutherford prism having a dispersion between *Hα* and *Hγ* of 6.3. The energy-curve of the lamp was again determined, and at ten places, by numerous comparisons of the spectrum of the glow-lamp No. 2 with the spectrum of the black body.

log *E'*

λ	W.	S.
0.451 μ	9.227	9.265
0.472	9.507	9.496
0.494	9.703	9.700
0.514	9.846	9.811
0.535	9.999	9.994
0.556	0.133	0.099
0.577	0.240	0.247
0.593	0.319	0.336
0.615	0.437	0.464
0.642	0.593	0.589

The spectrum of the Argand flame was then compared with that of the incandescent lamp, and the temperature of the flame was computed by Planck's formula from the measurements of each observer with the values of log E' found by him. The following table contains the differences between observation and computation:

DIFFERENCES BETWEEN OBSERVED AND COMPUTED VALUES OF

log E $(O.-C.)$

λ	W.	S.	Mean
0.451 μ	−0.039	−0.032	−0.036
0.472	+0.035	−0.004	+0.016
0.494	+0.016	+0.046	+0.013
0.514	−0.008	−0.012	−0.013
0.535	+0.001	+0.012	+0.007
0.556	−0.006	−0.019	−0.013
0.577	−0.011	−0.006	−0.009
0.593	+0.007	+0.006	+0.007
0.615	−0.028	+0.018	−0.005
0.642	+0.032	−0.004	+0.014
Number of observations	2	2	
Temperature	2050°	2050°	2050°

The residuals after the adjustment are represented within the uncertainty of the measurements, and the marked deviation in the green, which was previously observed, has disappeared entirely. This remark also applies to the measurements previously made in stellar spectra of the first type. For comparison with the earlier measures, they have been reduced with the value $c=14,600$, and with the uncorrected temperature indications of the thermo-element.

DIFFERENCES BETWEEN OBSERVED AND COMPUTED VALUES OF

log E $(O.-C.)$ Wilsing

λ	*ζ Pegasi*	*α Pegasi*	*α Leonis*	*α Lyrae*	*γ Geminorum*
0.451 μ	+0.010	−0.020	−0.043	−0.014	−0.026
0.472	+0.027	+0.082	+0.063	+0.010	+0.023
0.494	−0.015	+0.049	+0.052	−0.017	0.000
0.514	−0.017	−0.048	−0.036	−0.004	+0.027
0.535	−0.017	−0.068	−0.011	+0.010	+0.033
0.556	−0.001	−0.064	−0.033	+0.044	+0.008
0.577	−0.021	−0.015	−0.018	−0.006	−0.116
0.593	−0.008	−0.005	−0.028	+0.028	+0.019
0.615	−0.003	+0.025	−0.021	−0.029	+0.043
0.642	+0.041	+0.064	+0.072	−0.003	+0.027
Number of observations	4	2	4	2	2

DIFFERENCES BETWEEN OBSERVED AND COMPUTED VALUES OF log *E* (*O*. − *C*.) Scheiner

λ	*ζ Pegasi*	*α Pegasi*	*α Lyrae*	*γ Geminorum*
0.451 μ	−0.015	+0.039	−0.038	−0.048
0.472	−0.003	+0.026	−0.041	−0.052
0.494	+0.009	−0.036	−0.003	+0.027
0.514	−0.012	−0.033	+0.048	+0.050
0.535	+0.040	−0.017	+0.082	+0.082
0.556	+0.001	−0.052	+0.069	+0.046
0.577	+0.004	+0.030	−0.009	0.012
0.593	−0.004	−0.013	−0.064	−0.021
0.615	−0.022	+0.009	−0.012	−0.061
0.642	+0.003	+0.043	−0.027	−0.010
Number of observations	4	1	2	1
Weight		½		½

DIFFERENCES BETWEEN OBSERVED AND COMPUTED VALUES OF log *E* (*O*. − *C*.)

		MEAN	
λ	W.	S.	$\frac{\text{W.+S.}}{2}$
0.451 μ	−0.018	−0.019	−0.019
0.472	+0.041	−0.019	+0.011
0.494	+0.014	+0.001	+0.008
0.574	−0.015	+0.015	0.000
0.535	−0.011	+0.052	+0.020
0.556	−0.009	+0.022	+0.007
0.577	−0.035	+0.001	−0.017
0.593	−0.010	−0.028	−0.019
0.615	+0.003	−0.020	−0.009
0.642	+0.040	−0.003	+0.019

The following summary contains the temperatures computed from the measures of each observer (with $c = 14{,}600$); the mean of the same, T (with $c = 14{,}600$, and $c = 14{,}200$); the effective temperatures T' as formerly computed (pp. 63 ff.); the mean temperatures of the spectral classes T'_M (p. 65 of our paper); and, finally, the differences $T - T'$ and $T - T'_M$.

The second series of measures appears, therefore, to yield a value for the temperature of the stars of the first type about 1300° too low. This difference notably exceeds the accidental uncertainty following from the adjustment of the errors. But we must take into account an uncertainty of 0.01 to 0.02 in the logarithms of the separate values of the energy log E'_λ as computed from the internal agreement of the measurements; and that a difference of about 0.05 in the logarithms of the ratio of the energy of the positions of the spectrum situated at λ 0.448 and λ 0.638 is sufficient for explaining the difference of temperature in question. On the basis of the new measurements, we are therefore justified in the conclusion that the probable value of the corrections Δ_λ previously found differ only slightly from the true values. Furthermore, the difference between the temperatures derived from the two sources of measurements is diminished in proportion to the square of the temperature of the stars.

	W. $c=14600$	S. 14600	T		T' $c=14600$	T'_M $c=14600$	$T-T'$ $c=14600$	$T-T'_M$ $c=14600$
			$c=14600$	$c=14200$				
ζ Pegasi	7700°	8100°	7900°	9100°	9300°± 800°	9500°	−1400°	−1600°
α Pegasi	8900°	8500°	8700°	10500°	11500°±1100°	9600°	−2800°	− 900°
α Leonis	8700°		8700°	10500°	9400°± 800°	9500°	−700°	− 800°
α Lyrae	8200°	7900°	8100°	9500°		9600°		
γ Geminorum	6900°	6800°	6900°	8000°	10300°±1000°	8700°	−3400°	−1800°
					Mean.......	......	−2100°	−1300°

In the opinion of the authors, this statement shows that the results of the earlier measures agree as well as could be expected, in view of the difficulty of such measurements, with those of the second series based upon the new determination of the constant of radiation of the lamp. The observations are now to be resumed, and will be extended to all stars down to magnitude 4, north of the equator.

POTSDAM
May 1910

STUDIES ON THE EMISSION OF GASES

II. ON SPECTRAL TUBES FOR USE WITH DIRECT CURRENT

By H. KONEN and W. JUNGJOHANN

1. Mention has often been made of the desirability of employing spectral tubes adapted for direct current. The advantages of Geissler tubes fed by an induction coil are very great on account of their convenience and their serviceability in the case of very slight quantities of gas; they fail, however, when it is desired to make quantitative investigations, such as measurements of the energy in gaseous spectra, or to obtain the greatest possible brilliancy. In the first case, the average intensities are comparatively small, even when electric vibrations having a large maximum current-strength are employed; and further, the complicated electrical conditions of the discharge, which vary from one apparatus to another in a manner very difficult to control, prevent in general a separation of the different constants of the gaseous spectra and the establishment of the conditions under which such spectra appear. Furthermore, anyone who has investigated spectra of Geissler tubes with great resolving power, such as that of a large Rowland grating, will have felt the desire to increase the intensity of his source of light. We may even assert that the inadequate knowledge of many gaseous spectra, particularly beyond the limit of the visible spectrum, is for the most part to be attributed to the relative faintness of the spectral tubes.

2. It is an obvious idea to employ the direct current instead of the induction coil. If this is done, we shall obtain electrical conditions which are accurately controllable, and the different portions of the circuit will differentiate themselves clearly and constantly. We cannot say, however, that the earlier attempts with direct current gave the success awaited. On account of the very considerable kathode drop in many gases, particularly at higher pressures, it is necessary to employ high-tension batteries for supplying the direct-current tubes. But if these have the ordinary form determined by the expense, space, and maintenance, it is possible to obtain from them

only a few milliamperes of current. Direct-current dynamos of corresponding capacity are not only very expensive, but are difficult to handle, and can hardly come into question for use in ordinary laboratory practice.

Even if the source of the current of sufficient tension and capacity is available, the work done at the kathode with large current is so great that very special precautions have to be taken in order to protect the tubes and electrodes from melting. So far as we see, these are the reasons why there are but few experiments recorded in this direction: the notably excellent success obtained by Paschen,[1] with quartz tubes using direct current from a battery of exceptionally large capacity and number of elements, was published when we had already carried out our experiments to a large extent, so that we have independently from him followed a similar procedure.

3. The properties of the Wehnelt kathode suggest the avoidance of the inconvenience of the high-potential battery by employing as kathode incandescent metallic surfaces covered with oxide. In fact, aside from Wehnelt himself, a number of other observers, as Wiedemann, Geiger, Janicki, have employed such tubes, and they have indeed been placed on the market by different firms, Some have used for source of light the gaseous discharge, while others have employed the arclike process of discharge at the anode, which occurs in consequence of the drop at the anode when the current-density is sufficiently increased by diminishing the size of the anode. This procedure is essentially similar to that recently described by A. Hagenbach[2] of an arc under diminished pressure. It is easy in using Wehnelt kathodes with a luminous potential to reach gaseous discharges having an intensity of several amperes, kathodes of sufficiently large surface being employed. In spectroscopic work, the point desired is density of current, rather than strength of current, which seems to have been overlooked by different persons, and since most gases absorb only slightly, it ultimately depends on the density of the strata. From this point of view the density of current hitherto attained cannot be designated as even large, inasmuch as tubes of large cross-section have been employed generally.

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[1] *Annalen der Physik* (4), **27**, 532–570, 1908.

[2] *Physikalische Zeitschrift*, **10**, 649–657, 1909.

We have, ourselves, in part in common with Mr. J. Kyll, made a large number of experiments with spectral tubes having Wehnelt kathodes, in which we modified the shape very greatly. Thus we have finally reduced the diameter of the tube to less than one millimeter, and have attained current-density of over 60 amperes per sq. cm. The limit was by no means reached there, but it was practically impossible to employ glass tubes with such current-densities which were equivalent to heavy arc discharges. We have employed different forms of tubes which were inclosed in vessels of water, but we found it impossible to use currents of the magnitude mentioned for any length of time with glass tubes. Our experience with these tubes, with which measurements of intensity in the emission and absorption spectra were simultaneously carried out, will be described in detail elsewhere.

4. In spite of frequent successes, we have, however, finally abandoned the tubes above described having Wehnelt kathodes. The necessity of employing a special current for heating the kathodes, which must amount to 10 amperes for a larger surface of the kathode and produces much heat, calls for a rather large dimension of the tubes, which become inconvenient with their surrounding vessels. With such strong currents as we used, it was difficult to prevent the metal kathode from burning up occasionally despite every precaution. The disadvantages connected with this can only imperfectly be avoided by a previous equipment with a reserve of several incandescent kathodes, but above all, the glass vessels cannot withstand the necessary current-density. The tubes have to be made of considerable dimensions, inasmuch as the incandescent kathode, with the attachments necessary for its renewal, cannot be diminished beyond a certain limit. The use of quartz vessels is forbidden by the technical difficulties and the very large cost for institutions only moderately endowed.

5. We have now found that a simple modification of the tubes described furnishes a form adapted for many purposes which stands about midway between a tube planned for a high potential and a Wehnelt tube. We may, in fact, entirely do away with the heating current of the glowing kathode. In many gases it is sufficient with pressures of 10 mm and upward to use the induction impulse of a

small induction coil, or even merely to bring up a piece of ebonite, glass, or shellac, that has been rubbed, to a tube provided with an appropriate kathode, in order to produce a discharge, if only the potential available amounts to about 800 volts. It is then sufficient to attach to the kathode a small piece of platinum foil about 0.25 sq. cm in size, and 0.01 mm thickness, that has previously been covered with a thin sheet of oxide by dipping it in a solution of calcium nitrate +barium nitrate, and heating it in a Bunsen burner. The thinner this coat is, the more successfully will the tube operate, and the platinum sheet may be of much smaller size. If we bring up to such a tube, for instance, a stick of hard rubber that has been rubbed, we notice that with increase in potential a luminous preliminary discharge occurs which follows the arc discharge after a slight interval, for the most part almost directly. The arc discharge is characterized by the fact that a point of the platinum foil is raised to incandescence by the discharge itself. The fall of the kathode is thus reduced to a few volts and the discharge of the tubes sinks under some circumstances down to 100 volts and less. The use of energy is limited in effect to the positive column and the anode drop, so that the anode is most strongly heated, as in the case of the arc. The negative glow light is lacking.

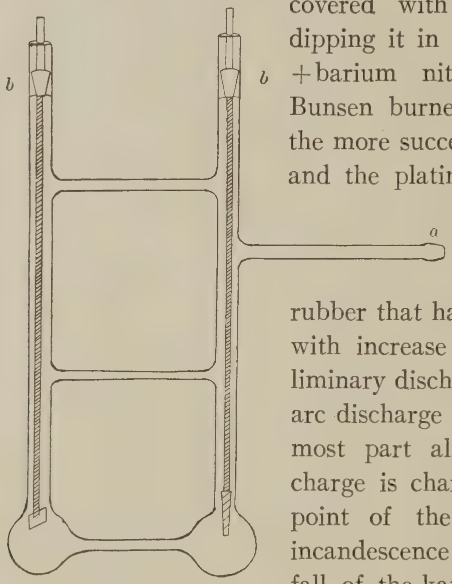

Fig. 1

6. After numerous preliminary experiments, we selected a tube of the form shown in Fig. 1. It was made by the firm of Heraeus from quartz glass, and can be connected with the pump by means of a ground attachment of glass. The length of the arms *bb* is about 22 cm. The spheres have a diameter of about 3 cm and the capillary is 60 mm long and 2 mm in internal width. It might be made narrower as well as longer. We have avoided this in order to be able to use the tubes also for experiments in absorption. At *b* are the

electrodes of nickel-steel with conical bearing, which can be made tight by pouring upon them a drop of mercury. The electrodes were made of steel wire of about 2 mm diameter, and they carry at their lower extremity a piece of steel wire several centimeters long. If necessary, this may be replaced by a piece of some other metal, such as aluminum. The end used as kathode carries a slit in which a small piece of platinum foil may be clamped.

7. As source of current, we used a dry battery of 420 elements, which in series gave a maximum current of 1.2 amperes or for a brief time 1.5 amperes. This was purchased with a grant from the Hittorf fund. In the hope that it might be enlarged in the future, it was very carefully set up and isolated by paraffin and thick plates of glass. Paraffin oil was poured over the acid in the ordinary way and the circuit leading to the battery was mounted on paraffin insulators. In spite of its relatively very small cost, this battery has served admirably without any interruption for several years. We have used it partly as a whole, and partly in parallel with 440 volts, in series, and partly also in groups of 140 elements arranged in series. We used the resistance bands made by Schniewindt of Neuenrade, i. W., which, in addition to their inexpensiveness, have the advantage of convenient mounting, slight self-induction, and high durability. The potential at the entrance of the tubes was measured with a standardized electroscope, as there was some difficulty in starting the tubes into action, when the measuring instruments did not take the current.

8. The tubes resembled very much the Wehnelt tubes. The discharge took place of itself at pressures which varied from gas to gas, and even at 840 volts. Where this was not the case, as already mentioned, an electrified rod of hard rubber was brought up. Its action was exhibited particularly at the anode. In many cases it was also sufficient to discharge the tubes by contact, or, better, by passing them through a flame. This procedure also proved to be necessary when the tubes had been running for a long time before the discharge could be produced by electrostatic action. The effect of the heat of a flame is felt in addition to that of conduction of the current; when the tube was heated to a faint redness by a Bunsen flame, the potential of discharge fell. It is natural to assume an action of ultra-violet light

here, since in these experiments an intense bluish-green phosphorescence of the quartz vessel was observed under the influence of the ultra-violet radiation of the Bunsen flame (the inner cone of which is particularly effective). This assumption, however, is not correct, as may be shown by check experiments with an arc lamp, and light concentrated upon a spark-gap by a quartz lens. The radiation from radium-bromide placed near by also failed to show an effect of reducing potential; but in our experiments, the radiation had first to penetrate 0.5 mm of glass and then the quartz glass, so that this may possibly explain the negative result. No effect was produced on bringing up a magnetic field, but we are still occupied with more accurate experiments on this point. Finally, it was also sufficient, as already mentioned, for starting the discharge, to allow a shock from a small induction coil to pass through the tube. For this, one tube was connected with the more remote electrode, the other with the platinum wire, which was set into the portion of glass connecting the tube with the pump.

9. It is not difficult with the means thus described to produce discharges at pressures of from 0.5 to 10 mm, hence just at the pressures which are involved in most spectroscopic work; the limits of pressure vary with the gas and with the tube. If it is desired to pass over to higher pressures, it is recommended that the discharge should be first introduced at lower pressure, and then to admit gas during the passage of the current. If exhaustion is carried below a definite limit, the glow discharge will first begin, then the current-strength falls, and finally the automatic passage of the current is extinguished. Within the range of pressure mentioned, the current-strength can be regulated in the simplest manner, however, and it remains constant to a fraction of a per cent. If, with tubes of the size described, the current-strength is appreciably less than 200 milliamperes, the glow discharge readily begins, the relatively small amount of heat produced at the kathode being conducted away and the remainder not being sufficient to produce the rise in temperature necessary to overcome the kathode drop. Under certain circumstances, we may then observe a periodic change from the arc to the glow discharge. After the glow discharge has lasted for a time, the temperature of the whole kathode rises in consequence of the kathode

drop, and the temperature of the platinum foil exceeds the limit necessary for the arc discharge. As soon as this happens and the arc discharge begins, the work done at the kathode decreases and the glow discharge begins again; and this process repeats itself. The glow discharge may be avoided by sufficiently reducing the oxidized surface and making it of thinner foil and connecting it with the steel kathode by a thinner wire. With an increase of current, the diameter of the incandescent spot at the kathode increases, but it is not until currents of 0.7 amperes and upward are reached that signs are seen of an increased evaporation of the oxides covering the kathode. There is then formed directly upon the kathode an aureole of reddish-yellow color which shows, along with a few lines of calcium and barium, the oxide bands of the two metals. This remark is based solely upon visual observation.

In the spectra of the capillary in general nothing is noticed of the lines of the metals mentioned. It is advisable to avoid excess of oxide at the kathode, for in consequence of the evaporation of a portion of it there forms in the capillary a coating of oxide which attacks the quartz glass at incandescence and darkens it. This is of no significance for the ordinary use of the tube viewed lengthwise, inasmuch as the discoloration is limited to the capillary and does not occur until after weeks of use; furthermore, it may be entirely avoided with proper precautions. For this reason it is desirable always to observe from the side of the anode. With an extended use and the employment of strong current, the kathode is dissipated and gradually forms a brown coating on the inner side of the tube consisting of iron and platinum; but this is readily removed with a weak nitric acid.

10. We used currents as high as 1.5 amperes in the quartz tubes, but a further increase is prevented by the increasing heat of the anode, which glows vigorously at 0.7 amperes. The capillary, too, is heated to a yellow glow at the current-strength mentioned. However, in consequence of the long arms of the tube, it is unnecessary to cool the electrodes.

With currents of from 0.2 to 1.5 amperes, and corresponding current-densities of 6 to 30 amperes per sq. cm, the tubes are illuminated with different gases with a brilliancy similar to that of the arc. A strong odor of ozone is noticed in the vicinity, particularly with

CO in the tube, as in the case of mercury vapor lamps. An exposure of from one to two minutes was sufficient to obtain the whole spectrum in the first and second order below λ 4800 on ordinary plates, with a nitrogen tube used with a Rowland concave grating of 180 cm focus and of a small ruled surface and moderate light-power. A concave grating of 4 meters radius gave a similar spectrum in 10 minutes in the third order. Similar exposure times were found for tubes with hydrogen, carbonic oxide, etc. These numbers have, of course, no absolute value, as they depend upon many variable conditions; they suffice, however, to give a comparison with ordinary spectral tubes with which exposures of hours are necessary when an induction coil fed with a primary current of about 8 amperes and having a striking distance of 50 cm is used.

11. The process of filling and cleaning the tubes does not differ from that of ordinary tubes, and we refer here particularly to the paper by Paschen, adding only a few remarks. Disturbing impurities in the tubes have been especially noticed in the case of hydrogen, water-vapor, mercury, and carbon and its compounds. The mercury lines are particularly disturbing in photometric measures. They come in part from the mercury vapor from the pump, but are only strong when the electrodes are not made very tight. On putting in the electrodes and pouring on the mercury the greatest care must be taken that no trace of the metal enters into the tube. In that case the mercury vapor can be removed only by frequent heating, washing, and complete exhaustion, and perhaps finally with the aid of liquid air and carbon. The hydrogen presumably comes from the electrodes, which give off appreciable quantities of the gas for a long time. The observer therefore generally obtains, in spite of careful drying, the spectrum of water-vapor, doubtless in consequence of the production of water by the oxide kathode simultaneously with the reduction of the oxide by the hydrogen. This production of hydrogen diminishes after a time, however, and with it vanishes the hydrogen spectrum. This explanation of the appearance of the bands of water-vapor is confirmed when the tubes are filled with hydrogen made and dried with the greatest care. We then obtained, along with the lines of the series spectrum, the "second" hydrogen spectrum (many-line spectrum) and at the same time lines of the strong bands of water-

vapor at λ 3064. The occurrence of the lines of hydrogen can further be brought about by strongly heating the tube with a flame of hydrogen or hydrocarbon; the hydrocarbon and hydrogen then diffuse into the interior of the tubes.

The spectra of carbon and its compounds are due in part to impurities in the tubes themselves, but to a greater extent to the carbon of the steel electrodes. On filling the tube with nitrogen we therefore obtained first the cyanogen spectrum and then the band spectrum of carbon (Swan spectrum) developed to a greater or less extent, and further the lines of carbon itself as impurities. If nitrogen is excluded, then we often find, besides the spectra named, the third Deslandres group of the so-called carbon-oxide bands present as an impurity. All these impurities are disturbing at the start, but they disappear with the passage of the current for a long time and with frequent exhaustion, washing out with oxygen and with the gas used for filling the tubes, which thus become very pure.

12. The precise determination of the electrical properties of the tubes described presents decided difficulties. The measurements of the fall of potential at the anode and kathode, as well as of the gradient in the positive column, have not hitherto been made with the current-densities like those used by us; they are also difficult because the conducting wires are introduced with difficulty and melt off. If we extrapolate from the measurements made with the strongest currents hitherto employed, then we find a drop at the anode of about 15 to 20 volts on the assumption that in the region in question the anode drop changes only slowly with the current-density, and that the changes with the pressure, etc., may be neglected. The kathode drop can be estimated from the measures made on the incandescent oxidized kathode and it is in any case very small. We obtained thus approximate values for the potential gradient, which at the same time make it possible to estimate the consumption of energy per unit of volume. The values so obtained agree for an unstratified discharge approximately with those derived from previous measurements on the dependence of the gradient in the positive column upon the width of the tube, pressure, and current strength. A conclusion as to these observations will follow later. They are mentioned here only to indicate the possibility of an estimate of the consumption of energy

in the tube and the usefulness of more precise measurements of the anode drop. According to the strength of the current and the pressure, we obtain stratified or unstratified discharges. The observations which we have made with the telephone and a rotating mirror indicate that these are continuous within the precision of the method. By introducing appropriate capacity and self-induction we obtain oscillating discharges. In its general character the discharge stands about half-way between a glow discharge and an arc, and resembles in different respects the arc at diminished pressure, which Hagenbach (*loc. cit.*) recently described, and into which it may be converted by a continuous increase of the current-strength.

13. We have employed tubes of the sort described for different purposes with success; as a source in photographing the spectra of gases, it appears that band spectra in particular are furnished in great intensity. With these, using various line spectra, photometric measurements have been made, particularly by Jungjohann and by Kyll, on the relation of absorption and emission to pressure, current-strength, and consumption of energy; a report of these experiments will be made very soon, as well as upon their employment for experiments on the distribution of emission along the path of the discharge, etc. Under all of these circumstances the tubes we have described have proved satisfactory. It will be agreed that the sort of tubes thus described is suited to replace in many cases the ordinary vacuum tubes, when we reflect that they are useful not only for direct current but also as metallic vacuum lamps and may be used as ordinary Geissler tubes in connection with an induction coil. Finally, the cost of a battery of small cells which brings an existing lighting circuit from 440 volts up to 840 volts, or of a correspondingly small generator, hardly exceeds the cost of a large induction coil.

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[1] EDITORIAL NOTE.—It was the intention of the authors that this bibliography should accompany their first article of this series, published in the *Journal* for June (**31**, 404–418, 1910). Through an oversight in the editorial office, however, that intention was not carried out, and the bibliography is accordingly appended here. See page 405 of the June number.

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SERIES IN THE SPECTRA OF CALCIUM, STRONTIUM, AND BARIUM

By F. A. SAUNDERS

Two abstracts have already appeared[1] of preliminary notes, presented to the Physical Society, on spectrum series in these elements. The following pages give a full account of these series, as far as they are known at present, together with tables of the arc spectra of the elements, which are more nearly complete than any that are already available.

The first series to be found in these spectra were the two main triplet series, first and second subordinate, of which Kayser and Runge gave both for *Ca* and the first for *Sr*. Rydberg[2] found the second *Sr* series, and the writer[3] recently sorted out the same two in *Ba*. One other triplet series in each spectrun is now known, called the "narrow triplets," the lines being closer together than the main triplets. These were found for *Sr* by Fowler,[4] and for *Ca* and *Ba* by the writer.[5] The wave-lengths for the *Ca* series have been measured with accuracy by Barnes,[6] the lines having been obtained sharp by reducing the pressure around the source of light. Without this precaution they are too diffuse to admit of accurate measurement. It was hoped that they could be obtained equally well by the same method in the case of *Sr* and *Ba* also, but the experiment has not yet succeeded, as far as the narrow triplet series are concerned. The lines are relatively fainter than in *Ca*, and fade away with reduction of pressure as fuzzy lines; if present at all in the low-pressure (5 mm) spectra, they have been too faint to impress themselves upon my photographs.

The other series in these spectra may be classified as pair and single-line series. These are considered in detail below.

[1] *Physical Review*, **28**, 152, 1909, and **30**, 270, 1910.

[2] *Wiedemann's Annalen*, **50**, 625, 1893.

[3] *Astrophysical Journal*, **28**, 223, 1908.

[4] *Ibid.*, **21**, 81, 1905.

[5] *Ibid.*, **21**, 195, 1905, and **28**, 228, 1908.

[6] *Ibid.*, **30**, 14, 1910.

Some of the apparatus employed has been described before.[1] A four-inch grating, purchased through the generosity of the Rumford Fund Committee, was used for the general survey of these spectra. For more exact wave-length measurements beyond λ 2400, I took a set of one-hour arc spectrum photographs with the four-inch concave grating (Rowland mounting, first order) at the Dominion Observatory, Ottawa, Canada. My best thanks are due to Dr. W. F. King, Director of the observatory, for permission to use this instrument, and to Mr. J. S. Plaskett, Chief Astronomer, for many favors in that connection. These spectra have excellent definition and considerable intensity, reaching in one case down to λ 1931.

For still shorter wave-lengths a quartz spectrograph was used, with Schumann plates at first, but it was later found that lantern slide plates were slightly more sensitive to the region in question (λ 2000), and they were accordingly substituted. The ordinary fast photographic plate is distinctly less sensitive than the lantern slide to these rays. The new lines that were found were faint and diffuse, and the wave-length measurements beyond λ 2150 may be considerably in error, possibly as much as a whole unit in the worst cases.

A few spark photographs were taken for comparison, but most of these series are obtained in far greater intensity from the arc, even the pair series, which are *relatively* (not absolutely) strong in the spark; so that the spark photographs were not much used.

A set of photographs was also obtained with the arcs inclosed in an iron box, from which the air could be largely removed by means of a Geryk pump. I am indebted to Mr. O. S. Blakeslee for working out the design of this very convenient apparatus, and constructing it. It is fitted with tubes. Two of these hold the terminals opposite one another, so that they can be turned about, or adjusted along their length, even when the vessel is exhausted. A third tube at right angles to these is sealed by a quartz plate at the farther end, and carries a quartz lens, which throws an image of the arc, outside of the apparatus, on the slit of the spectroscope. The terminals were usually graphite rods, saturated with chlorides of the elements, but, for *Ca*, I used the metal itself, and for *Ba*, also, on one or two occasions.

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[1] *Astrophysical Journal*, 20, 188, 1904.

SINGLE-LINE SERIES

In the spark spectrum of *Ca* beyond λ 2400 very little is to be found, except some members of the pair series; but, on changing to the arc, a remarkable difference is noted. The pairs are then relatively faint, and an isolated and very obvious series of single lines comes into prominence. These are reversed and unsymmetrically broadened. The first three lines are included in the list of Kayser and Runge. Since my report on this series before the Physical Society, Barnes[1] has also measured the first four of its lines with the arc under reduced pressure. The lines under these conditions, though sharp, become so faint that I have been unable to obtain measurable images of more than five of them. The wavelengths of the outer lines, obtained at normal pressure only, are therefore not very exactly known.

The spectrum of *Sr* shows a precisely similar series, the only important difference noted being that the pair and single-line series are more nearly of the same intensity in this case than in that of *Ca*. Reduction of pressure has the same effect on the *Sr* lines as on those of *Ca*. Table I below shows these new series, under the designation *SL1*.

The spectrum of *Ba* probably contains a similar series, but I have not been able to sort it out. The line λ 2597.26 has exactly the right aspect, and others look promising, but the series seems to be widened out, probably shifted toward the red, and reduced in intensity compared with that in *Sr*.

Barnes has shown that the light of the *Ca* λ 4226.9 comes more strongly from the positive terminal of the arc, while the opposite is the case with the pair-series lines. I have found that *Ca* λ 2398.6 shows the same peculiarity as λ 4226.9. Since the latter is also strongly reversed in the arc, and single, it would be natural to suspect that it was a member of the same series with these new reversed single lines. In the effort to test this, I have taken long-exposure photographs with the quartz spectrograph of an oxyhydrogen flame colored with calcium, as well as an ordinary Bunsen flame fitted with an attachment for spraying a solution of calcium chloride into it. In neither case did I obtain an image of any ultra-violet lines, though λ 4226 (which is a flame line) was strong. This may possibly be due to too short an exposure, but I am disposed to think that 4226 does not belong to this series, and that the ultra-violet lines are not flame lines. Experiments with the high-temperature oven spectra ought to settle this point easily, since 4226 then has an enormous intensity. A formula which would include 4226 with the others would have to be of an unusual type, if indeed it could be found at all. If, then, 4226 does not belong to this series, it is probably the first member of a principal series of single lines, to which *SL1* is a subordinate, and of which the other lines lie in the Schumann region, and have not yet been found. Similar remarks would apply also to the strong lines *Sr* λ 4607 and *Ba* λ 5535.

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[1] *Ibid.*, 30, 18, 1910.

Other single-line series have also been sorted out. Certain lines in the blue and violet have the aspect of outer series members, and have been long under suspicion, but it was difficult to prove their relationship to one another. This was finally done by making use of the "combination principle" of Ritz.[1] According to this idea, if one series is found, and its formula calculated, other series may reasonably be expected to be found, whose formulae are similar to the first, the differences amounting to simple alterations in one or two constants of the formula. The very simplest alteration possible is to change the "end-constant" only; that is, to shift the series as a whole to some other part of the spectrum without altering the separations of its lines on the wave-number scale. Having already plotted out these spectra on a wave-number scale on long rolls of paper, I had no trouble in searching for such repetitions of the series *SL1*, and one was soon found. The data for this series (called *SL2*) are given in Table I, together with the differences between the wave-numbers of corresponding lines in the two series. The corresponding series in *Sr*, which was readily picked out, is also given in the table. It might seem that these two series *SL1* and *SL2* together constitute one series of very wide pairs. This is unlikely since the appearance of the lines is so different. One set is strong and reversed; the other, weak and not reversed.

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[1] *Physikalische Zeitschrift*, 9, 521, 1908.

When the violet regions of these spectra were again examined, a few lines were noted that looked like series lines but were not included in the previous series. Most of these were seen to belong together to another new series, *SL3*, as shown in Table I. These lines are more diffuse than those of *SL2*, the long wave-length members stronger, the others weaker than those of *SL2*. The members of the two series alternate, just as two subordinate series do. It therefore seems probable that these form a pair of series of the subordinate type, running to the same end. If so, the last series, *SL3*, should be called a "first subordinate" series, from its relative strength and diffuse appearance. *SL3* may have a red member, which I cannot distinguish with certainty. There are several "applicants" for the vacancy in both *Ca* and *Sr*. λ 6718 looks very promising for *Ca*, but the corresponding *Sr* line is hard to select. This point cannot be settled until some observations of the Zeeman effect, or other crucial tests, are obtained for the lines concerned.

The list of lines in *SL3* is very short on account of the faintness of the outer members of this series. The fourth line of *Ca SL3* is due at 3973.3, but cannot be observed on account of the close proximity of the far stronger line 3973.89 which belongs to the second subordinate triplet series.

The wave-lengths for a few lines, given by Kayser and Runge, have been slightly altered. The reason for this is that these lines appear to shift when the pressure is reduced, the shift being measured, of course, with reference to other spectrum lines near by, which are sharp under all circumstances. If, therefore, the true wave-length is desired, the only accurate way of obtaining it is to put the arc *in vacuo*. This has been done, and some new values obtained. In no case was the line *in vacuo* outside of the space occupied by the widened line in air; but the density maximum was shifted in some cases by as much as 0.5 unit. For instance, one of the *Sr* lines has an apparent wave-length at normal pressure, measured by means of some of its strong and sharp neighbors, of

4406.8. When the pressure is reduced to 20 cm, and the wave-length (at normal pressure) deduced from this spectrum, it comes out 4406.45; similarly, from a 5 mm photograph, 4406.29. It appears to be often the case that the wave-lengths of diffuse or broadened lines obtained from arc in air are not accurate, even when measured by the projecting points obtained with an astigmatic grating. This is important in connection with series, as most of the outer series members are of this class and should be located with great accuracy, if series laws are to be studied.

In the table, the observer's name is indicated by initials: K & R signifying Kayser and Runge; EH, Exner and Haschek; B, Barnes, and S, the writer. In case two observers are given together, average values have been used for the wave-lengths. The column W.-N.D. gives the wave-number difference between the line of *SL1* and the corresponding line of *SL2;* that is, the amount by which the two series are separated. The constancy of these numbers is very satisfactory. The value of *n* in the sixth column is the one used in the formula. The last two columns give the calculated wave-number and the difference between the calculated and observed wave-lengths. The formulae used are presented below:

Ca SL1 and *SL2*

$$1/\lambda = \begin{Bmatrix} 49290.0 \\ 27441.2 \end{Bmatrix} - \frac{109675}{\left\{ n + 0.16 + \frac{112.444}{n^4} - \frac{4937.15}{n^8} \right\}^2}$$

Ca SL3

$$1/\lambda = 27441.2 - \frac{109675}{\left\{ n + 0.9344 - \frac{9.3989}{n^5} \right\}^2}$$

Sr SL1 and *SL2*

$$1/\lambda = \begin{Bmatrix} 45935.0 \\ 25786.0 \end{Bmatrix} - \frac{109675}{\left\{ n + 0.0554 + \frac{18.6419}{n^2} - \frac{124.57}{n^4} + \frac{263.26}{n^6} \right\}^2}$$

Sr SL3

$$1/\lambda = 25786 - \frac{109675}{\left\{ n + 0.9340 + \frac{50.149}{n^5} \right\}^2}$$

The series are similar to the one in *Mg* which Rydberg[1] discovered, for which he found a four-constant formula necessary.

TABLE I

Series	Wave-Length	Observer	Wave-No.	W.-N.D.	n	Calc. Wave-Number	C.−O. (λ)
Ca SL1	2398.66	K & R	41677.4	21849.1	3	41677.6	−0.01
	2275.60	K & R	43931.1	21848.4	4	43931.0	+0.01
	2200.80	B. K & R	45424.2	21849.0	5	45425.5	−0.06
	2151.00	B. S	46475.7	21846.3	6	46476.8	−0.05
	2118.99	S	47178.7	21846	7	47177.9	+0.04
	2097.8	S	47654	....	8	47653.8	+0.01
	2083.2	S	47988	....	9	47987.2	+0.03
	2072.8	S	48229	....	10	48229.9	−0.04
Ca SL2	5041.93	K & R	19828.3	....	3	19828.8	−0.12
	4527.17	K & R	22082.7	....	4	22082.2	+0.10
	4240.58	K & R	23575.2	....	5	23576.7	−0.27
	4059.06	S	24629.4	....	6	24628.0	+0.23
	3946.3	S	25333	....	7	25329.1	+0.61
Ca SL3	4878.34	K & R	20493.2	....	3	20493.2	0.00
	4355.41	K & R	22953.6	....	4	22953.6	0.00
	4108.70	S	24331.9	....	5	24330.1	+0.30
	....	....	....	....	6	25161.2	......
	3889.8	S	25701	....	7	25699.3	+0.26
Sr SL1	2931.98	K & R	34096.9	20149.3	2	34096.3	+0.05
	2569.60	EH. S	38905.2	20148.6	3	38905.3	+0.01
	2428.16	EH. S	41171.2	20149.1	4	41170.6	+0.04
	2354.40	EH. S	42460.8	20149.1	5	42460.8	0.00
	2307.5	S	43323	20146	6	43325	−0.06
	2275.5	S	43934	20148	7	43925	−0.4
	2253.5	S	44362	....	8	44349	−0.6
	2237.4	S	44682	....	9	44658	−1.1
	2226.0	S	44910	....	10	44887	−1.1
Sr SL2	7167.7	S	13947.6	....	2	13947.3	+0.15
	5330.00	S	18756.6	....	3	18756.3	+0.08
	4755.59	K & R	21022.1	....	4	21021.6	+0.11
	4480.73	K & R. S	22311.7	....	5	22311.8	−0.02
	4313.38	S	23177.3	....	6	23176	+0.24
	4202.95	S	23786.3	....	7	23776	+1.7
Sr SL3	5156.37	K & R	19388.2	....	3	19388.2	0.00
	4678.39	K & R	21369.0	....	4	21369.0	0.00
	4406.29	S	22688.6	....	5	22688.1	+0.1
	4253.7	S	23503	....	6	23509	−1.1

It is, therefore, not surprising to find that none of the usual three-constant formulae will fit any of them. For the *SL3* series a three-constant formula is used, involving n^5, but only a few terms are known, and a four-constant formula might be required for the entire series.

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[1] *Wiedemann's Annalen*, 50, 625, 1893.

For the series *SL1* and *SL2* one formula suffices with change only of the end-constant. A four-constant formula was found necessary for *Ca*, and for *Sr* I have not been able to obtain an expression with less than five adjustable constants to satisfy the nine observed values within experimental error. If no attention were paid to the first two or three lines, it could be done with three constants, and this may be looked upon as an argument against including the lines in question; but their appearance, relative intensities, and changes under altered conditions (arc to spark, reduced pressure, etc.), as well as the constancy of the numbers under the W.-N.D. column, all point strongly to including them in the series.

The difficulty with the formula in such a case is clearly seen by examining the various values that the quantity $\sqrt{\frac{109675}{a-1/\lambda}}$ takes for each line, *a* being the end-constant. This quantity equals the denominator (not squared) in the formula. If *a* is properly selected, the difference between each of these numbers and the corresponding integer *n* (to which they are generally nearly equal) ought to settle down to approximate constancy for the higher members of the series. It does so for most of the series found by Kayser and Runge. But in the case of *Sr*, *SL1* and *SL2*, these differences are at first rather large, then increase slightly to a maximum, and then drop, fast at first, more slowly later, settling down to their ultimate constant value. If we plot their curve, it has a point of inflection, which the corresponding curves for the simpler series have not. To represent these numbers properly, we must use an algebraic expression involving at least three constants, or more. It does not improve matters in this case to alter the "universal constant" 109,675; while the separate series can be represented equally well in this way (though no better), it becomes impossible to make the two series *SL1* and *SL2* end together. As it seems likely a priori that they have the same end-constant, I have abandoned this line of attack. Nor are we much better off if we examine the differences between the denominator squared and n^2. The same general form in the curve above mentioned is prominent in the case of the *Mg* series of single lines that Rydberg discovered, as well as elsewhere; so that one is tempted to look upon it as a general property of these series. It may still be possible to find a four-constant formula for all of them, and of the same type for all, but the search is laborious, and it is not unlikely that the result will be first reached (if at all) from theoretical investigations, rather than from the experimental side.

It is apparent that one of the constants (0.9344) for *Ca SL3* is practically the same (could easily be adjusted to be the same exactly) as the corresponding constant for *Sr SL3*. This agreement may be accidental, but it appears to be another of the many likenesses between the spectra of these two elements.

A further application of the combination principle is to be noted. The end-constants of *SL1* and *SL2* appear to be intimately related, as the following will show. According to Rydberg's simple theory, the end-constant itself takes the form $\frac{N}{(n+b)^2}$. If we consider this correct, as a first approximation, and put this fraction equal to 49,290, the end-constant for *Ca SL1*, we find that the end-constant for *SL2* is given by the expression $\frac{N}{(n+b+0.5)^2}$, the error being only 200 out of 27,441. A similar result is obtained for *Sr*. That the agreement is not exact is due to the fact that the expression $n+b$ is too simple, and must be enlarged by the addition of other terms, involving n and other constants. If n is increased by the addition of 0.5 again, we might hope in like manner to find the end-constant of another new series. This would then end near λ 5700 ($1/\lambda=17{,}500$) for *Ca*, and if such a series exists it might begin with the great line found by Paschen at λ 10,345, but I have not succeeded in sorting it out. In *Sr* the necessary ultra-red observations are lacking for such a search.

THE PAIR SERIES

It has been known for some time, from observations of the Zeeman effect,[1] that there were series of pairs in the spectra of *Ca*, *Sr*, and *Ba*. Only one pair was first found for each of these, but Eder and Valenta[1] measured a couple of pairs in *Ca* which (though the authors do not mention it) belong to the same grouping. Ritz[2] recognized this fact and summed up the series as far as known. Since the spectra of *Sr* and *Ba* are very like that of *Ca*, and consist, on the whole, of slower vibrations, it looked like a promising experiment to search for pairs in these series in the extreme ultra-violet, in the hope of finding enough of them to serve as a basis for the calculation of the formulae. This attempt has been partially successful, and the results are given in Table II, below. In the column of observers, H refers to Hermann,[3] and EV to Eder and Valenta.

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[1] *Astrophysical Journal*, 16, 123, 1902.

In *Ca* there are not enough terms for the calculation of the formulae, excepting in the case of the first subordinate series. Here, however, we have only three terms, including the one in the ultra-red, proposed by Paschen,[4] and even a three-constant formula would require all the lines for the determination of its constants. Ritz[5] calculated the formula for this series (without the ultra-red term) from two terms, along with the principal series pair, which his theory connects with this series, and obtained a value for the end-constant. This value is too low to suit the above three terms, since they converge too slowly in the ultra-violet. Other terms of this series were unsuccessfully sought for on my photographs.

For *Sr* we can calculate two formulae as follows, using in each case the line of greatest wave-length in the pair.

First subordinate series

$$Sr\ P1 \quad 1/\lambda = 59282 - \frac{109675}{\left\{ n - 1.79 + \frac{6.173}{n^2} \right\}^2} \qquad n = 3, 4, 5, \text{ etc.}$$

Second subordinate series

$$Sr\ P2 \quad 1/\lambda = 58782 - \frac{109675}{\left\{ n - 1.4533 + \frac{6.385}{n^2} \right\}^2} \qquad n = 2.5, 3.5, \text{ etc.}$$

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[1] *Sitzungsberichte der Wiener Akademie*, 1893; *Beiträge zur Photochemie und Spectralanalyse*, p. 57, 1904.

[2] *Physikalische Zeitschrift*, **9**, 521, 1908.

[3] *Annalen der Physik*, **16**, 684, 1905.

[4] *Ibid.*, **29**, 659, 1909.

[5] *Physikalische Zeitschrift*, **9**, 529, 1908.

TABLE II

Series	Observer	Wave-Length	Wave-Number Difference
Ca, principal..........	K & R	3968.63	222.9
	K & R	3933.83	
Ca, first subordinate ...	H	8662.42	222.8
	H	8542.48	
	H	8498.32	
	K & R	3181.40	223.1
	K & R	3179.45	
	K & R	3158.98	
	......	....	214.5
	EV	2113.01	
	EV	2103.47	
Ca, second subordinate.	K & R	3737.08	223.0
	K & R	3706.18	
	EV	2208.95	224.8
	EV	2198.03	
Sr, principal..........	K & R	4215.66	801.3
	K & R	4077.88	
Sr, first subordinate ...	K & R	3475.01	800.9
	K & R	3464.58	
	K & R	3380.89	
	S	2324.60	800.1
	EH	2322.47	
	EH	2282.14	
	......	....	778
	S	1995.7	
	S	1965.2	
Sr, second subordinate .	K & R	4305.60	801.6
	K & R	4161.95	
	EH	2471.71	801.6
	EH	2423.67	
	S	2053.3	791
	S	2020.5	
Ba, principal..........	K & R	4934.24	1690.7
	K & R	4554.21	
Ba, first subordinate ...	K & R	4166.24	1691.0
	K & R	4130.88	
	K & R	3891.97	
	K & R	2641.51	1691.0
	K & R	2634.91	
	EH	2528.52	
	S	2235.50	1691.2
	S	2232.90	
	S	2154.02	

TABLE II—*Continued*

Series	Observer	Wave-Length	Wave-Number Difference
	......	....	
Ba, first subordinate . . .	S	2055.0	1650
	S	1987.7	
Ba, second subordinate.	K & R	4900.13	
	K & R	4525.19	1690.5
	K & R	2771.51	
	K & R	2647.41	1690.9
	S	2286.21	
	S	2201.07	1691.3
	S	2082.8	
		....	

Some modification should apparently be made in the denominator of these formulae in order to bring the end-constants into agreement, but there are not enough terms known to test the formula. The end-constant derived by Ritz is, as in *Ca*, much too low.

For *Ba* four terms are known in each of two series, and, for these, three-constant formulae have been obtained, which give good agreement for all the lines. The line of greater wave-length in each group has been used for calculation, as before.

First subordinate series

$$Ba\ P1 \quad 1/\lambda = 58220 - \frac{390431}{\{n+0.37758\}^2} \qquad n=3,\ 4,\ 5,\ \text{etc.}$$

Second subordinate series

$$Ba\ P2 \quad 1/\lambda = 58425 - \frac{410890}{\{n+0.2873\}^2} \qquad n=3,\ 4,\ \text{etc.}$$

These may be considered to give a fair value for the end of the series, but it is over 8000 higher than the one found by Ritz. It seems, then, that in all three elements the pair series converge less rapidly than the theory of Ritz demands. Three-constant formulae involving the universal constant 109,675 could not be found to suit these *Ba* series. Four constants were then required.

Runge and Paschen[1] note also a pair of lines in *Ba*, λ 6497 and λ 5853 (wave-number difference, 1690.6), which have the Zeeman separations that are characteristic of first subordinate lines, but interchanged. They suggest the possibility of these being the first members of a principal series of a new sort, related to the first subordinate series instead of, as usual, to the second. Such a series should have a term or two within reach, but I have not been able to identify any of them.

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[1] *Astrophysical Journal*, 16, 123, 1902.

In all three elements there occurs a strong pair in the ultra-violet: *Ba* λ 2335 and 2304; *Sr* λ 2165 and 2152; *Ca* λ 1840 and 1837, which are reversed in *Sr* and *Ba*, and probably in *Ca* also, and the line of greater wave-length is the stronger in each. They therefore look like subordinate-series pairs in a series of great strength, the rest of which is in the Schumann region. This possibility would, perhaps, not be worth noting, were it not for the fact that the *Ca* spectrum shows a curious "pair" of lines (appearing only with arc at low pressures) near λ 5000, which may be related to these pairs. The one in the green is apparently accompanied by a satellite (very faint), just as first subordinate pairs always are, and the wave-number difference for the group is equal to that for the ultra-violet pair. Similar "vacuum pairs" appear in *Mg* (two in this case), but I have sought for them in *Sr* in vain; they must be very faint, if present at all. The *ratios* of the wave-number differences for these ultra-violet pairs in *Ca*, *Sr*, and *Ba* are nearly the same as for the regular pair series, which are related to the atomic weights. This would indicate that they were not accidental pairs. These facts taken together make it look probable that there are series of narrow pairs, as well as the wider ones, most of the lines being at present inaccessible. These spectra also show a few pairs of the same separation as the first two lines of the main triplets, but there is no obvious relation among them.

In regard to the unclassified lines in these spectra, their number is still fairly large, especially in the case of *Ba*. It is likely that several more series are to be found among them. Paschen[1] gives an ultra-red member of the *Ca* "Principal Series of Triplets" (λ 19,900), and seeks to combine this with lines at λ 9546.8 and 7288.0 into a series. Unfortunately the line λ 7288 is absent from my photographs, and this leaves the series rather short, having at the most but two terms of its own, and this is the more unfortunate as it should be (by analogy) one of the strongest series in *Ca*. He also suggests several single combinations, after Ritz (*op. cit.*, p. 658), which might lead to new series. Two of them involve lines (λ 6833 and λ 3653) which certainly do not occur on my photographs.

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[1] *Annalen der Physik*, 29, 656, 1909.

There are, finally, certain curious groups of lines, six to a group, which occur in all three spectra in the same sort of way. These six lines include a triplet of the same separation as the main triplets, a pair of the same separation as the first two lines of the triplet, and a single line. They are usually strong, and reversed. The groups occur in *Ca* near λ 3000 and λ 4300. The corresponding ones in *Sr* are near λ 3350 and λ 4800, and in *Ba* around λ 4500 and in the deep red. These look like some new sort of "complex" triplets, but there is no very obvious relation among them, or with other lines. They are "arc" lines, slightly stronger about the negative pole of a direct-current arc than the positive one, like the triplet series, and unlike the *SL* series; they seem to be physically related to the main triplet series in every way. They furnish the most striking feature among the unclassified lines, and it would be very interesting to find the key to their arrangement. Observations in the Schumann region ought to throw light on this group.

For the sake of completeness, I give, below, tables of the arc spectra of these elements. It is hoped that these are fairly accurate, and contain all the lines known to date. The wave-lengths given are those furnished by the observers mentioned, who are indicated by the same abbreviations as in the tables above, with the addition of the following: *R* for Rowland's table of the sun's spectrum; *P* for Paschen;[1] *J* for Janicki;[2] *K* for Kent;[3] *F* for Fowler,[4] and Ryd. for Rydberg.[5] Rowland's wave-length system is used. The column under *E* contains the error, estimated as well as the available data (sometimes lacking) permit. The inten-

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[1] *Annalen der Physik*, 26, 655, 1909.

[2] *Ibid.*, 29, 833, 1909.

[3] *Astrophysical Journal*, 28, 225, 1908.

[4] *Ibid.*, 21, 81, 1905.

[5] *Wiedemann's Annalen*, 52, 119, 1894.

CALCIUM ARC SPECTRUM

Intensity	Wave-Length	Observer	E	Series
2	22655.9	P	1.5	
2	24.6	P	2.5	
1	10.0	P	3	
1	19946.8	P	3	TH (?)
2	35.8	P	1	TH (?)
1	17.5	P	3	T1
4	19859.9	P	2	TH (?)
1	17.3	P	2	
3	19777.4	P	1	T1
2	19507.1	P	1	T1
3	19452.9	P	1	T1
2	19310.6	P	1	T1
2	16432.6	P	3	
2	16200.0	P	3	
2	16162.2	P	3	
2	44.8	P	3	
2	13038.0	P	3	
3	12821.6	P	1.5	
10	10345.0	P	1.5	
3	9694.5	P	3	
1	9546.8	P	3	TH (?)
2	9250.8	P	2	
7	8662.42	H	...	P1
8	8542.48	H	...	P1
6	8498.32	H	...	P1
6	7326.45	S	0.3	
6	7202.45	S	0.3	p
8	7148.5	S	0.3	p
1	6798.9	S	0.3	
1	89.6	S	0.3	
2	84.35	S	0.3	
8	18.1	S	0.3	
3	6573.0	S	0.3	
1	09.1	S	0.3	
5	6499.85	K & R	0.1	p
6	93.97	K & R	0.1	p
6	71.85	K & R	0.1	
7 r	62.75	K & R	0.1	
3	55.75	S	0.3	p
6	49.99	K & R	0.1	p
8 r	39.36	K & R	0.1	
1	17.95	S	0.3	
1	6395.4	S	0.3	
2	6261.7	S	0.3	
6	6169.87	K & R	0.1	
4	69.36	K & R	0.1	
4	66.75	K & R	0.1	
4	63.98	K & R	0.1	
8 r	62.46	K & R	0.1	T2
2	61.60	K & R	0.1	
7 r	22.46	K & R	0.05	T2
6 r	02.99	K & R	0.05	T2
2 hR	5867.94	K & R	0.1	
8 h	57.77	K & R	0.1	

CALCIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
6	5603.06	K & R	0.05	
6	01.51	K & R	0.05	
6 h	5598.68	K & R	0.05	
8	94.64	K & R	0.05	
6	90.30	K & R	0.05	
8	88.96	K & R	0.05	
9	82.16	K & R	0.05	
6	13.07	K & R	0.1	
8	5349.66	K & R	0.05	
1	42.4	S	0.2	
8	5270.45	K & R	0.05	
8	65.79	K & R	0.05	
6	64.46	K & R	0.05	
6	62.48	K & R	0.05	
6	61.93	K & R	0.05	
4	60.58	K & R	0.05	
7	5189.05	K & R	0.05	
7 hR	5041.93	K & R	0.05	SL2
0	21.45	S	0.1	vp
2	20.35	S	0.1	vp
1	01.87	S	0.1	vp
8 hR	4878.34	K & R	0.1	SL3
3 hV	47.22	K & R	0.2	
2 hV	33.85	K & R	1.0	
2 hV	23.04	K & R	1.0	
2 hV	07.47	K & R	1.0	
4 hV	4685.40	K & R	0.5	
1 hV	24.71	K & R	0.5	
2	4586.22	J	0.1	t
7	86.12	K & R	0.1	t
6	81.66	K & R	0.1	t
5	78.82	K & R	0.1	t
4	27.17	K & R	0.1	SL2
1 h	12.73	K & R	1.0	
1 h	09.89	K & R	1.0	
1 h	08.04	K & R	1.0	
4	4456.81	K & R	0.03	T1
6 r	56.08	K & R	0.03	T1
8 r	54.97	K & R	0.03	T1
7 r	35.86	K & R	0.03	T1
7 r	35.13	K & R	0.03	T1
7 r	25.61	K & R	0.03	T1
3 hR	4355.41	K & R	0.1	SL3
6 r. hV	18.80	K & R	0.03	T
6 r	07.91	K & R	0.03	
7 r	02.68	K & R	0.03	p
5 hV	4299.14	K & R	0.03	T
6 r. hV	89.51	K & R	0.03	T
6 r	83.16	K & R	0.03	p
2 h	40.58	K & R	0.1	SL2
10 r	26.91	K & R	0.03	?
1 h	4108.70	S	0.1	SL3
3 hR	4098.66	B	0.1	t
2 hR	95.05	B	0.1	t

Analogy indicates that *Ca* λ 4581 is double; separation about .03 t-m.

CALCIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
2 hR	4092.76	B	0.1	t
1 h	59.06	S	0.1	SL2
7 hR	3973.89	K & R	0.05	T2
10 r	68.63	K & R	0.03	PH
6 hR	57.23	K & R	0.05	T2
1 h	53.6	S	0.2	
5 hR	49.09	K & R	0.05	T2
0 h	46.3	S	0.2	SL2
10 r	33.83	K & R	0.03	PH
1 h	3889.8	S	0.2	SL3
3 h	75.85	B	0.1	t
2 h	72.60	B	0.1	t
2 h	70.57	B	0.1	t
2 h	3753.56	B	0.1	t
1 h	50.40	B	0.1	t
1 h	48.39	B	0.1	t
7	37.08	K & R	0.03	P2
6	06.18	K & R	0.03	P2
1 h	3678.46	B	0.1	t
1 h	75.53	B	0.1	t
1 h	73.49	B	0.1	t
4 r. hR	44.86	K & R	0.05	T1
8 hR	44.50	K & R	0.05	T1
7 hR	31.10	K & R	0.05	T1
7 r. hR	30.83	K & R	0.05	T1
7 r. hR	24.15	K & R	0.05	T1
5 hR	3487.76	K & R	0.05	T2
4 hR	74.98	K & R	0.05	T2
4 hR	68.68	K & R	0.05	T2
7 hV	3361.92	K & R	0.1	T1
6 hV	50.22	K & R	0.1	T1
5 hV	44.49	K & R	0.1	T1
4 hR	3286.26	K & R	0.1	T2
3 hR	74.88	K & R	0.1	T2
3 hR	69.31	K & R	0.1	T2
4 hV	25.74	K & R	0.5	T1
3 hV	15.15	K & R	0.5	T1
3 hV	09.68	K & R	0.5	T1
4	3181.40	K & R	0.03	P1
7	79.45	K & R	0.03	P1
1 h	70.23	K & R	0.5	T2
1 vh	66.95	K & R	2.0	T2
6	58.98	K & R	0.03	P1
2 h	50.85	K & R	0.5	T1
2 h	40.91	K & R	0.5	T1
1 h	36.09	K & R	0.5	T1
1 vh	17.74	K & R	1.0	T2
1 vh	07.96	K & R	1.0	T2
1 vh	01.87	K & R	1.0	T1
5	3009.30	R	0.05	T
6	06.98	R	0.05	p
4	00.91	R	0.05	
4	2999.75	R	0.05	T
5	97.44	R	0.05	p

CALCIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
5	2995.06	R	0.05	T
7	2398.66	K & R	0.1	SL1
2	73.3	S. EV	0.3	
1 h	54.7	S	0.3	
1 h	35.0	S	0.3	
1	2287.9	S	0.3	
5 r. h	75.60	K & R	0.1	SL1
1	49.8	S	0.3	
1	16.7	S	0.3	
2	08.95	EV	....	P2
3 r. h	00.80	B. S	0.2	SL1
2	2198.03	EV	...	P2
2 r. h	51.00	B. S	0.2	SL1
2 h	18.99	S	0.3	SL1
2	13.01	EV	...	P1
2	03.47	EV	...	P1
1 vh	2097.8	S	0.3	SL1
1 vh	83.2	S	0.5	SL1
1 vh	72.8	S	1.0	SL1
4	1839.8	S	1.0	vp
3	37.1	S	1.0	vp

STRONTIUM ARC SPECTRUM

Intensity	Wave-Length	Observer	E	Series
6	7309.7	S	0.3	
5	7232.6	S	0.3	
7	7167.7	S	0.2	SL2
8	7070.7	K & R	0.2	T2
6	6892.8	S	0.2	
8	78.8	K & R	0.2	T2
7	6791.4	K & R	0.2	T2
5	6643.8	S	0.2	
6	17.6	S	0.2	
8	6550.53	K & R	0.2	p
6	47.1	S	0.2	
1	21.3	S	0.2	
8	04.17	K & R	0.1	
2	6466.1	S	0.2	
2	47.0	S	0.2	
8	08.65	K & R	0.1	
7	6388.4	S	0.2	
8	86.74	K & R	0.1	p
7	80.95	K & R	0.1	
5	70.2	S	0.2	
4	64.3	S	0.2	
2	45.9	S	0.2	
4 hR	5970.38	K & R	0.05	
1	5848.01	K & R	0.1	
2	17.01	K & R	0.05	
3	5767.29	K & R	0.1	
3	5693.35	S	0.1	
3	74.25	S	0.1	

STRONTIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
3	5556.66	S	0.1	
6 h	43.49	K & R	0.05	
7	40.28	K & R	0.05	
9	35.01	K & R	0.05	
7	22.02	K & R	0.05	
8	04.48	K & R	0.05	
7	5486.37	K & R	0.05	
8	81.15	K & R	0.03	
7	51.08	K & R	0.03	
6	5330.00	S	0.1	SL2
7	5257.12	K & R	0.03	
7	38.76	K & R	0.03	
6	29.52	K & R	0.03	
6	25.35	K & R	0.03	
6	22.43	K & R	0.03	
4	13.23	K & R	0.03	
2 hR	5166.2	S	0.1	
8	56.37	K & R	0.05	SL3
3	4971.85	K & R	0.05	T1
6	68.11	K & R	0.03	T1
9 r	62.45	K & R	0.03	T1
7	4892.87	Ryd	0.03	t
7	92.20	K & R	0.03	t
7	76.49	Ryd	0.05	T
7 r	76.23	Ryd	0.05	T1
8 r	72.66	K & R	0.05	T1
6 h	69.41	K & R	0.05	t
6 h	68.92	K & R	0.03	t
5 h	55.27	K & R	0.05	t
7 r	32.23	K & R	0.03	T1
7 r	12.01	K & R	0.03	p
6	4784.43	K & R	0.05	T
4	55.59	K & R	0.1	SL2
6	42.07	K & R	0.03	T
3 h	29.93	K & R	0.1	
6	22.43	K & R	0.03	p
2	14.2	S	0.1	
1	07.4	S	0.1	
1	04.2	S	0.1	
1	4688.8	S	0.1	
1	84.6	S	0.1	
5 h	78.39	K & R	0.1	SL3
10 r	07.52	K & R	0.03	SL ?
3	4531.54	K & R	0.03	
3 h	4480.73	K & R. S	0.1	SL2
5 h	38.22	K & R	0.03	T2
3	12.82	K & R	0.03	
3 hR	06.29	S	0.1	SL3
5 h	4361.87	K & R	0.03	T2
6 hR	38.00	K & R	0.05	t
4	26.60	K & R	0.03	T2
5 hR	19.39	K & R	0.05	t
2 h	13.38	S	0.1	SL2
4 h R	08.49	K & R	0.1	t

STRONTIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
6	4305.60	K & R	0.1	P2
1 hR	4253.7	S	0.3	SL3
10 r	15.66	K & R	0.03	PH
1 h	02.95	S	0.1	SL2
5	4161.95	K & R	0.03	P2
2	40.50	S	0.1	
2 h	4087.67	F	....	t
10 r	77.88	K & R	0.03	PH
2 h	71.01	F	....	t
2 h	61.21	F	....	t
1 hR	51.1	S	0.2	
3	33.25	K & R	0.05	T1
5 hV	32.51	K & R	0.05	T1
7 hV	30.45	K & R	0.05	T1
5 h	3970.15	K & R	0.05	T1
5	69.42	K & R	0.05	T1
2 h	62.75	S	0.2	
2 h	50.96	F	....	t
4 hV	40.91	K & R	0.05	T1
1 h	35.33	F	....	t
1 h	26.27	F	....	t
0 h	3867.3	S	0.3	
4	65.59	K & R	0.1	T2
4	07.51	K & R	0.1	T2
3	3780.58	K & R	0.1	T2
7 vh	05.88	K & R	0.2	T1
5 h	3653.90	K & R	0.1	T1
6 h	53.32	K & R	0.1	T1
5 h	29.15	K & R	0.1	T1
3	28.62	K & R	0.1	T2
2	3577.45	K & R	0.1	T2
2	53.6	S	0.1	T2
3 vh	47.92	K & R	0.3	T1
2 vh	04.70	K & R	0.3	T2
3 h	3499.40	K & R	0.2	T1
2 h	77.33	K & R	0.1	T1
3 h	75.01	K & R	0.05	P1
6	64.58	K & R	0.03	P1
2 vh	57.70	K & R	0.2	T1
1	56.78	K & R	0.2	T2
1	34.95	K & R	0.5	T2
1 h	11.62	K & R	0.5	T1
1 h	00.39	K & R	1.0	T1
1 h	3390.09	K & R	0.5	T1
5	80.89	K & R	0.2	P1
1	71.10	EH	....	
4	66.43	K & R	0.03	T
5 r	51.35	K & R	0.05	p
3	30.15	K & R	0.03	
3	22.32	K & R	0.03	T
3 r	07.64	K & R	0.05	p
3	01.81	K & R	0.05	T
1 h	3200.4	K & R	0.2	
1 h	3199.1	K & R	0.2	

STRONTIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
1 vh	3190.1	K & R	0.2	
1 vh	89.4	K & R	0.2	
1 vh	82.4	K & R	0.2	
1 vh	72.3	K & R	0.2	
4	2931.98	K & R	0.03	SL1
3 r. h	2569.60	EH. S	0.1	SL1
1	49.62	S	0.1	
2	2471.71	EH. S	0.1	P2
2 h	28.16	EH. S	0.1	SL1
1	23.67	EH. S	0.1	P2
1 r. h	2354.40	S	0.1	SL1
1	24.60	S	0.1	P1
2	22.47	EH. S	0.1	P1
1 h	07.5	S	0.2	SL1
2	2282.14	EH. S	0.2	P1
1 h	75.5	S	0.3	SL1
1 h	53.5	S	0.5	SL1
1 h	37.4	S	1.0	SL1
1 h	26.0	S	1.0	SL1
3 r	2166.11	EH. S	0.2	vp ?
2	52.82	EH. S	0.2	vp ?
1 h	2053.3	S	0.5	P2
0 h	20.5	S	0.5	P2
0 h	1995.7	S	1.0	P1
0 h	65.2	S	1.0	P1

BARIUM ARC SPECTRUM

Intensity	Wave-Length	Observer	E	Series
2	8915.19	H	0.4	
1	8861.40	H	0.3	
0	8799.86	H	0.3	T ?
1	8654.33	H	0.3	
1	8560.20	H	0.3	p
6	8210.73	H	0.3	
0	8148.32	H	0.4	T ?
0	20.84	H	0.3	
0	7961.23	H	0.2	p
0	39.21	H	0.2	
4	11.53	H	0.3	T ?
7	06.13	H	0.2	T2
0	7878.13	H	0.2	
1	39.57	H	0.2	
5	7780.70	H	0.1	
2	51.92	H	0.1	
1 h	09.96	H	0.1	
1	06.82	H	0.1	
5	7672.42	H	0.1	
3	43.42	H	0.1	
1 h	37.47	H	0.1	
2	10.74	H	0.1	
6	7488.38	H	0.1	
6	60.27	H	0.1	

BARIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
6	7417.80	H	0.1	
1	10.6	S	0.3	
7	7392.83	H	0.1	T2
4	76.10	H	0.1	
3	59.67	H	0.1	
10	7280.58	H	0.1	
10	29.40	H	0.2	
2	13.83	H	0.1	
2	08.50	H	0.1	
9	7195.71	H	0.2	T2
5	53.72	S	0.3	
8	20.73	H	0.2	
9	7090.51	H	0.1	
1	69.64	H	0.1	
10 r	60.26	H	0.1	
1	6989.82	H	0.3	
3	87.27	H	0.3	
0 h	62.77	H	0.3	
2	6874.35	H	0.1	
4	68.04	H	0.1	
8	65.93	H	0.1	
4	6772.07	H	0.1	
1	61.90	H	0.1	
7	6694.2	S	0.2	
7	75.30	K & R	0.2	
4	54.4	S	0.2	
8	6595.55	K & R	0.1	
3	81.0	S	0.2	
7	27.56	K & R	0.1	
7	6498.93	K & R	0.1	
10 r	97.07	K & R	0.1	
7	83.10	K & R	0.1	
3	51.05	K & R	0.1	
7	6341.88	K & R	0.1	
2 vh	23.0	S	0.5	
2 vh	6258.7	S	0.5	
2 vhR	33.9	S	0.5	
10 r	6141.93	K & R	0.03	
6	11.01	K & R	0.03	
1	6083.63	K & R	0.15	
5	63.33	K & R	0.03	
4	19.69	K & R	0.03	
4	5997.31	K & R	0.03	
1	78.72	K & R	0.20	
4	71.94	K & R	0.03	
3 h	65.06	K & R	0.20	
1 h	62.8	S	0.2	
4	07.88	K & R	0.05	
8 r	5853.91	K & R	0.1	
7 r	26.50	K & R	0.03	
4 h	19.21	K & R	0.05	T1
5 r	05.86	K & R	0.05	|
7 r	00.48	K & R	0.05	T1
3 h	5784.24	K & R	0.15	

BARIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
9 r	5777.84	K & R	0.03	T1
1 h	21.66	K & R	0.3	
3 hR	13.62	K & R	0.15	
3 hR	09.82	K & R	0.15	
4 r	5680.34	K & R	0.05	
1 h	64.5	S	0.2	?Ba
1 h	40.8	S	0.2	?Ba
4 vhR	20.41	K & R	0.5	
3 h	5593.45	K & R	0.1	
7	36.07	K	0.05	T1
10 r	35.69	K & R	0.05	SL?
9 r	19.37	K & R	0.05	T1
4 h	5473.94	K & R	0.1	
4 h	37.66	K & R	0.1	
9 r	24.82	K & R	0.05	T1
4 h	05.2	S	0.2	?Ba
1 hR	5393.47	K & R	0.1	
1 hR	81.25	K & R	0.5	
1 hR	79.05	K & R	1.0	
2 hR	65.46	K & R	0.5	
2 hR	09.20	K & R	0.3	
2	05.99	K & R	0.2	
4 hR	02.97	K & R	0.2	
3	5294.40	K & R	0.1	
2 h	91.16	K & R	0.2	
1 h	79.72	K & R	0.2	
3 vh	77.84	K & R	0.15	
5 vh	67.20	K & R	0.1	
1 vh	53.94	K & R	0.1	
2 h	5177.60	K & R	0.05	
3 h	75.74	K & R	0.05	
4 h	60.27	K & R	0.1	
3 h	5055.12	K & R	0.1	
3 h	4947.50	K & R	0.1	
10 r	34.24	K & R	0.03	PH
4 hR	03.11	K & R	0.05	T2
8	00.13	K & R	0.05	P2
4 vh	4877.99	K & R	0.15	
1 vh	07.6	S	0.2	
1 h	4739.40	S	0.2	
1 h	32.53	S	0.2	
6 r	26.63	K & R	0.05	
1 h	24.98	K & R	0.1	
5 hR	00.64	K & R	0.05	T2
2 h	4698.30	S	0.2	
6 r	91.74	K & R	0.03	T
6 hV	73.69	K & R	0.05	
1 h	70.95	S	0.2	
1 h	68.76	S	0.2	
1 h	63.80	S	0.2	
1 h	51.60	S	0.2	
1 hR	42.38	K & R	0.1	
2 hR	36.80	K & R	0.1	
5 hV	28.45	K & R	0.05	

BARIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
1 h	4624.38	S	0.2	
4 hR	20.19	K & R	0.05	T2
3 hV	05.11	K & R	0.05	
5 hR	00.02	K & R	0.05	
3 hV	4591.88	K & R	0.05	
3 hV	89.82	K & R	0.05	
6 r	79.84	K & R	0.03	p
5 r	74.08	K & R	0.05	
10 r	54.21	K & R	0.03	PH
5	25.19	K & R	0.05	P2
5 r	23.48	K & R	0.1	
5	06.11	K & R	0.1	T
5 hV	4493.82	K & R	0.2	T1
6 hV	89.50	K & R	0.5	T1
4	67.36	K & R	0.05	
5 r	32.13	K & R	0.03	T
3	13.96	K & R	0.05	
4 hR	07.10	K & R	0.05	
6 r	02.75	K & R	0.05	p
1 vh	4376.05	S	0.2	
3	59.80	K & R	0.05	
7 r	50.49	K & R	0.1	
5 hV	33.04	K & R	0.05	T1
4	25.38	K & R	0.05	
5 hV	23.15	K & R	0.05	T1
4	4291.32	K & R	0.05	
7 r	83.27	K & R	0.03	
5 h	64.45	K & R	0.05	T1
1 vh	52.6	S	0.2	
3 hR	42.83	K & R	0.05	
4 hR	39.91	K & R	0.05	T2
4	24.11	K & R	0.05	
3 hR	4179.57	K & R	0.2	
5	66.24	K & R	0.05	P1
4	32.60	K & R	0.05	
8 r	30.88	K & R	0.05	P1
2	10.46	K & R	0.1	
2 vh	4087.53	K & R. S	0.1	T1
3 vh	84.94	K & R. S	0.1	T1
2 h	81.1	K & R. S	0.1	
2	26.57	S	0.1	T2
4	3995.92	K & R	0.1	t
8 r	93.60	K & R	0.03	t
2 hR	75.55	K & R	0.1	T2
1 vh	47.6	S	0.2	T1
2 vh	45.6	S	0.2	T1
4	38.09	K & R	0.05	t
7 r	35.87	K & R	0.05	t
3	17.42	K & R	0.05	
7 r	10.04	K & R	0.05	t
2 h	06.20	K & R	0.05	
2 h	00.54	K & R	0.05	
1 h	3895.2	S	0.2	T1
5 h	92.93	K & R	0.1	

BARIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
9 h	3891.97	K & R	0.05	P1
5	89.45	K & R	0.05	
4 h	61.87	K & R	0.15	
1 h	41.72	S	0.2	T2
4 h	3794.77	K & R	0.2	
0 vh	87	S	1.0	T1
0 vh	67.5	S	1.0	T1
2 h	01.87	K & R	0.15	
4 hR	3689.28	K & R	0.15	
2 h	64.76	K & R	0.1	
4	62.62	K & R	0.05	
2	40.72	S	0.1	
3 vh	37.10	K & R	0.5	
7	30.84	S	0.1	
4 hR	11.17	K & R	0.1	
4	3599.60	K & R	0.05	
4 hR	93.58	K & R	0.05	
3 h	88.33	K & R	0.1	
3	86.64	K & R	0.1	
6 r. h	79.97	K & R	0.1	
4 h	77.79	K & R	0.05	
3	76.20	K & R	0.1	
1 h	72.0	S	0.2	
3 h	66.90	K & R	0.05	
3 h	62.23	K & R	0.1	
1 h	52.5	S	0.2	
3 h	48.14	S	0.05	
6 r. h	44.94	S	0.1	
2 vh	31.8	S	0.2	
2	29.6	S	0.2	
5 r. h	25.23	K & R	0.1	
8 r	01.29	K & R	0.05	
1 vh	3464.5	S	0.3	
5 r. hR	20.48	K & R	0.1	t
4 r. hR	3377.19	K & R	0.1	t
3 r. hR	57.00	K & R	0.1	t
3 hR	23.06	K & R	0.1	
1 h	15.90	K & R	0.1	
1 hR	3298.25	K & R	0.1	
3 hR	81.96	K & R	0.1	
3 hR	62.57	K & R	0.1	
2 vh	23.11	K & R	0.2	t
1 vh	04.09	K & R	0.2	
1 vh	3184.45	K & R	0.3	t
0 vh	74	S	1.0	
0 vh	65	S	1.0	
0 vh	3119.48	K & R	0.5	t?
1 h	08.37	K & R	0.5	
7 r	3071.71	K & R	0.03	
4 h	2785.22	K & R	0.15	
4	71.51	K & R	0.1	P2
2	39.12	S	0.1	
3 r	02.78	S	0.03	
3	2647.41	S	0.05	P2

BARIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
1 h	2646.65	S	0.1	
2 h	41.52	K & R	0.05	P1
6	34.91	K & R	0.05	P1
2 r. vh	2597.26	K & R. S	0.2	
0 vh	92.40	S	0.2	
0 vh	44	S	2.0	
5	28.52	EH	...	P1
1 h	00.27	S	0.1	
0 h	2473.20	S	0.1	
5	2347.67	K & R	0.05	
9 r	35.33	K & R	0.05	vp?
8 r	04.32	K & R	0.05	vp?
2	2286.21	S	0.1	P2
3	54.80	K & R	0.1	
2	45.72	K & R	0.1	
0	35.50	S	0.1	P1
2	32.79	S	0.1	P1
2	14.79	S	0.1	
0	01.07	S	0.2	P2
1	2154.02	S	0.3	P1
0	2082.8	S	1.0	P2
1	55.0	S	1.0	P1
0	1987.8	S	1.0	P1

sities are on a scale of 10 to 0, subject to the usual inaccuracies. The abbreviations in the first column are: *h* for hazy; *vh*, very hazy; *hR*, broadened on the long-wave side; *hV*, broadened on the short-wave side; and *r*, reversed. The abbreviations under the column headed "Series" are as follows: *TH*, (possible) principal series (Hauptserie) of main triplets; *T1*, first subordinate series of main triplets; *T2*, second subordinate series of the same; *t*, narrow triplets; *T*, triplets occurring in the groups of six; *PH*, principal series of pairs; *P1*, first subordinate series of pairs; *P2*, second subordinate series of pairs; *p*, pairs occurring in the groups of six; *vp*, the "vacuum pair" in *Ca*, and those which there is reason to believe are related to it; *SL1*, the ultra-violet series of single lines; *SL2*, the related series in the visible spectrum; *SL3*, the single line series, alternating with *SL2*.

Bands are prominent in these spectra, especially at low pressures, but they are omitted from the tables.

Syracuse University
August 1910

THE BRIGHTNESS OF HALLEY'S COMET AS MEASURED WITH A SELENIUM PHOTOMETER

By JOEL STEBBINS

Anticipating that Halley's comet would become a brilliant object when near the earth, I planned to measure its light with the selenium photometer which is in regular use at this observatory. Visual estimates of the total light of a comet are necessarily rough, and the advantage of the selenium cell lies in the fact that a comet may be compared with the extra-focal image of a star, and the relative light found from galvanometer deflections.

I have in preparation a paper which describes in detail the methods and precautions necessary in observing with selenium. Let it suffice to state here that the apparatus used for the comet includes a selenium cell attached to the 12-inch refractor, the cell being placed 4 inches (10 cm) outside the focus of the large objective. The cell has an effective aperture of $\frac{3}{4}$ inch (18 mm), but the field is limited by a diaphragm, which it is found admits all the light over a circle of about 7 minutes of an arc in diameter. This is nearly one-fourth of the moon's apparent diameter, and it is probable that eye estimates of the comet's brightness never included a larger area. Under the conditions of the observations, a 10-second exposure to a 3.0 magnitude star gives a galvanometer deflection of approximately 3.5 mm, and the probable error of a single deflection is of the order of 10 per cent or 0.35 mm. For brighter stars the deflections are increased, but the errors remain about the same, so that the relative accuracy becomes much greater.

As is well known, the color sensibility of selenium is different from that of the eye, being especially great near the red end of the visible spectrum. In a previous number of this *Journal*,[1] I have already published a determination of the color sensibility of the cell in question, Giltay No. 93; and unless the comet had a most peculiar spectrum, I think it probable that this selenium cell gives the visual magnitudes of the total light of the comet with less systematic error than is present in the eye comparison of a luminous surface with a point source of light.

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[1] **27**, 183, 1908.

In observing, the usual procedure was to take alternate exposures on the comet and comparison star, at intervals of about 1 minute, which is necessary to allow the cell time for recovery. In all of the early morning observations there was a bright sky background, and the exposures were made in the order: sky, star, sky, comet, sky, etc., until the effect from the sky became predominant. When the moon was bright, similar precautions were necessary, and of course in the reductions the extra light of the sky was allowed for.

Although the effect of differential atmospheric absorption may be practically eliminated under proper conditions, it was not possible to do this in the comet observations. The correction for absorption is based upon the Potsdam tables,[1] and I allowed for the increase of absorption on poor nights by multiplying by an arbitrary factor, which was determined from the general appearance of the sky at the time of the observations. This is, of course, only a guess at the true value of the absorption, but a check is secured in the size of the galvanometer deflections given by the same star on different nights.

In the following table are given the results of the observations, the headings of the different columns being self-explanatory. The magnitudes of the comparison stars are from Vol. **50** of the *Harvard Annals*. The difference of magnitude between the comet and star follows from the ratio of the observed galvanometer deflections.

The results of this paper are shown in the column of adopted magnitudes, the values on three nights, when the atmospheric conditions were poor, being in parentheses. Unfortunately, the only really clear nights between May 1 and 24 were those of May 3, 4, 17, and 18, the head of the comet not being visible on the latter two dates. Smoke from forest fires spoiled what would otherwise have been a clear sky. On May 20, a few readings were taken in what seemed to be perfectly clear spaces between passing clouds, but there was no opportunity to use a comparison star. On this date my direct visual estimate of the comet was magnitude 1.0. Not enough material is here available to exhibit the variation of the comet's brightness from night

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[1] Müller, *Photometrie der Gestirne*, p. 515.

OBSERVATIONS OF HALLEY'S COMET WITH THE SELENIUM PHOTOMETER

DATE 1910	G.M.T.	EXPOSURES		ABSORPTION FACTOR	COMPARISON STAR	MAGNITUDE OF STAR	DIFF. OF MAG. COMET – STAR	ABSORPTION	MAGNITUDE OF COMET		REMARKS
		Comet	Star						Observed	Adopted	
May 3	21h 37m	9	7	1	*γ Pegasi*	2.87	−0.74	−0.14	1.99	2.0	Sky perfect
4	21 41	5	4	1	*γ Pegasi*	2.87	−0.95	−0.07	1.85	1.8	
9	21 39	4	4	2	*γ Pegasi*	2.87	−1.25	−0.42	1.20	(1.2)	Sky thick and smoky
11	21 31	6	7	3	*γ Pegasi*	2.87	−1.26	−1.05	0.56	(0.6)	Sky still worse
20	14 4	4	..	1	*β Geminorum*	1.21	+0.79	−0.92	1.08	(1.1)	Between clouds, comparison star on May 24
24	14 38	6	4	1	*β Geminorum*	1.21	+0.64	−0.03	1.82	1.8	
25	15 36	5	4	1.5	*ζ Hydrae*	3.30	−0.74	+0.07	2.63	2.6	Sky only fair
26	15 39	6	2	1	*ζ Hydrae*	3.30	−0.81	+0.10	2.59		
26	15 39	..	5	1	*o Leonis*	3.76	−0.71	−0.21	2.84		
26	15 39	..	2	1	*α Leonis*	1.34	+1.68	−0.34	2.68	2.7	
29	14 48	5	3	1	*ζ Hydrae*	3.30	+0.27	+0.15	3.72		
29	15 19	9	12	1	*o Leonis*	3.76	−0.20	−0.11	3.45	3.5	
30	15 51	11	12	1	*o Leonis*	3.76	−0.04	−0.07	3.65	3.6	Cell irregular
31	15 9	13	12	1	*o Leonis*	3.76	−0.36	−0.05	3.35	3.4	
June 1	15 35	15	12	1	*o Leonis*	3.76	−0.15	−0.05	3.56	3.6	

to night, but it is possible that these measures will be of use for comparison with the work of other observers. As determined with the selenium photometer, Halley's comet became brighter than first magnitude, but at no time when we could observe it did it reach magnitude 0.0.

I am indebted to Mr. Percy F. Whisler for his assistance in reading the galvanometer throughout all of the observations.

University of Illinois Observatory
June 15, 1910

MINOR CONTRIBUTIONS AND NOTES

VARIABLE RADIAL VELOCITIES OF TWO STARS IN THE *TAURUS* STREAM

The following two stars were included among six reported as having variable velocities by Professor Frost[1] at the Baltimore meeting of the American Association in 1909. The measures have been made on plates taken with the Bruce spectrograph with one prism.

B.D. 15°637 *Tauri* ($\alpha=4^h\ 25^m$; $\delta=+15°\ 59'$; Mag. = 5.0)

Plate	Date	G.M.T.	Taken by	No. Lines	Velocity	Quality
1B 1822.....	1908 Nov. 2	$21^h\ 57^m$	B	7	+41 km	Good
1878.....	Nov. 30	20 34	B	6	32	Good
1882.....	Dec. 4	14 15	F, B	14	30	Good
1911.....	Dec. 21	15 43	L	15	50	Good
1945.....	1909 Jan. 1	16 52	B	13	49	Good
2143.....	Sept. 20	20 26	B	6	34	Weak

B = Barrett; F = Frost; L = Lee.

The spectrum of this star, as well as the following one, is of Vogel's type Ia2, or A5 according to the system adopted in the Harvard Revision.

90 c_1 *Tauri* ($\alpha=4^h\ 38^m$; $\delta=+12°\ 19'$; Mag. = 4.7)

Plate	Date	G.M.T.	Taken by	No. Lines	Velocity	Quality
1B 1855.....	1908 Nov. 13	$18^h\ 10^m$	B	20	+32 km	Good
1877.....	Nov. 30	19 21	B	20	62	Good
1883.....	Dec. 4	15 27	B	11	52	Good
1903.....	Dec. 14	19 38	B	24	85	Good

Evidence for the occasional duplicity of many of the lines in these two stars seems almost conclusive, but consistent results for the two components have not as yet been obtained. The reliability of the measures is somewhat impaired by this apparent complexity.

STORRS B. BARRETT

YERKES OBSERVATORY
June 20, 1910

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[1] *Astrophysical Journal*, **29**, 233, 1909.

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Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

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THE
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THE MEASUREMENT OF THE LIGHT OF STARS WITH A SELENIUM PHOTOMETER, WITH AN APPLICATION TO THE VARIATIONS OF *ALGOL*

By JOEL STEBBINS

Some three years ago I became interested in the possible application of selenium to astronomical photometry, and with Dr. F. C. Brown, I began to experiment on selenium cells, with the hope of using them for accurate measures of the light of stars. The idea is an attractive one: the proposed method being to expose a selenium surface to the light of a star, focused by a large lens, and to note the change of resistance by means of a galvanometer: the brighter the star, the larger the galvanometer-deflection. In theory this is very simple, but at the outset we met some of the difficulties which confront everyone who tries to work with selenium. Other agencies than light affect the resistance, and apparently no experimenter has solved, to his own satisfaction, the mysteries of this peculiar element.

After some futile attempts to obtain results from starlight at the focus of a 12-inch refractor, we found that a selenium cell gave large effects when exposed directly to the moon. This led to a study of the variation of the moon's light throughout a lunation,[1] and it was also found that various selenium cells have widely different curves of color-sensibility.[2] One reason that physicists and electricians have not used these cells in photometry is that selenium is relatively most sensitive to red light, but in certain kinds of astronomical work, such as the study of eclipsing variable stars, it is quite immaterial what part of the spectrum is used.

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[1] *Astrophysical Journal*, 26, 326, 1907.

[2] *Ibid.*, 27, 183, 1908.

During several years of experimenting I have tried the selenium cells of different makers, whose processes are secret, also a number of cells made by Dr. Brown and myself. The best cells known to me are those by Giltay of Delft, Holland. In the form which he uses, two wires are wound in a double spiral about a flat insulator, and the spaces on one face are filled with selenium which is properly treated to make it sensitive. In response to my inquiry, Mr. Giltay has said that his method of sensitizing is indeed a secret, but he adds that it is often a secret even to himself, for after thirty years of experience in making cells he frequently has surprises, and usually of the disagreeable kind.

In working with faint lights, we found that a small variation in the temperature of a cell gives a greater change of resistance than is produced by the light of the brightest stars. I made numerous attempts to maintain a constant temperature with thermostats, but although some improvement was noted, the irregularities remained almost hopelessly large. One of the fundamental obstacles is the heating effect of the current which is necessarily passed through the selenium to measure the resistance. It is unnecessary to enumerate the various experiments which were made, but after a long time I learned that there are three simple precautions necessary to insure success, and they are given in what seems to be the relative order of importance:

1. The selenium should be kept at a uniform, low temperature, 0° C., or lower.

2. The current should pass continually through the selenium.

3. Exposures to light should be short, say 10 seconds, with longer intervals for recovery.

Such success as I have secured has been through the observance of these simple precepts, and although other experimenters have not formulated such rules, I have been quite unable to do anything in measuring starlight without adhering to them. In discussing the reasons for these precautions, it is not possible to give numerical results which are applicable to different cells, but the general principles seem to hold wherever faint lights are to be measured.

First, the effect of low temperature is best illustrated by an example. A cell of about 1,000,000 ohms resistance at 20° C. increases to 3,000,000 ohms at 0° C. It is found that small variations in temperature produce relatively much less effect upon the higher resistance, and, in terms of galvanometer-deflections, the irregularities are in the ratio of about 50 to 1 in favor of the low temperature. Also, the light-action is about twice as great at 0° as at 20°, and we have the rather astonishing result that it is possible to increase the effective sensibility of a cell 100 fold merely by reducing its temperature. In practice it is not difficult to surround a cell with an ice-pack, but moisture must be kept from the cell and from the insulation of the connecting wires.

Second, the current which passes through the selenium should not be interrupted. If an E.M.F. of 6 volts be applied to a cell of 3,000,000 ohms, the resistance decreases slowly, until at the end of half an hour a steady condition is reached where the resistance is about 10 per cent less than at the start. This change is presumably due to the heating of the current, which is enough to account for the decrease. In terms of galvanometer-deflections, this heating effect is of the order of 100 times the light-effect from a bright star, and therefore the cell should not be disturbed by breaking the circuit.

Third, as is well known, selenium requires some time for recovery from light-action, and it is found that if a cell be exposed for 10 seconds to starlight, it is best to wait about 1 minute to insure recovery and allow the cell to regain its original sensibility.

Other precautions are the same as must be taken in any work where high resistances and a sensitive galvanometer are used. Good insulation is essential, and the connecting wires must be shielded from the effect of static charges, which are especially large in cold weather.

As the character of the galvanometer used is of considerable importance, I shall give the constants of the one I have at present, which was made by the Weston Electrical Instrument Co. It is of the d'Arsonval type, with flat mirror 8 mm in diameter, and is used with a view telescope magnifying 25 times, and a millimeter scale placed at from 2 to 4 meters distance. The resistance is

512 ohms. The period of complete free swing is 5.0 seconds, and when connected in circuit the instrument is practically dead beat. The "figure of merit," or current required to produce 1 mm deflection at 1 meter scale-distance, is 2.4×10^{-10} amperes, but because of the excellence of the optical system a current of 5×10^{-12} amperes can be detected with certainty. Although this sensibility is not the highest possible in a galvanometer, it is the best I have ever seen in one of the d'Arsonval type, and there is the great advantage that the instrument is practically unaffected by external conditions.

In the arrangement for measuring starlight the selenium cell is connected as one arm of a Wheatstone bridge. Current is supplied by a few dry cells, which are ample for the purpose. The resistances of the selenium cell, galvanometer, and battery being fixed, the other arms of the bridge should be chosen to give the maximum current through the galvanometer for a small change in the resistance of the selenium. Convenient formulae for the best arrangement have been derived by Heaviside;[1] but as megohm resistance boxes are somewhat expensive, I have found that with resistances of 10,000 ohms and less it is possible to obtain fully three-fourths of the computed maximum sensibility. Precaution must be taken that the resistances are so arranged that the galvanometer is not damped excessively, in order that full deflection may be secured.

Fig. 1 shows the ice chamber containing a cell, attached to our 12-inch telescope. In warm weather the ice is renewed every day, but in winter the end of the telescope is wrapped up in a blanket, and may not be disturbed for a month at a time. Exposures to starlight are made by opening a small shutter which is operated by hand, the observer listening to the beats of a chronometer. In Fig. 2 are shown the galvanometer, scale, and resistance boxes which are located in a room some distance from the dome, the two wires leading from the telescope being easily carried to where it is convenient to mount the galvanometer. In practice we always have two observers, one to point the telescope and make the exposures, while the other reads the galvanometer and records the measures. Contrary to astronomical custom, the recorder is the real observer in this work, and the assistant is stationed at the telescope. In fact, it often happens that the man at the galvanometer can tell that the sky is growing less transparent, before the observer in the dome can ascertain the same by direct inspection.

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[1] *Philosophical Magazine*, 45, 114, 1873.

This brief outline of the method of using selenium in photometry will perhaps indicate the general way in which the observations are made, but the procedure will be better illustrated by giving the actual figures and results derived in an exhaustive study of a variable star.

APPLICATION TO THE LIGHT-VARIATIONS OF *ALGOL*

As soon as I began experimenting with selenium, I had it in mind as one of the problems to take up, when it became possible to measure starlight with accuracy, to investigate the light-curve of *Algol*, especially for a secondary minimum, which, if present, would give us valuable information concerning the companion. The history of the investigations of *Algol* is so well known, even to the general reader, that it is superfluous to mention the names of the astronomers who have contributed to our knowledge of this star. In fact, we may say that if any star in the whole sky has been subjected to an exhaustive study, *Algol* is that one. For this very reason, however, it seemed advisable to try the selenium photometer, for this star would furnish an excellent test of the improvement of our work over visual methods.

It was found on trial that with good conditions a 10-second exposure of our best selenium cell to *Algol* at maximum light gave a scale deflection of 8.0 mm. The probable error of a single deflection was of the order of 0.16 mm, or about 2 per cent, and as this was a better accordance than can be secured in visual work, I felt ready to adopt a program for observing this star during several months. In illustration of the accuracy which was insisted upon, it may be said that when the cell happened to become so irregular that the probable error of one deflection became higher than 5 per cent, observations were not undertaken.

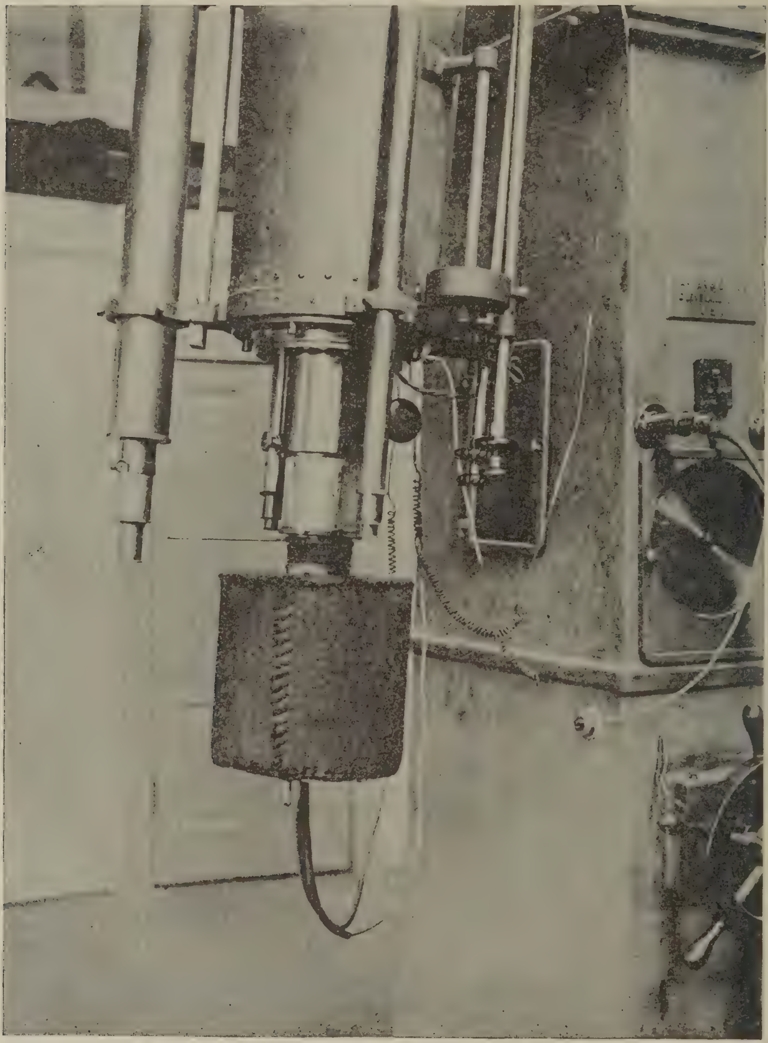

Fig. 1.—Selenium Cell in Ice-Pack Attached to 12-Inch Refractor

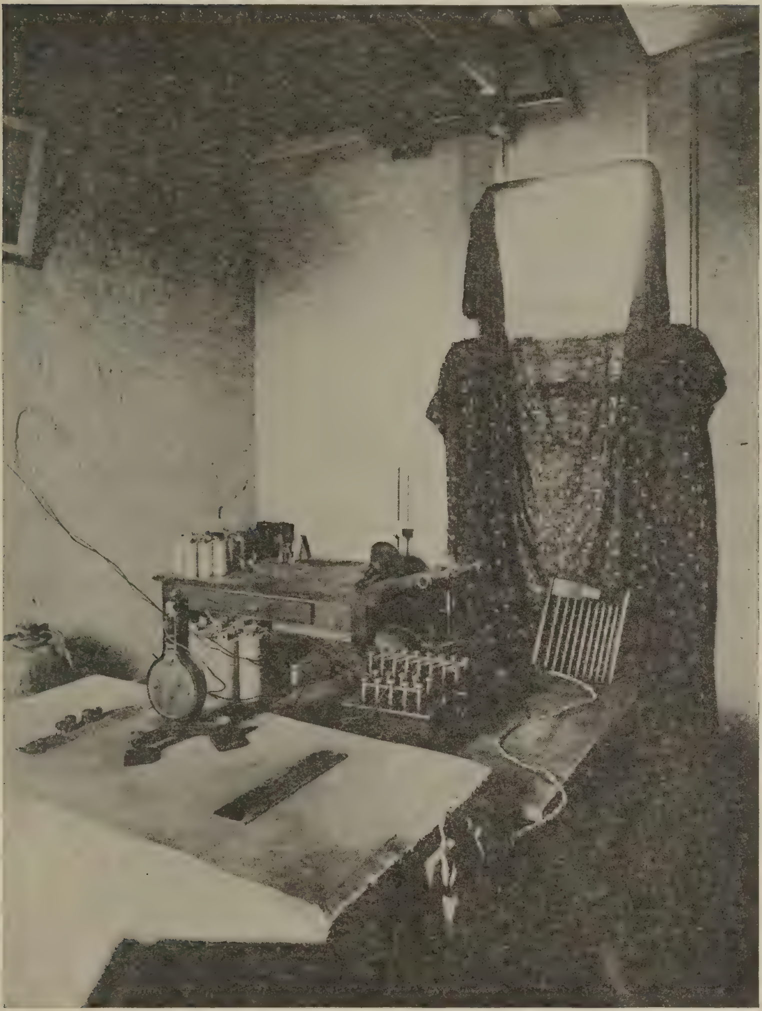

FIG. 2.—Galvanometer and Accessories for the Selenium Photometer

On account of the atmospheric absorption, there is of course no method of measuring the light of a star, which does not include observations of one or more comparison stars. On consideration, I decided to use *a Persei* and *δ Persei*. After making this decision, I found that these same stars had been chosen by Müller,[1] whose determination of the light-curve is perhaps the best that has ever been made visually. The normal plan with selenium was to make 4 exposures on *a Persei*, then 8 on *Algol*, and finally 4 on *a Persei*. Such a series of 8 readings on each star will be called a set. When *Algol* was near minimum, both *a* and *δ* were used, and a set of readings usually consisted of 4 on *a*, 4 on *δ*, 8 on *Algol*, 4 on *δ*, and 4 on *a*. When only two stars were observed, the 16 deflections of a set were secured in about 20 minutes, including the time for moving the dome and telescope. Ordinarily it was considered that 4 to 6 sets of measures were sufficient for one night, but when the phase of *Algol* was near principal minimum, or the supposed secondary minimum, the observations were continued all night, or as long as conditions would permit. The method of observation and reduction is best illustrated by an actual sample, and the following represents a typical page of the notebook.

FRIDAY, JANUARY 7, 1910

Observers, J. S. and P. F. W.
Cell, Giltay No. 93, area 18 × 26 mm.
Extra-focal star image, 7 mm diameter.
Ratio arms, 10,000 and 10 ohms.
Resistance of cell, 5,000,000 ohms.
Dome temperature, −5° C.
Galvanometer, Weston.
Scale-distance, 2.3 meters.
Battery, 7 Edison cells, 6 volts.
Exposures, 10 seconds.
Sky, fair; absorption factor, 2.
Wind, south.
Watch ΔT, +4 seconds.

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[1] *Astronomische Nachrichten*, 156, 178, 1901.

Time	Readings	Deflections
	α Persei	mm
$8^h\ 18^m$	280.0–87.3	7.3
19	281.7–88.9	7.2
20	282.9–90.2	7.3
21	284.4–91.6	7.2
		7.25
	Algol	
$8^h\ 24^m$	278.0–84.7	6.7
25	279.9–86.4	6.5
26	281.9–88.2	6.3
27	283.3–89.7	6.4
28	284.3–90.4	6.1
29	285.2–91.3	6.1
30	285.7–92.1	6.4
31	286.8–93.1	6.3
		6.35
	α Persei	
$8^h\ 34^m$	282.1–90.0	7.9
35	285.6–93.3	7.7
36	288.3–96.0	7.7
38	290.2–97.8	7.6
		7.72 7.25
		Mean 7.48

REDUCTION

Mean time	$8^h 28^m$	
Sidereal time	3 42	
Star	*α Persei*	*Algol*
Deflection, D	7.48	6.35
Drift	−0.02	−0.02
Corrected D	7.46	6.33
Log. D	0.873	0.801
Log. ratio	0.072	
Difference of magnitude	0.18	
Absorption	0.00	
Corrected difference	0.18	

RESULTS ON JANUARY 7, 1910

Time	Difference of Magnitude	Residual	Time	Difference of Magnitude	Residual
$8^h 28^m$	0.18	0.00	$9^h 26^m$	0.16	−0.02
8 47	0.17	−0.01	10 00	0.22	+0.04
9 07	0.20	+0.02	10 19	0.16	−0.02
Means 8 47	0.18		9 55	0.18	±0.02

The ratio of the scale-deflections gives at once the difference of magnitude of the two stars, the main corrections which need to be applied being due to progressive drift of the galvanometer, and differential atmospheric absorption. At the beginning of each night's work, a series of readings was taken to determine the rate of change of the galvanometer zero, due to small temperature-variation in the selenium cell, this being repeated about once an hour, and the drift was then interpolated for the times of the measures of the stars. Near minimum, the difference in the deflections between *Algol* and *α Persei* was so large that the drift was of much consequence, and measures of *δ Persei* were utilized for the determination of drift. From the observations on a number of nights, the difference of magnitude between *α* and *δ Persei* was found to be 1.10 magnitudes, and this value was used for computing the drift.

The correction for atmospheric absorption was based upon the Potsdam tables by Müller.[1] A table of the correction was computed for *Algol*, *α* and *δ Persei* for each half hour of sidereal time, and the interpolated value from this table was multiplied by the factor, 1.0, 1.5, or 2.0, depending upon my judgment of the transparency of the sky. This may seem like a crude method, but in four-fifths of the cases the correction was either 0.00 or 0.01 magnitude. When *Algol* became relatively low in the west, additional observations for the absorption-factor were taken on *β Andromedae*, but there were only 3 sets of measures where the correction was greater than 0.04 magnitude.

Near the time of full moon, the bright background of the sky gives an appreciable deflection of the galvanometer; but where necessary this effect was carefully determined, and allowed for in the reductions.

Table I contains the results of observations which were taken of *Algol* from September 1909, to March 1910. The heliocentric phase was derived from Hartwig's ephemeris, which is based upon Chandler's elements. After combining the observations near principal minimum, I found a correction to the ephemeris of $-1^h\ 16^m$, and this has already been applied in Table I. The

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[1] *Die Photometrie der Gestirne*, Leipzig, 1897, p. 515

TABLE I

Observations of *Algol* near Principal Minimum

Date	G.M.T.	Heliocentric Phase	Difference of Magnitude	Date	G.M.T.	Heliocentric Phase	Difference of Magnitude
September 9,'09	18^h22^m	$+1^h11^m$	1.06	September 29,'09	22^h16^m	$+3^h24^m$	0.36
	19 04	+1 53	0.66	October 25	15 37	+1 27	1.13
	19 22	+2 11	0.68		16 14	+2 04	0.73
	19 56	+2 45	0.40		16 51	+2 41	0.59
	20 22	+3 11	0.43		17 20	+3 10	0.40
	21 08	+3 57	0.23		17 59	+3 49	0.30
	21 26	+4 15	0.21		18 26	+4 16	0.22
	21 56	+4 45	0.16	October 28	16 25	+5 26	0.17
September 12	16 42	+2 42	0.66		16 44	+5 45	0.18
	17 13	+3 13	0.46		17 06	+6 07	0.21
	17 44	+3 44	0.37		17 25	+6 26	0.18
	18 10	+4 10	0.25	November 14	13 36	−2 16	0.63
	18 28	+4 28	0.25		14 05	−1 47	0.87
	18 53	+4 53	0.16		14 35	−1 17	1.03
September 15	16 36	+5 48	0.14		15 09	−0 43	1.26
	17 14	+6 26	0.12		15 41	−0 11	1.39
	17 34	+6 46	0.12		16 14	+0 22	1.34
	17 53	+7 05	0.21		16 44	+0 52	1.19
September 26	16 40	−5 23	0.14		17 15	+1 23	1.11
	17 11	−4 52	0.24	December 7	13 07	−1 18	1.01
	17 48	−4 15	0.16		13 45	−0 40	1.20
	18 09	−3 54	0.31		14 18	−0 07	1.35
	18 48	−3 15	0.40		14 51	+0 26	1.36
	19 32	−2 31	0.61		15 29	+1 04	1.06
	20 12	−1 51	0.76		16 15	+1 50	0.84
	21 24	−0 39	1.17	February 5,'10	13 48	−5 52	0.17
September 29	17 35	−1 17	0.95		14 06	−5 34	0.14
	18 03	−0 49	1.28		14 25	−5 15	0.20
	18 46	−0 06	1.30		14 43	−4 57	0.19
	19 19	+0 27	1.42		15 05	−4 35	0.22
	19 50	+0 58	1.01		15 23	−4 17	0.22
	20 18	+1 26	0.95		15 39	−4 01	0.23
	21 11	+2 19	0.66		15 53	−3 47	0.30
	21 41	+2 49	0.45				

Observations near Secondary Minimum

Date	G.M.T.	Heliocentric Phase	Difference of Magnitude	Date	G.M.T.	Heliocentric Phase	Difference of Magnitude
September 16,'09	18 45	31 57	0.13	October 15,'09	18 37	39 41	0.12
	19 07	32 19	0.15		18 55	39 59	0.10
	19 28	32 40	0.14		19 13	40 17	0.11
	19 56	33 08	0.17	November 4	15 21	34 44	0.19
	20 24	33 36	0.18		15 43	35 06	0.15
	20 42	33 54	0.16		16 05	35 28	0.19
	21 00	34 12	0.20		16 39	36 02	0.18
	22 00	35 12	0.20		16 59	36 22	0.19
	22 20	35 32	0.15		17 19	36 42	0.16
September 25	16 08	38 54	0.15		17 47	37 10	0.12
	16 28	39 14	0.14		18 15	37 38	0.14
	16 50	39 36	0.12	November 24	19 58	37 38	0.13
	17 09	39 55	0.17		20 16	37 56	0.14

TABLE I—*Continued*

Date	G.M.T.	Heliocentric Phase	Difference of Magnitude	Date	G.M.T.	Heliocentric Phase	Difference of Magnitude
November 24,'09	20^h35^m	38^h15^m	0.16	December 20,'09	16^h42^m	39^h00^m	0.10
	20 53	38 33	0.14		17 00	39 18	0.12
	21 12	38 52	0.14	January 6, '10	15 07	32 29	0.15
December 17	13 12	32 19	0.16		15 39	33 01	0.15
	13 30	32 37	0.13		16 43	34 05	0.14
	13 51	32 58	0.21		17 07	34 29	0.19
	14 11	33 18	0.14		17 34	34 56	0.19
	14 45	33 52	0.24	February 1	12 28	34 27	0.16
	15 06	34 13	0.23		12 46	34 45	0.22
	15 28	34 35	0.21		13 05	35 04	0.20
	17 57	37 04	0.18		13 23	35 22	0.19
	18 16	37 23	0.14		13 42	35 41	0.17
	18 40	37 47	0.11		14 00	35 59	0.17
	19 04	38 11	0.16		14 31	36 30	0.15
	19 27	38 34	0.14		14 50	36 49	0.17
December 20	11 49	34 07	0.17		15 15	37 14	0.17
	12 11	34 29	0.18		15 34	37 33	0.13
	12 34	34 52	0.21		15 53	37 52	0.15
	12 52	35 10	0.19	February 18	14 32	31 34	0.12
	13 28	35 46	0.20		14 52	31 54	0.15
	13 47	36 05	0.17		15 31	32 33	0.15
	14 11	36 29	0.12		15 55	32 57	0.17
	14 32	36 50	0.18		16 11	33 13	0.19
	14 56	37 14	0.18	March 13	13 12	31 39	0.18
	15 36	37 54	0.13		13 25	31 52	0.14
	15 57	38 15	0.12		13 46	32 13	0.18
	16 15	38 33	0.13		14 15	32 42	0.20

OBSERVATIONS BETWEEN MINIMA

Date	G.M.T.	Heliocentric Phase	Difference of Magnitude	Number of Sets	Date	G.M.T.	Heliocentric Phase	Difference of Magnitude	Number of Sets
Sept. 10,'09	17^h15^m	24^h04^m	0.13	2	Nov. 29,'09	13^h54^m	13^h56^m	0.17	3
11	18 52	49 41	0.14	3	29	15 01	15 03	0.17	3
13	16 58	26 58	0.15	4	Dec. 8	14 02	23 37	0.14	3
17	17 29	54 41	0.16	4	8	15 13	24 48	0.13	3
23	17 02	60 36	0.18	4	9	13 55	47 30	0.12	3
24	16 45	15 31	0.14	4	9	15 00	48 35	0.14	3
27	17 34	19 31	0.15	4	18	15 33	58 40	0.17	3
28	16 57	42 54	0.15	4	18	16 40	59 47	0.17	3
Oct. 1	16 44	45 52	0.12	4	19	12 12	10 30	0.15	3
6	17 26	28 57	0.14	4	19	13 13	11 31	0.15	3
14	16 00	13 04	0.17	3	Jan. 7,'10	14 47	56 09	0.18	3
18	17 32	41 49	0.14	4	7	15 55	57 17	0.18	3
21	15 52	43 20	0.13	4	8	12 34	9 08	0.17	3
21	17 29	44 57	0.13	4	8	13 42	10 16	0.17	3
29	16 06	29 07	0.13	4	31	14 36	12 35	0.17	3
30	16 02	53 03	0.16	4	Feb. 12	13 43	24 24	0.13	3
Nov. 3	17 00	12 23	0.14	4	12	14 21	25 02	0.13	3
5	16 49	60 12	0.15	4	19	13 31	54 33	0.15	3

difference of magnitude is uniformly in the sense: magnitude of *Algol* minus magnitude of *a Persei*. Near minima the result for each set is given, but between minima, where there could be no rapid change in light, I have averaged the several sets of each night.

The observations of Table I were arranged according to phase, and each successive 3 or more sets were combined, the normal magnitudes containing sets as follows:

Near principal minimum	3 sets
Near secondary minimum	6 sets
Between minima	9 or more sets

It was afterward found that the duration of the eclipsing phase is less than 10 hours, and the first and last normals near principal

TABLE II

NORMAL MAGNITUDES

Near Principal Minimum		
Phase	Difference of Magnitude	O.−C.
$-5^{h}18^{m}$	0.18	+0.01
−4 22	0.20	−0.02
−3 54	0.28	0.00
−2 41	0.55	0.00
−1 39	0.88	−0.01
−1 08	1.09	0.00
−0 41	1.21	−0.05
−0 08	1.35	−0.02
+0 25	1.37	+0.04
+0 48	1.15	−0.06
+1 13	1.08	+0.02
+1 34	0.97	+0.04
+2 03	0.69	−0.06
+2 34	0.64	+0.05
+2 55	0.42	−0.07
+3 16	0.42	+0.02
+3 50	0.30	+0.01
+4 14	0.23	0.00
+4 42	0.19	+0.01
+6 14	0.17	0.00

Near Secondary Minimum		
Phase	Difference of Magnitude	O.−C.
$31^{h}52^{m}$	0.150	0.000
32 30	0.147	−0.014
33 00	0.182	+0.012
33 49	0.172	−0.011
34 24	0.195	+0.007
34 54	0.193	+0.005
35 24	0.182	−0.002
36 07	0.172	−0.003
36 51	0.160	−0.003
37 27	0.148	−0.004
37 59	0.142	0.000
38 37	0.137	+0.004
39 38	0.122	−0.006
Between Minima		
$9^{h}58^{m}$	0.163	−0.002
12 10	0.153	−0.007
14 01	0.170	+0.012
19 33	0.143	−0.002
24 25	0.130	−0.005
27 01	0.138	+0.006
42 41	0.140	+0.008
46 06	0.123	−0.017
50 26	0.147	−0.001
55 40	0.168	+0.008
59 49	0.168	0.000

minimum include 6 sets of measures. In Table II are listed the normal magnitudes, together with the residuals formed by a comparison with the final light-curve computed from the elements of the system of *Algol* which I have derived.

Except near principal minimum, the accordance of the observations justifies carrying the normal magnitudes to the third decimal, and it will be seen that the accuracy obtainable with the selenium photometer, for bright stars at least, is greater than has been attained in any visual or photographic method.

From the residuals in Table II we have

Probable error of a normal magnitude near principal minimum = ±0.023 mag.
Probable error of a normal magnitude near secondary or between minima = ± 0.006 mag.

One peculiarity of the selenium photometer is that the errors, expressed in magnitudes, increase for faint stars, but in light-units the accordance is practically the same for any intensity. In visual observations, the residuals in magnitudes are about the same over a wide range of brightness. Where a variable star has a range of 1 magnitude and eye observations are expressed in light-units, the probable error near minimum is only $\frac{2}{5}$ that at maximum, and according to the usual method the weights should be assigned as 6 to 1. So far as I know, no computer has taken this fact into account, although there have been many elaborate calculations of the elements of different variable stars. In this connection, I may say that I am quite unable to see the justification for publishing the elements of a system like that of *Algol* to 4 or even 5 significant figures, when the original data are often not exact to 2 places, and when even the first figure of some of the results may be in error.

The normal magnitudes of Table II are shown graphically in Fig. 3. The first peculiarity of the light-curve which will attract those familiar with *Algol* is the existence of a secondary minimum. This has been sought for in vain by visual observers, though in my opinion if *Algol* were not so bright, the variation of 0.06 magnitude might have been detected. Of perhaps equal interest is the continuous variation in the light between minima, showing this star to be a distant relative of *β Lyrae*, though the maximum brilliancy does not seem to occur midway between minima, the curve being highest just before and after secondary minimum. The fact that a smooth, symmetrical light-curve fits in with the observations is perhaps sufficient reason for believing this continuous variation to be real.

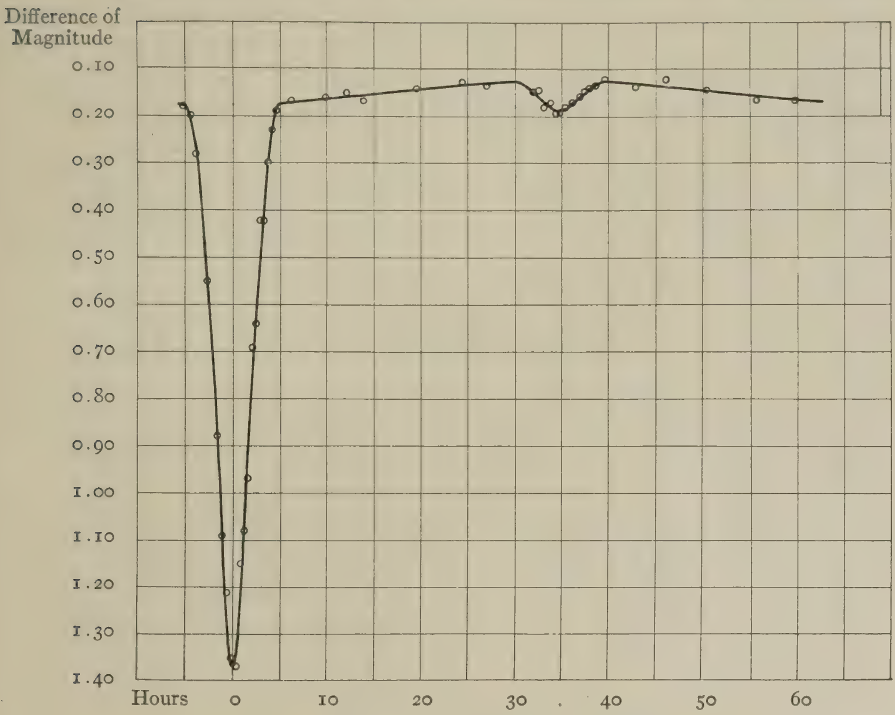

FIG. 3.—The Light-Curve of *Algol*

THEORY OF THE SYSTEM OF *ALGOL*

On the basis of the observations which have been tabulated, let us now consider what conditions in the system of *Algol* will account for the variations in light. Since Vogel's classic determination of the radial velocities before and after light-minimum, the eclipse theory has been regarded as established. The presence of the secondary minimum now proves that the companion is not wholly dark, and it is evident that we have to deal with a bright and a relatively faint body, which in what follows I shall designate as *Algol* and the companion.

As a first approximation let us assume that the orbit is circular, that both bodies are spheres, and further that they are devoid of extensive atmospheres which would cause a decrease of intensity from center to limb, such as we know does exist in the case of the sun. The most obvious explanation of the continuous variation of light between minima is that the companion keeps one face toward *Algol*, and is brighter on that side due to radiation received from the primary. I therefore assume that the companion rotates uniformly once in the period of revolution, and that it is divided into two hemispheres each uniformly intense. While this last assumption is probably far from the truth, it is sufficient for the accuracy of the observations.

In the calculations which follow, I have converted stellar magnitudes into relative light, changing the unit as convenient. First, taking the light of *α Persei* as unity, let

ϕ = the angular phase, or true anomaly counted from minimum,
L = the corresponding total light of the system, as seen from the earth,
L_1 = the light due to *Algol* + the faint hemisphere of the companion,
$2s$ = the excess of light emitted by the bright hemisphere over the faint hemisphere of the companion.

It then easily follows that

$$L = L_1 + s(1 - \cos\phi). \tag{1}$$

Each normal magnitude between minima in Table II furnishes a value of L for the corresponding ϕ, and solving the 11 equations by the method of least squares, I obtain for the two unknowns:

$$L_1 = 0.8507 \pm 0.0033,$$
$$s = 0.0201 \pm 0.0029,$$

or in terms of stellar magnitude, referred to *α Persei*, we have

$$L_1 = 0.176 \text{ mag.} \pm 0.004 \text{ mag.}$$
$$s = 0.025 \text{ mag.} \pm 0.003 \text{ mag.}$$

Using these values of L_1 and s, the light-curve between minima was computed, and is shown in Fig. 3. If we assume no variation between minima, the squares of the residuals give $[vv] = 0.001691$, and the addition of the term in s reduces this to $[vv]=0.000470$, or to the order of one-fourth.

It is somewhat tempting to introduce another unknown which will express the ellipticity of *Algol* due to tidal action, but the residuals between minima are now so small that I should consider the result of such a computation as illusory. For the present it seems justifiable to state that, within the limits of error, the observations are represented by the simple assumption that both bodies are spheres, and that the companion is uniformly bright over each of the two hemispheres.

We are now ready to take up the elements of the system of *Algol*, as they may be determined from the variation throughout the eclipsing phase. In 1880 Professor E. C. Pickering[1] first worked out a determination of the relative distance and dimensions of the components of *Algol*, and his theory was extended by Harting.[2] I shall follow the method of Harting, with some changes necessitated by the influence of the light of the companion.

Let the radius, the surface-intensity, and the total light of *Algol* each be taken as unity, then the elements to be determined are:

$\kappa =$ the radius of the companion,
$r =$ the radius of the relative orbit,
$i =$ the inclination of the orbit,
$\lambda =$ the surface-intensity of the faint hemisphere of the companion,
$\lambda + \lambda_1 =$ the surface-intensity of the bright hemisphere of the companion.

During the eclipse, we may consider the disks of the two stars projected on a plane perpendicular to the line of sight, and in this projection let

$2\eta =$ the angle at the center of *Algol*, subtended by the common chord,
$2\xi =$ the similar angle at the center of the companion,
$\rho =$ the distance between centers,
$M =$ the area common to the projected disks,
$v =$ the projection of the true anomaly in the apparent orbit,
$P =$ the period $= 68.816$ hours,
$t =$ the time in hours from minimum,
$L =$ the total light as seen from the earth.

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[1] *Proceedings of the American Academy of Arts and Sciences*, 16, 1, 1880.

[2] *Untersuchungen über den Lichtwechsel des Sternes β Persei*, Munich, 1889.

Then the following equations give the light L at any instant, when the elements are known:

$$\begin{aligned}
\phi &= 360^\circ \frac{t}{P} \\
\operatorname{ctn} v &= \cos i \operatorname{ctn} \phi \\
\rho &= r\frac{\sin \phi}{\sin v} \\
\cos \eta &= \frac{1+\rho^2-\kappa^2}{2\rho} \\
\sin \xi &= \frac{1}{\kappa} \sin \eta \\
M &= \kappa^2\xi-\rho \sin \eta+\eta \\
L &= 1-\frac{M}{\pi}+\kappa^2\lambda, \text{ near principal minimum,} \\
&= 1-\frac{M}{\pi}(\lambda+\lambda_1)+\kappa^2(\lambda+\lambda_1), \\
&\qquad\qquad \text{near secondary minimum.}
\end{aligned} \tag{2}$$

In practice it is best to assume a set of elements, compute L for the phase of each normal magnitude, and from the residuals derive corrections to the elements. Confining ourselves to the principal eclipse, the corrections $\delta\kappa$, δr, and δi are given by equations of the form

$$\frac{\pi}{2}\delta L=\kappa(\pi\lambda-\xi)\delta\kappa+\frac{\rho \sin \eta}{r}\delta r-\frac{r^2 \sin 2i \cos^2 \phi \sin \eta}{2\rho}\delta i. \tag{3}$$

From equations (2) and (3) the elements may be derived after λ is known. We can now make use of the observations outside of the principal eclipse, and let L with different subscripts represent the light received from the system by an observer on the earth; also put $M=M_\circ$ at either minimum. We have then

$L_1=1+\kappa^2\lambda$, *Algol*+faint side of companion,

$L_2=1+\kappa^2(\lambda+\lambda_1)$, *Algol*+bright side of companion,

$L_M=1+\kappa^2\lambda-\frac{M_\circ}{\pi}$, at principal minimum,

$L_m=1+\kappa^2(\lambda+\lambda_1)-\frac{M_\circ}{\pi}(\lambda+\lambda_1)$, at secondary minimum.

On referring to (1) it is seen that with the proper unit

$$\kappa^2\lambda_1 = 2s.$$

The values of L_1 and L_2 are already known in terms of the light of *a Persei*. L_M and L_m may be found graphically, but when approximate elements are known, $M_\circ$ and L_M are derived from the light-ephemeris.

The solution of the foregoing equations gives

$$\begin{aligned} \frac{M_\circ}{\pi} &= L_1 - L_M \\ \lambda_1 &= \frac{L_2 - L_1}{\kappa^2} \qquad (4) \\ \lambda &= \frac{\pi(L_2 - L_m)}{M_\circ} - \lambda_1 . \end{aligned}$$

The unit of light to which the L's must be referred is defined by

$$L_1 = 1 + \kappa^2\lambda ,$$

and it was found from (1) that L_1 corresponds to a light which is 0.176 magnitude fainter than *a Persei*. Hence, each successive value of κ or λ requires that all computed and observed lights be referred to a new unit.

The approximation of the elements by (2), (3), and (4) is then as follows. Obtain preliminary values of κ, r, and i and substitute κ in (4), which gives λ_1 and λ; compute by (2) the light-ephemeris for a number of epochs, and form the residuals, δL; find the corrections $\delta\kappa$, δr, and δi from three or more observation equations (3); substitute the corrected κ in (4), changing the light-unit, and the new values for λ_1 and λ complete the set of elements. The ephemeris is now recomputed, and the process of correction repeated until satisfactory elements are secured.

Some valuable suggestions for finding preliminary values for κ, r, and i may be found in a paper by Schlesinger.[1] In the first approximations it is unnecessary to use all of the observations to form equation (3), perhaps the easiest method being to draw a light-curve and select 3 points which are sufficient to give κ, r, and i. When sufficiently approximate elements have been derived, the final solution may be made, using all of the observations, and forming normal equations in the usual way.

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[1] *Publications of the Allegheny Observatory*, 1, 123, 1909.

In applying this method to the case in hand I first assumed that the light-curve is symmetrical near principal minimum, and I arranged the normal magnitudes in Table II in order, without regard to the sign of the phase. Averaging each successive pair of normals, I obtained the results in Table III. The phase is now given in decimals of an hour, and the light is referred to difference of magnitude 0.244 as unity, which is the final adopted brightness of *Algol*. The residuals are derived from a comparison with the light-ephemeris computed from the final elements.

TABLE III

COMBINED NORMAL MAGNITUDES NEAR PRINCIPAL MINIMUM

Phase	Difference of Magnitude	Light	O.−C. Light
$0^{\mathrm{h}}28$	1.360	0.358	−0.002
0.74	1.180	0.422	+0.021
1.18	1.085	0.461	−0.004
1.61	0.925	0.534	−0.008
2.31	0.665	0.679	−0.001
2.80	0.485	0.801	+0.024
3.55	0.360	0.899	−0.014
4.07	0.255	0.990	−0.001
4.53	0.195	1.047	+0.004

It is not necessary to set down here the details of the numerical work by which I arrived at different sets of elements. Let it suffice to give merely the results of the successive approximations. Starting with the magnitudes in Table III, and using equations (2), (3), and (4) as indicated, the different elements are as follows:

TABLE IV

ELEMENTS

	I	II	III
κ	1.00	1.15	1.14 ±0.05
r....................	4.54	4.85	4.77 ±0.05
i....................	84°0	82°6	82°3 ±0°3
λ_1	0.049	0.038	0.038±0.007
λ	0 045	0.054	0.050±0.010
$[vv]$...............	0 002318	0.001810	0.001315

An inspection of the numerical solution for Elements III shows that one more approximation will reduce $[vv]$ by about 2 per cent, but I consider this refinement as unnecessary. The probable errors of κ, r, and i were computed, in the usual way, from the weights derived from the normal equations; but for the probable errors of λ and λ_1, I have assumed that the probable errors of L_m and L_M are respectively ±0.003 and ±0.005 light-units. In Fig. 4 the system of *Algol* as seen from the earth has been drawn from the data of Elements III.

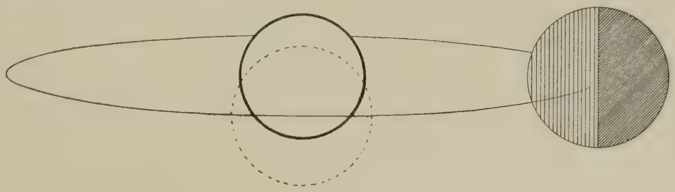

FIG. 4.—The System of *Algol* as Seen from the Earth

As these elements are considerably different from those previously derived for *Algol*, it may be of interest to note the reason for this result. My value for κ shows that the companion is about one-seventh larger in diameter than *Algol*, while previous determinations have yielded a value of about 0.75. The question at once arises as to whether there is something in the selenium photometer which gives an unusually large range of magnitude for this star. In Table V is a comparison of this range with that determined at Harvard,[1] and by Müller.[2] For the maximum with the selenium photometer I have taken an average magnitude between minima.

TABLE V

RANGE IN MAGNITUDE OF *ALGOL*

	Harvard	Müller	Selenium Photometer
	Mag.	Mag.	Mag.
Maximum....	2.1	2.43	0.15 fainter than *a Persei*
Minimum	3.2	3.55	1.37 fainter than *a Persei*
Range	1.1	1.12	1.22

There is no evidence of a large difference in scale between my results and those derived from visual observation, but in any event it is my opinion that the selenium photometer gives more nearly the absolute scale than can be obtained visually.

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[1] *Harvard Annals*, 50, 202, 1908.

[2] *Op. cit.*, p. 193.

Assuming for the moment that the companion gives no light, and that the range of variation is 1.22 magnitudes, it follows that 0.675 of the bright disk is obscured at minimum, which gives 0.82 as the least possible value of κ. It is evident that if the companion is not wholly dark, or if it is not wholly projected against the bright star at principal minimum, then the radius, κ, may be considerably larger than unity.

From Elements III the adopted light-curve has been computed from (1) between minima, and from (2) during eclipse. The light L has been converted into stellar magnitudes, and as thousandths of a magnitude determined with the selenium photometer would have no meaning in either the Harvard or Potsdam systems, both the computed and observed magnitudes are still referred to *α Persei.*

TABLE VI

ADOPTED LIGHT-CURVE OF *ALGOL*

Phase	Difference of Magnitude	Phase	Difference of Magnitude
$\pm 0^{\mathrm{h}}0$	1.37	$30^{\mathrm{h}}0$	0.129
±0.5	1.31	31.0	0.138
±1.0	1.15	32.0	0.152
±1.5	0.95	33.0	0.170
±2.0	0.77	34.0	0.186
±2.5	0.61	34.67	0.189
±3.0	0.47	35.0	0.188
±3.5	0.35	36.0	0.177
±4.0	0.265	37.0	0.160
±4.5	0.200	38.0	0.142
±4.90	0.174	39.0	0.131
+5.0	0.174	39.6	0.128
7.5	0.170	40.0	0.128
10.0	0.166	45.0	0.136
15.0	0.156	50.0	0.147
20.0	0.144	55.0	0.158
25.0	0.134	60.0	0.168
29.8	0.128		

This light-curve has already been shown in Fig. 3, and it should be understood that the curve there drawn has been derived entirely by computation from the elements, except that I made a graphical determination of the times of minima.

The duration of the eclipse is easily found from the equation

$$\cos \phi_1 = \frac{\sqrt{r^2-(1+\kappa)^2}}{r \sin i},$$

where ϕ_1 represents the true anomaly at first or last contact. Using Elements III, I obtain

$$\text{Duration of eclipse} = 9^{\text{h}}.80.$$

The conclusion of Müller that the principal minimum lasts 12 or 13 hours has therefore not been verified, but he was not fortunate in securing enough observations near the critical times of beginning and ending to establish this longer time. Rödiger[1] has used Müller's light-curve as a basis of deriving the elements of *Algol* with allowance for an assumed decrease in intensity of the bright disk from center to limb, but likewise his results are not confirmed. Judging from the beautiful accordance of Müller's observations near the time of greatest eclipse, it is my opinion that if he had been able to push his observations to the limit throughout the entire period of the star's variation, he would have detected the secondary minimum.

COMPARISON OF *ALGOL* WITH THE SUN

As any direct determination of the intrinsic luminosity of a star requires that its distance be known, we are fortunate in having a pretty fair determination of the parallax of *Algol*. I am indebted to Professor Comstock for a statement of his best judgment as to the parallax of this star, together with an unpublished determination by Flint at Madison. In Table VII the results by Chase and Pritchard have been increased by 0.″01 on account of assumed parallax of the comparison stars.

TABLE VII

PARALLAX OF *ALGOL*

Observer	π	Weight
Chase........	+0.″05	3
Pritchard	+0.07	1
Flint.........	+0.12	1
Adopt........	+0.″07	

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[1] *Untersuchungen über das Doppelsternsystem Algol*, Königsberg, 1902.

With this value of the parallax it is now easy to compute the luminosity, or total light, of *Algol* as compared with the sun. Let l be the luminosity, π the parallax, and M the stellar magnitude of the bright component of *Algol;* also let S be the stellar magnitude of the sun. From simple considerations we have

$$\log l = -2 \log \sin \pi - 0.4\,(M - S).$$

Adopting $M = 2.2$, and $S = -26.6$, there follows:

TABLE VIII

		☉=1	Stellar Magnitude
l	Total light of *Algol*	26	2.2
$l\kappa^2\lambda$..........	Total light of faint hemisphere of companion	1.7	5.2
$l\kappa^2(\lambda+\lambda_1)$	Total light of bright hemisphere of companion	3.0	4.6

The principal source of error in the comparison with the sun lies of course in the parallax. If instead of $\pi = 0''.07$ we adopt the results of Chase's heliometer measures, $\pi = 0''.05$, the computed luminosities in Table VIII will be doubled, and I am inclined to look upon the tabulated values as too small. In any case we have the result that the light from one side of the companion is nearly twice, and from the other, three times that of the sun. If the bright star could be extinguished, the companion would appear to us as varying on its own account, the range being from 4.6 to 5.2 magnitudes.

DIMENSIONS OF THE SYSTEM OF *ALGOL*

As is well known, Vogel's determination of the variable radial velocity of *Algol* led to the first approximation of the actual diameter in kilometers of any of the fixed stars; but unfortunately Vogel's numerical results are often quoted without due emphasis on his necessary assumption that the two components are of the same density. For a comparison with my light-curve there are available the results by Schlesinger and Curtiss[1] from the observations in 1906 and 1907. In the conventional notation, the elements which are applicable are as follows:

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[1] *Publications of the Allegheny Observatory*, 1, 25, 1908.

TABLE IX

ALLEGHENY SPECTROSCOPIC ELEMENTS

	Schlesinger	Curtiss	Adopted
e	0.031	0.060	0.05
ω	+44°	−3°	+21°
$a \sin i$	1,570,000 km	1,630,000 km	1,600,000 km

There was a discrepancy between the time of light-minimum as determined photometrically and as demanded by these velocity determinations, amounting to $1\frac{1}{2}$ to 2 hours. Since my correction to the light-ephemeris in 1909–1910 amounts to $-1^{h}\,16^{m}$, a portion of this discordance is removed; but as I have learned from Director Schlesinger that the orbit of *Algol* is still being investigated at Allegheny, it seems best to wait for a comparison of spectrographic and photometric observations taken near the same epoch.

As only one spectrum was discernible on the plates, the spectroscopic elements refer to the motion of the bright component about the center of mass. Evidently, when $\omega=0°$ the distances between centers of the projected disks are equal at the two minima, and in view of the small value derived for ω, my determination of the relative brightness of the components has not been materially affected by neglecting the eccentricity. Likewise a simple calculation shows that the assumption of a circular orbit introduces no appreciable error into the computed variation of the light during the eclipses. In my judgment, it is as yet impossible to determine the eccentricity of this orbit from photometric observations, and all the numerical results which have been derived from the asymmetry of the light-curves by visual observers are spurious. Nevertheless, it may be of passing interest to compare the intervals between minima as determined graphically from my light-curve, and as computed from the Allegheny elements. The total period being $68^{h}.816$, I find the following times by which the secondary follows the principal minimum:

$e=0$	$34^{h}.41$
From light-curve	34.67
From Allegheny elements	36.46

It is apparent that the photometric result is not in agreement with the Allegheny determination of 3 years previous, but in view of the difficulty of finding the exact time of secondary minimum from a variation of only 0.06 magnitude, I attribute little significance to this fact.

Before deriving plausible values of the dimensions in the system of *Algol*, on the basis of the spectroscopic orbit, let us consider the probability of various assumptions as to the relative masses of the two bodies. Although we have many theories of stellar evolution, to my mind the most direct evidence which bears upon this question is that found by Schlesinger and Baker[1] in a comparative study of spectroscopic binaries. They have announced the rule thus far without exception, that where the spectra of both components of a close binary are visible, the brighter star is always the more massive. Therefore when the spectrum of the fainter component is not visible, the mass of the bright star is presumably much the greater. With this in mind, I have thought it best to compute the dimensions on two assumptions: first, that the two bodies are of equal density, and second, that *Algol* has twice the mass of the companion.

The sun being taken as unity, let m and m_s represent the masses, and d and d_s the densities of the two bodies, the subscript referring to the companion. Changing to conventional notation, we have $r=a+a_s$. The combined mass is given by

$$m+m_s=\frac{4}{10^{20}}\ \frac{(a+a_s)^3}{P^2},$$

where the distances are expressed in kilometers and the period in days. From $a \sin i=1{,}600{,}000$ km and $i=82^\circ\!.3$ it follows that $a=1{,}610{,}000$ km. To compute $(a+a_s)$ we have the two assumptions

$$(1),\ d=d_s \qquad\qquad (2),\ m=2m_s$$

$$a+a_s=a\left(1+\frac{1}{\kappa^3}\right) \qquad\qquad a+a_s=3a$$

The radius of the sun being 697,000 km, I find from simple computations the results in Table X, where each quantity is referred to the sun.

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[1] *Publications of the Allegheny Observatory*, 1, 135, 1910.

TABLE X

		(1), $d=d_s$	(2), $m=2m_s$
R	Radius of *Algol*	0.81⊙	1.45⊙
R_κ	Radius of companion	0.92	1.66
m	Mass of *Algol*	0.04	0.37
m_s	Mass of companion	0.06	0.18
d	Density of *Algol*	0.07	0.12
d_s	Density of companion	0.07	0.04
σ	Surface-intensity of *Algol*	40	12
$\sigma\lambda$	Surface-intensity of faint hemisphere of companion	2.0	0.6
$\sigma(\lambda+\lambda_1)$	Surface-intensity of bright hemisphere of companion	3.5	1.1

On the first assumption there results a great surface-intensity for a small mass, while in the second case, where the companion is supposed to be relatively rare, the masses and radiative powers are much nearer those of the sun. On the basis of these figures alone, it would seem that the second assumption is nearer the truth.

The low density of *Algol* stars was shown independently by Roberts[1] and Russell,[2] though the first determination of the density of *Algol* itself seems to be due to Mériau.[3] From my elements, I find

Mean density of the system	0.07⊙
Limiting density of *Algol*	0.18
Limiting density of companion	0.12

It is to be hoped that a means of detecting the spectrum of the companion may be found, which will at once give us the actual masses, diameters, and densities of the two bodies, subject only to the errors of observation. In this connection, it is barely possible that the extra sensibility of selenium at the red end of the spectrum[4] may have been a factor in the detection of the secondary minimum in the light-curve, and it would be of interest to know the spectral type of each hemisphere of the companion.

We now have numerical data on which to base a plausible explanation of the greater luminosity of the companion on the side toward the primary. Two reasons at once suggest themselves: first, that the companion reflects the light of the bright star, and second, that the radiation from *Algol* is so intense that it is sufficient to heat the near hemisphere of the companion up to incandescence.

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[1] *Astrophysical Journal*, 10, 308, 1899.

[2] *Ibid.*, 10, 315, 1899.

[3] *Comptes Rendus*, 122, 1254, 1896.

[4] The color-sensibility curve of the selenium cell, Giltay 93, which was used in the observations of *Algol*, will be found in the *Astrophysical Journal*, 27, 185, 1908.

From the values of κ and r, and the solid angle at the center of *Algol* subtended by the companion, I find the fraction of the total light intercepted by the companion to be 0.015. Hence it is quite impossible that the relative light reflected from the bright side of the companion should amount to the previously computed value, $\kappa^2\lambda_1 = 0.049$. It seems, therefore, that the increase in brightness of one hemisphere must be due to the heating effect of the intense radiation emitted by *Algol*.

Consider the sphere concentric with *Algol*, and of radius equal to that of the relative orbit. The radiation received per unit surface of this sphere is $1/r^2 = 1/22.8$ times the radiation emitted per unit surface of *Algol*. This fraction represents roughly the relative radiation received by the companion. On the hypothesis of equal density, the surface-intensity of *Algol* is 40, and it would follow that the companion receives 40/22.8, or nearly twice as much light per unit surface as is emitted by the sun. Presumably this value is too high, and it refers only to a limited portion of the spectrum. Of course the determination of the total energy of the radiation received by the companion is an entirely different problem, but in any event these figures are suggestive.

The increase of luminosity on one side of the companion, as shown by my light-curve, is a striking independent confirmation of the results of Nordmann.[1] From measures of the relative intensity of different portions of the visible spectrum, he has derived a value of the effective temperature of *Algol*, and from this he concludes that the radiation at the distance of the companion is sufficient to raise a body to incandescence. "Il est donc probable que, par le seul effet de ce rayonnement et independamment de sa chaleur propre, une partie de la surface du satellite d'*Algol* tournée vers l'étoile principale, est portée à l'incandescence." Although all determinations of stellar temperatures are as yet open to question, and the parallax of *Algol* is so small as to introduce uncertainty into the results, the main conclusion of Nordmann is well confirmed by my observations.

The question of the interaction of the radiation of two components of a close variable star has not been considered heretofore, because of the small accuracy obtainable in photometric measures. It is evident that such an effect will not be discernible where the two bodies involved are of the same order of brightness, but in many *Algol* variables where the primary is a star of great intrinsic brilliancy, and the companion relatively faint, there is every reason to suppose that future observations will demonstrate that the conditions in *Algol* are typical of this class of stellar system.

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[1] *Bulletin Astronomique*, 27, 145, 1910.

SUMMARY

1. It has been demonstrated that for bright stars the selenium photometer, attached to a 12-inch telescope, yields results which are considerably more accurate than have ever been obtained by visual or photographic methods.

2. An application of this new device to observations of *Algol* has led to the discovery that the companion, far from being dark, gives off more light than our sun, and in addition is much brighter on the side which is turned toward the primary.

3. A discussion of the photometric and other results for *Algol* gives the following principal facts concerning this system:

FROM THE LIGHT-CURVE

..	Radius of *Algol*	1.00
κ	Radius of companion	1.14 ± 0.05
r	Distance between centers	4.77 ± 0.05
i	Inclination of orbit	$82^\circ.3\pm0^\circ.3$
..	Surface-intensity of *Algol*	1.00
λ	Surface-intensity of faint hemisphere of companion	0.050 ± 0.010
$\lambda+\lambda_1$	Surface-intensity of bright hemisphere of companion	0.088 ± 0.012
..	Total period	$68^h.816$
..	Duration of eclipse	$9^h.80$
..	Mean density of the system	0.07⊙
..	Limiting density of *Algol*	0.18⊙
..	Limiting density of companion	0.12⊙

FROM $\pi=0''.07$; SUN $=-26.6$ MAG., *Algol* $=2.2$ MAG.

		⊙=1	Stellar Magnitude
l	Total light of *Algol*	26	2.2
$l\kappa^2\lambda$	Total light of faint hemisphere of companion	1.7	5.2
$l\kappa^2(\lambda+\lambda_1)$	Total light of bright hemisphere of companion	3.0	4.6

From $a \sin i = 1{,}600{,}000$ km, Two Assumptions

		$d=d_s$	$m=2m_s$
R	Radius of *Algol*	0.81⊙	1.45⊙
$R\kappa$	Radius of companion	0.92	1.66
m	Mass of *Algol*	0.04	0.37
m_s	Mass of companion	0.06	0.18
d	Density of *Algol*	0.07	0.12
d_s	Density of companion	0.07	0.04
σ	Surface-intensity of *Algol*	40	12
$\sigma\lambda$	Surface-intensity of faint hemisphere of companion	2.0	0.6
$\sigma(\lambda+\lambda_1)$	Surface-intensity of bright hemisphere of companion	3.5	1.1

In conclusion, I beg to acknowledge my indebtedness to Dr. F. C. Brown, in collaboration with whom many of the preliminary experiments were made; also to Dr. F. W. Reed, and Mr. Percy F. Whisler for efficient assistance in taking the observations of *Algol*. I also desire to thank the Rumford Committee of the American Academy of Arts and Sciences for successive grants of $200 and $350 which enabled me to carry on this work.

University of Illinois Observatory
August 1, 1910

NOTE ADDED AUGUST 12, 1910

I have just received the list of parallaxes by Kapteyn and Weersma in *Groningen Publications No. 24*, which contains an additional determination by H. N. Russell, who finds for *Algol*, $\pi = +0''.01$. Kapteyn rejects Pritchard's result, and using only the values of Chase and Russell, he adopts $\pi = +0''.029$; also he uses -26.1 as the sun's magnitude in the derivation of luminosities. Recomputing the results of Table VIII on this basis, we have

Total light of *Algol*	240⊙
Total light of faint hemisphere of companion	16
Total light of bright hemisphere of companion	28

These luminosities are nearly 10 times as large as those which I adopted, and demonstrate that at the present time it is not possible to fix a reliable value for the light of *Algol*, except that we may say that the companion, even on the faint side, is much more luminous than the sun.

SECONDARY STANDARDS OF WAVE-LENGTH, INTERNATIONAL SYSTEM, IN THE ARC SPECTRUM OF IRON

ADOPTED BY THE SOLAR UNION, 1910

At the fourth meeting of the International Union for co-operation in Solar Research, held at Mount Wilson August 31, 1910, it was unanimously agreed to establish as secondary standards of wave-length in the "International System" the means of the values obtained by three independent observers of the wave-lengths of certain lines in the arc spectrum of iron. These observers were Fabry and Buisson, working at Marseilles, Eversheim at Bonn, and Pfund at Baltimore. The committee of the Union which made the report on this subject was authorized to publish these secondary standards, and the following values—together with the individual results of the different investigations—are accordingly presented. It was also voted at this meeting

Adopted Wave-Lengths	Fabry and Buisson	Eversheim	Pfund
4282.408	.407	.408	.408
4315.089	.089	.089	.089
4375.934	.935	.934	.934
4427.314	.314	.313	.316
4466.556	.554	.557	.558
4494.572	.572	.571	.572
4531.155	.155	....	.155
4547.853	.854	.853	.853
4592.658	.658	.658	.657
4602.947	.944	.948	.948
4647.439	.437	.441	.439
4691.417	....	.419	.416
4707.288	.287	.292	.286
4736.786	.785	.787	.787
4789.657	.657	.658	.657
4878.225	.226	.224	.225
4903.325	.324	.327	.324
4919.007	.006	.007	.008

Adopted Wave-Lengths	Fabry and Buisson	Eversheim	Pfund
5001.881	.880	.885	.879
5012.073	.072	.074	.072
5049.827	.827	.827	.827
5083.344	.343	.346	.343
5110.415	.415	.414	.416
5167.492	.492	.491	.492
5192.363	.362	....	.364
5232.957	.958	.958	.956
5266.569	.568	.569	.569
5371.495	.498	.493	.494
5405.780	.780	.780	.780
5434.527	.530	.524	.528
5455.614	.616	.611	.614
5497.522	.521	.523	.523
5506.784	.783	.785	.784
5569.633	.632	.636	.631
5586.772	.770	.773	.772
5615.661	.658	.662	.663
5658.836	.835	.838	.835
5763.013	.013	.013	.014
6027.059	.059	....	.059
6065.492	.493	.493	.491
6137.701	.700	....	.702
6191.568	.569	.568	.567
6230.734	.732	.736	.735
6265.145	.147	....	.143
6318.028	.029	.028	.026
6335.341	.343	.342	.337
6393.612	.612	.613	.612
6430.859	.859	.862	.855
6494.993	.994	.994	.992

of the Union that wave-lengths measured in this system should be designated in the future by using the symbol "I.A." to indicate the unit used.

Members present of the Committee on Standards of Wave-Length
H. KAYSER
CH. FABRY
J. S. AMES

MOUNT WILSON, CALIFORNIA
September 1, 1910

STANDARDS OF THIRD ORDER OF WAVE-LENGTHS ON THE INTERNATIONAL SYSTEM

By H. KAYSER

The following table contains measurements of the arc spectrum of iron photographed in the second order with an excellent Rowland concave grating of twenty-one feet radius. The plates were made within the past five years, in part by Professor Konen, and in part by Professor Eversheim, Dr. Bachem, and myself. The measurements were made by myself, with a dividing engine constructed by Wolz.

I have employed as standards the arithmetical means of the measurements of Fabry and Buisson, Eversheim, and Pfund. The portions of spectrum measured were always between three or four successive standards, so that each line is referred to as many neighboring standard lines as possible. It turned out that measurements of the same sharp line on different plates yielded differences of not more than from 0.001 to 0.002 Å, when the same standards were employed; but if different standards were employed, differences as great as about 0.006 occurred. This proves that some of the standards still contain errors of from 0.004 to 0.005 Å, and that the measurements of the best plates in the second order give a greater degree of accuracy than that of the secondary standards. Each line was measured from ten to twenty times, and from all the measurements the arithmetical mean is taken, the probable error also being given.

This method of measurement has also yielded values for the standards themselves, which I regard as somewhat more accurate than the secondary international standards, inasmuch as they are adjusted. They are in general in good accord and only in a few cases is the difference large, in the case of λ 6430 reaching the amount of 0.012 Å. It remains to be investigated whether my value is more accurate than the other.

A comparison with Rowland's solar spectrum wave-lengths shows that the differences vary irregularly between 0.15 and 0.22 Å. By substracting about 0.19 Å, all measurements on Rowland's system can be reduced to the international system with sufficient accuracy.

λ	Intensity	Probable Error	Rowland	Difference	Fabry and Buisson	Evers-heim	Pfund
4118.553	3	1	.708	0.155			
21.807	1	1	.963	.156			
27.615	2	1	.767	.152			
32.064	6	1	.235	.171			
34.684	3	1	.840	.156	.685		
37.003	3	1	.156	.153			
43.421	4	1	.572	.151			
43.873	7	1	.038	.165			
47.674	2	1	.836	.162	.677		
54.506	2	1	.667	.161			
56.806	2	1	.970	.164			
70.906	2	1	.068	.162			
75.642	2	1	.806	.164			
84.894	2	1	.058	.164			
87.050	4	1	.204	.154			
87.807	4	1	.943	.136			
91.441	4	1	.595	.154	.441		
91.677	1	2	.843	.166			
98.307	4	2	.494	.187			
99.099	4	1	.267	.168			
4202.030	5	1	.198	.168			
10.357	2	1	.494	.137			
16.185	3	1	.351	.166			
19.365	3	1	.516	.151			
22.220	3	1	.382	.162			
27.440	2	1	.606	.166			
33.610	3	1	.772	.162	.615		
38.824	3	1	.970	.146			
47.437	3	1	.591	.154			
50.129	4	1	.287	.158			
50.789	5	1	.945	.156			
54.334	2	1	.505 *Ca*	.171			
60.484	7	1	.640	.156			
67.828	2	1	.985	.157			
71.163	6	1	.325	.162			
71.745	7R	1	.934	.189			
71.838	2	2	.045	.207			
82.404	4	1	.565	.161	.407	.408	.410
94.124	4	1	.301	.177			
99.243	5	1	.410	.167			
4307.872	6R	2	.907 *Ca*	.135			
07.945	4R	2	.081	.136			
15.089	4	1	.262	.173	.089	.089	.092
25.763	8R	..	.939	.176			
37.053	4	1	.216	.163			
52.740	4	1	.908	.168	.741	.741	.742
58.500	2	1	.670	.170			
69.767	3	2	.941	.174			
75.930	4	1	.107	.177	.935	.934	
83.552	10R	1	.720	.168			
90.950	2	1	.123	.173			
4404.753	8R	1	.927	.174			
15.125	6R	1	.293	.168			

λ	Intensity	Probable Error	Rowland	Difference	Fabry and Buisson	Eversheim	Pfund
4422.562	3	1	.741	0.179			
27.314	4	1	.482	.168	.314	.313	.314
30.618	3	1	.785	.167			
42.343	4	1	.510	.167			
47.723	4	1	.892	.169			
54.382	3	1	.552	.170			
59.120	3	1	.301	.181			
61.652	4	1	.818	.166			
66.553	4	1	.727	.174	.554	.557	
69.382	4	1	.545	.16;			
82.163	3	1	.338	.175			
82.266	4	1	.438	.172			
89.744	3	1	.911	.167			
90.088	2	1	.253	.165			
94.571	4	1	.738	.167	.572	.571	.572
4514.192	1	1	.358	.166			
17.530	2	1	.702	.172			
25.152	3	1	.314	.162			
28.622	4	2	.798	.176		.622	
31.159	3	1	.327	.168	.155		.154
47.859	3	1	.024	.165	.854	.853	
56.127	3	1	.306	.179			
66.526	1	1	.693	.167			
74.729	1	1	.899	.170			
81.527	2	1	.693	.166			
87.135	1	1	.308	.173			
92.656	3	1	.840	.184	.658	.658	
98.136	2	3	.303	.167			
4602.004	2	2	.183	.179			
02.945	4	2	.126	.181	.944	.948	
11.295	3	1	.469	.174			
13.225	2	3	.386	.161			
19.295	3	3	.468	.173			
25.064	4	1	.227	.163			
32.917	3	1	.100	.183			
37.517	3	1	.685	.168			
38.019	3	1	.193	.174			
47.436	3	1	.617	.181	.437	.441	
54.495	3	1	.672	.177			
54.639	3	1	.800	.161			
67.457	3	2	.626	.169			
68.148	3	1	.331	.183			
73.167	3	2	.347	.180			
78.854	4	2	.027	.173	.855		
83.580	2	1	.745	.165			
91.415	3	1	.602	.187			
4707.289	3	1	.457	.168	.287	.292	
07.492	1	2	(.672)	.180			
10.280	2	1	.471	.191			
20.999	1	1	.179	.180			
27.438	1	1	.582	.144			
33.592	2	1	.779	.187			
36.787	4	2	.963	.176	.785	.787	

λ	Intensity	Probable Error	Rowland	Difference	Fabry and Buisson	Eversheim	Pfund	
4741.530	2	1	.718	0.188				
45.802	2	2	.992	.190				
54.049	..	1	.225	.176	.046	.049		*Mn*
57.576	2	1	.771	.195				
62.371	..	1	.567	.186				*Mn*
66.422	..	2	.621	.199				*Mn*
72.816	2	2	.007	.191				
79.439	1	2	.634	.195				
83.436	..	2	.613	.177				*Mn*
86.811	2	2	.003	.192				
89.651	2	1	.849	.198	.657	.658		
98.278	2	3	.453	.175				
4800.653	2	2	.842	.189				
02.886	1	2	.072	.186				
07.735	1	3	.900	.165				
23.523	..	1	.697	.174	.521	.523		*Mn*
32.734	2	1	.905	.171				
39.546	2	1	.735	.189				
43.161	2	1	.336	.175				
45.652	1	1	.843	.191				
55.693	1	1	.859	.166				
59.754	4	1	.928	.174	.756	.758	.755	
63.664	1	1	.833	.169				
71.333	5	1	.512	.179				
72.155	4	1	.332	.177				
78.229	4	1	.407	.178	.226	.224	.223	
81.724	2	2	.904	.180				
82.166	2	2	.336	.170				
85.445	3	1	.620	.175				
90.771	5	1	.948	.177				
91.510	6	1	.683	.173				
4903.323	4	1	.502	.179	.324	.327	.325	
10.031	2	1	.198	.167				
19.014	5	1	.174	.160	.006	.007	.005	
20.523	7	1	.685	.162				
24.777	2	1	.956	.179				
30.348	1	1	.486	.138				
38.195	2	1	.350	.155				
38.830	3	1	.997	.167				
46.408	2	1	.568	.160				
50.140	1	2	.291	.151				
57.303	4	1	.480	.177				
57.609	5	1	.785	.176				
66.105	3	1	.270	.165	.104	.105		
70.506	1	1	.671	.165				
78.620	2	1	.785	.165				
82.529	3	1	.682	.153				
85.274	3	1	.432	.158				
85.569	3	1	.730	.161				
94.139	2	1	.316	.177				
5001.884	3	1	.044	.160	.880	.885	.883	
05.732	3	1	.896	.164				
06.136	4	1	.306	.170				

λ	Intensity	Probable Error	Rowland	Difference	Fabry and Buisson	Evers-heim	Pfund
5012.070	3	2	.252	0.182	.072	.074	
14.958	3	2	.123	.165			
22.258	2	1	.414	.156			
28.136	2	2	.308	.172			
39.264	2	1	.428	.164			
41.079	2	1	.255	.176			
41.758	2	2	.936	.178			
49.825	3	3	.008	.183	.827	.827	
51.640	4	1	.825	.185			
68.786	3	1	.944	.158			
74.740	3	1	.932	.192			
79.231	2	1	.409	.178			
79.751	2	1	.921	.170			
83.346	2	1	.518	.172	.343	.346	
90.782	2	1	.954	.172			
96.999	2	1	.175	.176			
5107.462	3	1	.619	.157			
07.650	3	1	.823	.173			
10.412	3	1	.574	.162	.415	.414	
23.729	2	1	.899	.170			
27.366	2	1	.533	.167			
31.482	3	1	.642	.160			
33.675	1	1	.870	.195			
39.268	3	1	.427	.159			
39.482	4	1	.644	.162			
42.934	2	1	.111	.177			
50.841	2	1	.020	.179			
51.915	2	2	.087	.172			
62.310	3u	2	.449	.139			
67.488	4	1	.678	.190	.492	.491	.493
74.184	1	1	....	....			
87.928	1	1	.079	.151			
91.477	3	1	.629	.152		.473	
92.362	4	1	.523	.161	.362		
94.946	3	1	.941	.195			
98.717	2	1	.888	.171			
5202.341	2	1	.516	.175			
08.619	1	1	.776	.157			
15.193	2	1	.353	.160			
16.275	2	1	.437	.162			
17.403	2	1	.552	.149			
26.874	3	1	.043	.169			
27.185	4	1	.362	.177			
29.843	2	1	.030	.187			
32.960	5	1	.122	.162	.958	.958	.956
42.495	2	1	.658	.163			
54.959	1	1	.121	.162			
59.740	1	1	(.912)	.172			
63.319	2	1	.486	.167			
66.574	4	1	.738	.164	.568	.569	
69.535	6	1	.723	.188			
70.354	4	2	.558	.204			
73.176	2	1	.339	.163			

λ	Intensity	Probable Error	Rowland	Difference	Fabry and Buisson	Evers-heim	Pfund
5273.378	2	1	.558	0.180			
81.806	4	1	.971	.165			
83.635	4	1	.802	.167			
5302.314	3	1	.480	.166	.316	.316	
07.364	2	1	.541	.177			
24.197	5	2	.373	.176	.196	.196	
28.040	5	1	.236	.176			
28.532	4	1	.696 .747	.189			
32.904	3	1	.089	.185			
39.947	3	1	.121	.174			
41.027	4	1	.213	.186			
53.390	2	2	.571	.181			
64.862	3u	1	.069	.207			
65.404	2	1	.596	.192			
71.490	3	2	.734	.244	.498	.493	
79.583	2	2	.775	.192			
83.360	5	2u	.578	.218			
89.482	2	3u	.683	.201			
93.186	4	1	.375	.189			
5400.510	2	2	.711	.201			
05.778	5	1	.989	.211	.780	.780	
10.902	3	1	.124	.222			
15.184	4u	1	.416	.232			
24.050	5u	2	.290	.240			
29.701	5	1	.911	.211			
34.527	4	1	.740	.213	.530	.524	.529
45.037	2	1	.259	.222			
46.918	4	1	.130	.212			
55.439	2	1	.671	.232			
55.616	3	1	.834	.218	.616	.611	.615
62.964	2	1	.174	.210			
63.272	3	1	.494	.222			
66.424	2	1	.609	.185			
73.910	2	1	.113	.203			
76.294	2	1	.500	.206			
76.581	3	1	.778	.197			
87.768	2	2	.959	.191			
97.521	4	1	.735	.214			
5501.469	4	1	.683	.214			
06.783	4	1	.000	.217			
09.529	1	1	(.756)	.227			
25.559	2	1	.765	.206			
35.420	3	1	.644	.224	.418		
43.184	2	1	.414	.230			
43.944	2	2	.157	.213			
54.878	2	1	.122	.244			
63.608	3	1	.824	.216			
65.685	3	2	.931	.246			
69.630	4	1	.848	.218	.632	.636	
72.858	5	1	.075	.217			
76.102	3	1	.320	.218			
86.774	7	1	.991	.217	.770	.773	.770

λ	Intensity	Probable Error	Rowland	Difference	Fabry and Buisson	Evers-heim	Pfund	
5598.292	2	2	.524	0.232				
5602.962	3	2	.186	.224				
15.667	8	1	.877	.210	.658	.662	.657	
24.562	4	1	.769	.207				
38.289	2u	2	.488	.199				
48.323	2	1	(.503)	.180				
58.846	4	1	.052	.206	.835	.838		
62.537	3	2	.744	.207				
75.305	2u	2	....	....				
82.222	1u	2	.427	.205				*Ni*
92.220	2	1	....	....				
5701.561	3	1	.772	.211				
09.394	4	1	.601	.207	.396			
09.584	..	1	.775	.191				*Ni*
15.111	3	1	.308	.197				
17.863	2	1	.055	.192				
31.774	3	3	.984	.210				
48.381	..	3	.576	.195				*Ni*
53.142	3	1	.344	.202				
54.687	..	1	.881	.194				*Ni*
63.014	4	1	.218	.204	.013	.013		
82.156	1	3	....	....				
91.044	1	3	.243	.199				
5805.212	..	3	.441	.228	.211			*Ni*
08.750	1u	2	....	....				
17.044	1u	3	....	....				
30.562	1u	3	....	....				
32.783	1u	3	....	....				
42.461	1u	3	....	....				
46.996	..	3	.221	.225				*Ni*
54.420	1u	3	....	....				
57.753	..	3	.976	.223	.760	.759		*Ni*
62.322	4	2	.582	.260				
72.897	1	3	....	....				
77.959	1	3	....	....				
83.822	2	3	.028	.206				
92.897	4	3	.097	.200	.882	.881		*Ni*
5905.661	3	3	.895	.234				
14.142	4u	3	.335	.193				
16.241	2	3	.475	.234				
30.152	3u	3	.406	.254				
34.668	3	3	.881	.213	.683			
52.739	3	3	.943	.204	.739			
56.695	2	3	.923	.228				
75.354	2	2	.575	.221				
76.800	2	2	.007	.207				
83.709	3	2	.908	.199				
84.795	4	1	.040	.245				
87.045	3	1	.290	.245				
6003.029	3	1	.239	.210	.039			
07.953	3	1	.186	.233				
08.580	4	1	.785	.205				
13.516		1	.715	.199				*Mn*

λ	Intensity	Probable Error	Rowland	Difference	Fabry and Buisson	Evers-heim	Pfund	
6016.666	..	1	.861	0.195				*Mn*
20.167	4u	1	.401	.234				
21.821	..	1	.016	.195				*Mn*
24.047	5	1	.281	.234				
27.062	4	1	.274	.212	.059			
42.092	2	1	.315	.223				
55.992	3	1	.227	.235				
65.489	5	2	.709	.220	.493	.493		
78.476	3	2	.710	.234				
89.581	2	2	.787	.206				
90.229	..	2	.429	.200				
96.708	..	3	.880	.172				
6102.182	4	3	.392	.210				
03.192	3	1u	.400	.208				
27.927	2	1	.124	.197				
36.628	5	1	.829	.201				
37.012	1	1	.210	.198				
37.703	5	1	.915	.212	.700			
47.844	2	1	.040	.196				
51.633	3	1	.834	.201				
57.740	4	1	.945	.205				
65.387	1	2	.577	.190				
73.350	3	1	.553	.203				
80.230	3	1	.420	.190				
91.568	5	1	.779	.211	.569	.568	.566	
6200.330	5	1	.527	.197				
13.438	4	1	.644	.206				
15.153	3	1	.360	.207				
19.289	4	1	.494	.205				
29.251	2	2	.437	.186				
30.734	6	1	.943	.209	.732	.736	.729	
32.669	3	1	.856	.187				
40.666	2	1	.863	.197				
46.345	4	1	.535	.190				
52.567	5	1	.773	.206				
54.269	3	1	.456	.187				
56.377	3	1	.572	.195				
65.145	3	1	.348	.203	.147			
70.245	1u	3	.442	.197				
80.631	2u	3	.833	.202				
97.804	3	1	.007	.203				
6301.528	4	1	.718	.190				
02.522	3	1	.709	.187				
15.323	2	1	.517	.184				
15.822	2	1	.028	.206				
18.031	5	1	.239	.208	.029	.028		
22.198	2	1	.907	.209				
35.339	5	1	.554	.215	.343	.342		
36.850	5	1	.048	.198				
44.164	1u	2	.371	.207				
55.045	2	1	.246	.201				
58.693	2u	2	.898	.205				
75.206	1	3	(.455)	.249				

λ	Intensity	Probable Error	Rowland	Difference	Fabry and Buisson	Evers-heim	Pfund
6376.198	1	2	(.385)	.187			
80.754	2	1	.958	.204			
93.612	6	1	.820	.208	.612	.613	.609
6400.036	8	3	.217	.181			
08.045	3	2	.233	.188			
11.673	4	1	.865	.192			
21.354	3	1	.570	.216			
30.848	4	3	.066	.217	.859	.862	
62.734	2	1	.965	.231			
72.631	1	1	(.823)	.192			
81.896	2	2	(.098)	.202			
94.994	7	..	.213	.219	.994	.994	.990

The observations will soon be published *in extenso*. I intend to continue the measurements as far as λ 7900.

September 1910

ON THE APPARENT PERIODICITY IN THE SPACING OF THE SATELLITES OF SOME OF THE MERCURY LINES

By G. F. HULL

During the year 1905–1906 the writer, while working in the Cavendish Laboratory in Cambridge, observed that the satellites of the mercury lines λ 5461, 4359, and 4047 differed in their positions from one another by quantities which had a rather large common divisor. The chief problem[1] in hand at that time, however, was not concerned with this observation, so the records were set aside for later consideration. It was expected too that an instrument of greater resolving power could be obtained by means of which the observations could be subjected to a severer test. In the meantime, other observers have published their results, so that the hypothesis that there is a periodicity in the spacing of the satellites of the mercury lines may now be examined.

Periodicity in the spacing of so-called satellites may be due to two causes. Faulty construction of the analyzing apparatus, whether interferometer, grating, single or multiple plate echelon, may produce "ghosts" of the main line which would probably be spaced in some kind of periodic order. In this case, there might be a variation in the intensities of the "satellites" which would suggest the cause of the phenomenon. To this class some of the satellites of the mercury lines in the early observations of Lummer and Gehrcke belong.[2] That there were a number of "ghosts" in those apparent satellites has since been shown by Gehrcke and v. Baeyer, using crossed echelon plates. The other possible cause of the periodicity in spacing of the satellites would lie in the structure of the atom or molecule. Let us assume that the atom consists of a sphere of positive electricity with electrons arranged in concentric circles. Given the number, p, of the electrons at the center, the number on a ring required for equilibrium would lie between certain limits $n \pm e$. Thus we might have atoms with p electrons at the center and $n \pm 1$, $n \pm 2$, $n \pm 3$ electrons on a ring. Professor Thomson finds that when n is large, the frequency of the vibration arising from a tangential displacement of an electron takes the form

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[1] G. F. Hull, "On the Influence of Electric Fields on Spectral Lines," *Astrophysical Journal*, 25, 1, 1907.

[2] *Annalen der Physik*, 10, 457, 1903.

[3] *Ibid.*, 20, 269, 1906.

$$q^2 = A\left(1 - \frac{1}{2}\frac{S^2\pi^2}{n^2}\right)$$

where S has the values 1, 2, 3, etc.

And for a radial displacement

$$q^2 = B + C\left(1 - \frac{1}{2}\frac{K^2\pi^2}{n^2}\right)$$

where B has as a factor p, the number of electrons at the center. Giving to K or S the integral values 1, 2, 3, etc., the frequencies would produce a band spectrum according to Deslandres' law. If we were to assume that the number of electrons in a ring varied slightly for different atoms, we would get variations in q which might account for the phenomenon of satellites. Our theory of the Zeeman effect is not complete enough to lead us to predict whether these different vibrations would be influenced by a magnetic field all in the same way. Consequently, we are not forced to reject this explanation of the origin of the satellites on account of the fact that they do not exhibit the same Zeeman effect.

It must be noted that n would have to be very large in order to account for the small variations in frequency which we find to exist. At best, we can only consider this theory as pointing to a possible cause for the existence of satellites.

In the data below are included measurements made by various observers, viz.: O. v. Baeyer,[1] L. Janicki,[2] Gehrcke and v. Baeyer[3] (the mean of three sets), Gale and Lemon.[4] The numbers +228, −122, etc., are the differences in thousandths of an Ångström unit between the satellites and the main line.

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[1] *Physikalische Zeitschrift*, **9**, 831, 1908.

[2] *Annalen der Physik*, **19**, 36, 1906; **29**, 823, 1908.

[3] *Ibid.*, **20**, 278, 1906.

[4] *Astrophysical Journal*, **31**, 78, 1910.

5790.000	v. B.			G. & L. −8		
		+228=+45×5	+3	+229	+221 = 32×7	− 3
		+133=+45×3	—2	+135	+127 = 32×4	− 1
		−122=−60×2	−2	−119	−127 =−32×4	+ 1
		−180=−60×3	0	−181	−189 =−32×6	+ 3
				−930 } ?	−938 } ?=−32×2	9−10 } ?
				−988 }	−996 } =−32×3	1− 4 }
5769.000	v. B. +2			J. −14		
	+44	+46		+48	+34 =+32×1	+2
	−48	−46		−52	− 66=−32×2	−2
				−114	−128=−32×4	+0
5461.000	J.			G. & L. +6		
		+133= 44×3	+1	+217	+223= 45×5	−2
		+ 88= 44×2	+0	+130	+136= 45×3	+1
		− 66=−33×2	+0	+ 83	+ 89= 45×2	−1
		− 99=−33×3	+0	− 54	− 48=−45×1	−3
		−232=−33×7	−1	− 94	− 88=−45×2	+2
				−233	−227=−45×5	−2
4358.000	G. & L. −40			+2		
	+194	+154= 77×2	+0		196=11× 18	− 2
	+118	+ 78= 77×1	+1		120=11× 11	− 1
	+ 40	0			42=11× 4	− 2
	− 88	−128=−65×2	+2		− 86=−11× 8	+ 2
	−155	−195=−65×3	+0		−153=−11×15	+12
4047.000	G. & v. B. −11					
	−115	−126=−64×2	+2			
	− 53	− 64=−64×1	+0			
	+ 75	+ 64=+64×1	+0			
	+138	+127=+64×2	−1			

In v. Baeyer's measurements for λ 5790 two divisors have been used, 45 on one side and 60 on the other. Evidently 15 might have been used on both sides, but so small a divisor has no significance unless the remainders are small by comparison. A similar observation may be made concerning Janicki's values for λ 5461 and concerning Gale and Lemon's values for λ 4358. The small divisor 11 has no significance. In Gale and Lemon's values for λ 5790, the lines at 930 and 988 probably ought to be discarded. They are not obtained by other observers; moreover, they are so far away from the main line that they may be regarded —if they belong to the mercury spectrum—as separate lines.

The data above given, with no pretension of completeness, show how divisors may be found for the distances between the satellites and the main line. The question arises, To what extent is this a matter of chance? In other words, given six numbers, as in Gale and Lemon's values for λ 5461, between +250 and −250, what is the probability that an integer can be found which when added to all the numbers will allow the numbers so obtained to be divided by an integer of the order of 50 so that the remainders will be less than 10 per cent of the divisor? The rather tedious arithmetical solution shows that the upper limit to the resulting probability is so small that we must regard whatever periodicity appears in the data in the table as having an existence apart from the laws of chance.

It appears, then, that for some of the mercury lines the satellites measured by some observers show periodicity not to be accounted for as accidental. Thus Gale and Lemon's values for λ 5790 (leaving out the two satellites already discussed), Janicki's values for λ 5769, Gale and Lemon's values for λ 5461, and Gehrcke and v. Baeyer's values for λ 4047 show periodicity. V. Baeyer's values for λ 5790, Janicki's for λ 5461, and Gale and Lemon's for λ 4358 show a periodicity for satellites on one side of the main line and another periodicity for satellites on the other side.

The interpretation to be given to the quantity which has been added or subtracted from the measurements is that the main line, through unsymmetrical broadening or for some other reason, has been shifted one way or the other by the amount added or subtracted. In the case or λ 4358, however, this amount shifts the zero point of reference to the satellite +40.

An inspection of the results obtained by various observers shows that the structure of the mercury lines must depend on conditions of the source not fully known. Not only does the number of the satellites vary, but the positions of those evidently the same differ by quantities larger than the experimental errors. It is evident that if any complete law is to be found for the structure of the lines of an element, not only must we examine the lines of various elements, but we must also be able to control the physical condition determining the radiation.

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PROBABLE ERRORS OF RADIAL VELOCITY DETERMINATIONS

By J. S. PLASKETT

The magnitude of the probable errors attending the spectrographic determination of stellar radial velocities has always been with me a question of much interest, and considerable work[1] along the line of the dependence of probable error upon the width of the slit employed has already been accomplished. It is proposed in this paper to give a general discussion of the probable errors of radial velocities as affected by changes in the dispersion of the instrument, and in the type of stellar spectrum observed.

It may not be amiss to point out that in measuring stellar spectra, as in practically all scientific measurements, we have two classes of errors to deal with or guard against: first, the accidental errors of setting upon the lines of the spectra due partly to imperfect definition, and partly to the unavoidable differences in successive settings which are always present even with the most careful observers; second, the systematic errors, due in this case generally to instrumental conditions which give rise to spurious relative displacements of stellar and comparison lines. Among such conditions may be cited flexure or changes of temperature of the spectrograph, non-uniform illumination of the collimator, prisms, and camera by the light from star or spark, faulty focal adjustments of collimator or camera objectives, and so on. The former can readily be evaluated from the measures of the plates themselves, but no evidence of the latter appears in such measurements, and its magnitude can only be determined from the comparison of a number of plates of the same star. In the latter case, however, the errors so obtained will not be entirely systematic but will be affected by the accidental errors present in the measured velocities. In the discussion to follow, relative measures of the accidental errors are given by the probable errors of single lines or regions on a plate, while for the systematic effect the probable error of a single plate, obtained by the discussion of several plates of the same star, is probably the best that can be done, although, as stated above, such result has also included in it the effect of the accidental errors.

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[1] *Astrophysical Journal*, **28**, 259, 1908; and *Trans. Roy. Soc. Can.*, **3**, 209, 1909.

In considering the effect of change of dispersion one would naturally expect to find the actual linear errors of the same magnitude for all dispersions and, as errors are always expressed in velocity values, the probable errors in kilometers per second would hence be inversely proportional to the linear dispersion. That is to say, as we have at the Dominion Observatory three different dispersions 10.1, 20.2, and 33.4 tenth-meters per millimeter at $H\gamma$, practically as 3, $1\frac{1}{2}$, and 1, we should expect to find the probable errors in kilometers per second inversely proportional to these latter numbers, or as 1, 2, and 3.

Most of the radial velocities at the Dominion Observatory have been obtained from spectra made with the lowest dispersion, a single-prism spectrograph, on spectroscopic binary stars of early type, in which the spectral lines have generally been broad and diffuse. The probable errors of the velocity determinations of single plates have been consequently high, so high as to lead to the belief that the relation above expressed was not the true one but that the probable errors increased more rapidly than the dispersion diminished.

It seemed therefore worth while to make a definite test of the matter especially as, although considerable data as to the probable errors of high-dispersion star spectrographs are available, there is so far as I know not much published information in regard to the probable errors of one-prism instruments. In order to avoid, so far as possible, any effect due to diffuseness of the spectral lines, it was decided to use spectra of the solar type for the comparison. Further, to eliminate difficulties of identification of wave-lengths in the blends of lines always present in low-dispersion second-type spectra, it was essential to measure the plates by the spectro-comparator, an instrument in which the actual displacement of the star lines due to velocity is compared with those in a standard plate of the sun whose velocity is known. By this method no knowledge of wave-length is necessary and errors due to the loss of purity inherent with small dispersion cannot affect the measurements.

The brightest solar-type star, *Arcturus*, was selected as a test object for the obvious reason that only short exposures would be required. To produce spectra of good quality for measurement on *Arcturus* exposures are necessary of about ten minutes with the three-prism long-focus spectrograph designated as III L, 10.1 Ångströms per mm, four to five minutes with the three-prism short-focus designated as III R, 20.2 Å per mm, and one and a half minutes with the single-prism spectrograph designated as I, 33.4 Å per mm. at $H\gamma$, and consequently the plates required may be quickly made. Fortunately when this investigation was begun we had already obtained nearly thirty plates with III L for another purpose and only plates with III R and I were required. Eleven were made with III R and about fifty with I.

Of these plates 24 of III L, 11 of III R, and 38 of I were measured by myself on the spectro-comparator and from these measures the results to be discussed were derived. In order to explain clearly how the probable errors were obtained it is necessary to describe briefly the comparator and the method of measurement. In the first place a standard spectrum of the sun is obtained by the same spectrograph, and this plate has impressed upon it one on each side of the solar spectrum a strip of the same comparison as in the star spectrum. This standard sun spectrum and the star spectrum are viewed by a special double objective, single ocular microscope, with a Lummer Brodhun cube in the ocular, which serves to superpose the two spectra so that a narrow strip of star spectrum is seen between and touching two strips of sun spectrum, while on each side a narrow strip of the star comparison lies between and touching strips of sun comparison. The standard sun spectrum is moved by a micrometer screw until the corresponding lines of the star and sun spectra are in exact coincidence and then again moved until the comparison lines of the two spectra are coincident. The difference in the micrometer readings evidently gives us the displacement, due to radial velocity, of the star lines with respect to the sun lines, which on multiplication by a constant gives, after adding with the proper sign the known velocity of the sun, the velocity of the star with reference to the observer.

The measures were made as usual on the spectra comparator by determining the coincidences at a number of chosen regions, marked by dots on the sun spectrum, which correspond, in a sense, to the lines in an early-type spectrum, on which the cross wire is set. The accidental errors can thus evidently be determined from the probable error of the determination of the points of coincidence in these regions. After the spectra have been measured with the violet end to the left, for example, they are reversed on the comparator, and the same regions remeasured. The differences of displacement, corrected for a systematic effect due to reversal, are evidently wholly due to accidental errors of setting, and from these the probable error of a region is readily obtained in the well-known way, giving a measure of the purely accidental error.

In addition to the purely accidental errors of setting are others also of an accidental character due to irregular arrangement of the silver grains or distortion of the film, to the forming of the coincidences to one side or other of the dot or center of the region and consequent incorrect value of the velocity-constant by which the displacement is multiplied, and to numerous other causes. A measure of the total accidental error is evidently obtained by computing the velocities separately for the mean of the two measures red right and red left of each region, by obtaining the residuals from the mean velocity of the plate, and the probable error in the usual way. This probable error should be and is, as is seen below, somewhat greater than the purely accidental error of setting.

The systematic errors due to instrumental peculiarities can only be obtained from the discussion of plates of the same star in sufficiently large numbers to insure that the systematic displacements for this particular spectrograph become accidental in character. It is possible, however, that there may be slight constant systematic differences in the values given by different spectrographs. This is shown in the results below, but is likely due to accidental errors of the fundamental spectrum. We have, in the three dispersions, plates to the number of 24, 11, and 38. By treating the residuals from the mean velocities, we can obtain the probable error of a single plate in which we get a good relative idea of the systematic errors, although as before stated we have the accidental errors of measurement included. This certainly gives us an accurate idea as to the total error involved, which in the ultimate analysis is what we wish to know.

The following tables contain a summary of the measures in the three dispersions.

SPECTROGRAPH III L. 10.1 per mm

Plate No.	No. of Regions	Velocity	Probable Errors of Single Regions			Residual
			Errors of Setting		Total Accid'l Errors	
		km	Rev.	km	km	km
1426.......	11	−5.74	±0.0015	±0.59	±0.82	0.23
1455.......	11	4.97	.0008	.32	.59	0.54
1456.......	16	5.64	.0009	.35	.52	0.13
1514.......	12	5.91	.0017	.66	.93	0.40
1515.......	13	5.78	.0008	.31	.57	0.27
1529.......	10	6.14	.0010	.41	.65	0.63
1585.......	13	5.59	.0020	.78	.71	0.08
1586.......	10	5.94	.0012	.49	.61	0.43
1588.......	11	6.93	.0013	.53	.62	1.42
1595.......	12	4.14	.0009	.37	.53	1.37
1596.......	13	5.43	.0010	.39	.69	0.08
1597.......	13	6.12	.0011	.43	.66	0.71
1606.......	12	5.42	.0009	.37	.61	0.09
1615.......	12	6.24	.0011	.42	.74	0.73
1616.......	12	6.86	.0009	.37	.76	1.35
1619.......	16	5.14	.0011	.40	.32	0.37
1620.......	14	5.80	.0011	.43	.64	0.29
1622.......	15	4.58	.0015	.57	.61	0.93
1635.......	14	3.97	.0014	.53	.48	1.54
1645.......	16	5.00	.0015	.54	.64	0.51
1662.......	12	4.98	.0011	.45	.72	0.53
1671.......	16	6.10	.0009	.32	.61	0.59
1672.......	14	5.18	.0013	.51	.45	0.33
1709.......	15	4.62	.0009	.35	.74	0.89
Means	13	−5.51	±0.00117	±0.453	±0.628	

Average probable error of setting single region = ±0.45 km.
Average probable total accidental error single region = ±0.63 km.
Average probable total accidental error single plate = ±0.17 km.
Total probable error (accidental + systematic) single plate = ±0.50 km.

SPECTROGRAPH III R. 20.2 t.-m. per mm

PLATE NO.	NO. OF REGIONS	VELOCITY	PROBABLE ERRORS OF SINGLE REGIONS			RESIDUAL
			Errors of Setting		Total Accidental Errors	
		km	Rev.	km	km	km
3288.......	10	−6.64	±0.0011	±0.80	±1.22	2.09
3289.......	9	4.94	.0009	.67	1.04	0.39
3290.......	9	5.58	.0009	.67	0.83	1.03
3291.......	10	4.83	.0013	.95	1.18	0.28
3311.......	11	3.27	.0011	.78	1.02	1.28
3312.......	10	4.16	.0013	.95	0.64	0.39
3313.......	10	3.47	.0013	.95	1.25	1.08
3314.......	10	5.78	.0010	.73	0.87	1.23
3315.......	10	3.33	.0006	.44	1.11	1.22
3316.......	10	4.52	.0008	.58	0.88	0.03
3317.......	10	3.54	.0012	.88	0.99	1.01
Means	10	−4.55	±0.00105	±0.75	±1.00	

Average probable error of setting single region = ±0.75 km.
Average probable total accidental error single region = ±1.00 km.
Average probable total accidental error single plate = ±0.32 km.
Total probable error (accidental+systematic) single plate = ±0.75 km.

SPECTROGRAPH I. 33.4 Ångströms per mm

PLATE NO.	NO. OF REGIONS	VELOCITY	PROBABLE ERRORS OF SINGLE REGIONS			RESIDUAL
			Errors of Setting		Total Accidental Errors	
		km	Rev.	km	km	km
3126.......	6	−6.14	±0.0019	±2.25	±2.49	0.13
3127.......	8	7.87	.0009	1.02	1.25	1.86
3128.......	9	5.49	.0007	0.78	1.37	0.52
3129.......	7	6.58	.0016	1.86	1.02	1.57
3130.......	7	5.42	.0012	1.39	1.11	0.59
3146.......	8	8.54	.0008	0.90	1.57	2.53
3147*a*......	9	6.42	.0014	1.57	1.89	0.41
3147*b*......	9	5.49	.0011	1.23	1.39	0.52
3147*c*......	9	5.43	.0008	0.90	1.30	0.58
3148*a*......	11	7.56	.0007	0.76	1.52	1.55
3148*b*......	11	6.84	.0010	1.08	1.21	0.83
3148*c*......	11	6.19	.0006	0.65	1.80	0.18
3149*a*......	9	5.49	.0008	0.90	1.83	0.52
3149*b*......	9	4.44	.0011	1.23	1.31	1.57
3149*c*......	9	4.87	.0012	1.34	1.79	1.14
3150*a*......	9	4.44	.0007	0.78	1.25	1.57
3150*b*......	9	4.44	.0011	1.23	1.87	1.57
3150*c*......	9	5.99	.0007	0.78	1.45	0.02

SPECTROGRAPH I. 33.4 Ångströms per mm—*Continued*

Plat No.	No. of Regions	Velocity	Probable Errors of Single Regions			Residual
			Errors of Setting		Total Accidental Errors	
		km	Rev.	km	km	km
3238*b*......	9	5.51	.0008	0.90	1.16	0.50
3238*c*......	9	6.20	.0009	1.01	1.61	0.19
3239*a*......	9	5.58	.0009	1.01	1.00	0.43
3239*b*......	9	5.64	.0008	0.90	1.68	0.37
3239*c*......	9	5.64	.0005	0.56	1.15	0.37
3240*a*......	9	4.34	.0012	1.34	1.35	1.67
3240*b*......	9	5.08	.0010	1.12	0.93	0.93
3240*c*......	9	5.51	.0012	1.34	1.79	0.50
3272*a*......	9	6.21	.0011	1.23	0.91	0.20
3272*b*......	9	6.98	.0008	0.90	1.64	0.97
3272*c*......	9	5.96	.0006	0.67	1.17	0.05
3273*a*......	9	6.83	.0011	1.23	1.63	0.82
3273*b*......	9	7.76	.0011	1.23	1.39	1.75
3273*c*......	9	5.22	.0010	1.12	1.80	0.79
3274*a*......	9	5.22	.0009	1.01	1.06	0.79
3274*b*......	9	6.83	.0008	0.90	1.45	0.82
3274*c*......	9	6.15	.0011	1.23	0.86	0.14
3275*a*......	9	5.90	.0013	1.46	1.36	0.21
3280*c*......	9	8.07	.0009	1.01	1.71	2.06
3281*c*......	9	4.23	.0005	0.56	0.69	1.78
Means.....	8.84	−6.01	±0.00097	±1.09	±1.41	

Average probable error of setting single region = ±1.09 km.
Average probable total accidental error single region = ±1.41 km.
Average probable total accidental error single plate = ±0.47 km.
Total probable error (accidental+systematic) single plate = ±0.70 km.

Collecting in the following table the final values, we obtain the relative errors for the three dispersions.

Spectrograph	Scale Å per mm	No. of Plates	Mean Velocity	Errors of Single Region		Errors of Single Plates		
				Errors of Setting	Total Accidental Errors	Total Errors, Accidental and Systematic	Accidental	Systematic
III L......	10.1	24	−5.51	±0.45	±0.63	±0.50	±0.17	±0.47
III R......	20.2	11	−4.55	0.75	1.00	0.75	0.32	0.68
I..........	33.4	38	−6.01	1 09	1.41	0.70	0.47	0.52

We have in the first column the spectrograph employed; in the second the dispersion in Ångströms per millimeter at Hγ; in the third the number of plates measured; and in the fourth the mean velocity of these plates.

The fifth column contains the average probable error of the estimation of the coincidences in a single region, while the sixth contains the average total accidental error as obtained from the final kilometer values of the displacement for each region.

Under the heading of Errors of Single Plates we have in the seventh column the total error obtained from the mean velocities of all the plates, and in the eighth the accidental error obtained by dividing the total accidental error of a single region by the square root of the number of regions, while the ninth column is obtained from the two preceding columns by taking the square root of the difference of their squares.

It is difficult to account for the results obtained, especially in the errors of single plates, for, as before stated, one would expect the kilometer values of the probable errors to be inversely proportional to the linear dispersion. This is approximately true so far as the errors of single regions are concerned, and the discrepancy can be satisfactorily explained by the greater ease and accuracy in the determination of coincidences in the single-prism spectrograph owing to the decidedly smaller curvature of the spectral lines. But when we come to the total errors of a single plate as determined from the measured velocities of the plates, we find the errors instead of being in the ratio of 1, 2, and 3, as we should expect, areas 1, $1\frac{1}{2}$, and $1\frac{1}{2}$ approximately.

So far as III L and III R are concerned, it must be remembered that, although the linear dispersions are as 1 to 2, the angular dispersions and the resolving powers are equal and the decrease of the ratio from 1:2 to $1:1\frac{1}{2}$ can thus be accounted for. In the single-prism spectrograph, however, the linear dispersion, angular dispersion, resolving power, and purity of spectrum are practically only one-third of those of III L, and the errors should be three times as great. Instead of that the total error of a plate, which is of course the most important quantity to be determined, is only 40 per cent greater with the lower dispersion and an examination of the last two columns shows that the systematic (due to changing instrumental factors) part of this error is nearly the same in III L and I, while the accidental part is nearly in proportion to the dispersions.

This would seem to indicate either that the single-prism spectrograph is less likely to give systematic displacements of the spectrum lines than III L or that the kilometer rather than the linear value of the systematic displacements remains constant. So far as the first supposition is concerned, although the single-prism instrument[1] is undoubtedly less affected by flexure than III L and is probably better controlled and regulated as regards temperature, these two factors will not have much influence in the short exposures required on *Arcturus*. There is no ground for supposing on the other hand that kilometer rather than linear values of the systematic displacement should remain constant except in the case of displacements due to temperature changes in the prisms.

A partial explanation of the relative superiority of the low-dispersion instrument is that it may be due to the fact that the three-prism plates were mostly made on different dates and at varying hour angles, though never far from the meridian; while the one-prism plates were made in four groups only, the plates being obtained consecutively and probably under similar conditions in each of the groups.

In any case it seems to be evident that radial velocity determinations of second-type stars may be made with low-dispersion instruments with an accuracy not much less than that obtainable with the high-dispersion instruments at present in use. This fact, if considered established by the present investigation, is one of much importance, as it admits the carrying of the spectrographic survey of the heavens to stars more than a magnitude fainter than those at present available.

It may be of interest to compare the values of the probable errors of a region and plate in the solar-type star *Arcturus* with those obtained in my investigations on the effect of slit-width,[2] of the probable errors of an average line and of a single plate in the case of the early-type star *β Orionis*. I will in this case take the mean of the values obtained for the three slit-widths 0.025, 0.038, and

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[1] *Journal R.A.S. Canada*, **3**, 287, 1909.

[2] *Astrophysical Journal*, **28**, 259, 1908; *Trans. Royal Society of Canada*, **3**, 209, 1909.

0.051 mm, thus using 18 plates with III L, 30 plates with III R, and 30 plates with I.

PROBABLE ERRORS

Spectrograph	Line of Average Weight	Single Plate
III L. 10.1	±2.20	±1.17
III R. 20.2	±2.47	±1.53
I 33.5	±3.18	±2.16

These results are relatively to one another in substantial agreement with those obtained in the present investigation though not quite so favorable to the low dispersion. There seems hence to be no reasonable doubt that the accuracy of radial velocity determinations does not by any means proportionately diminish with decrease of dispersion.

It is evident therefore that the high probable errors of single observations obtained in our work on spectroscopic binary orbits must be due, not to the small dispersion employed giving results relatively less accurate than those obtained with high-dispersion instruments, but to the character of the lines in the spectra, with the resultant high errors of measurement. In many cases also they are probably due to abnormal conditions in the orbit causing deviations from velocity-curves due to simple elliptic motion, thus giving higher residuals.

I have tabulated below the probable error of an average observation of the velocity of the brighter stars of some of the spectroscopic binary systems as determined here and at the Allegheny and Lick observatories and it will be noticed at once that the accuracy very rapidly diminishes as the spectrum lines become broader and with orbits containing abnormal secondary or other effects. The slightly lower values obtained at Allegheny for the probable errors are likely due to the fact that many of their spectra were made on the fine-grained Seed 23 instead of the coarser-grained Seed 27 plate used at Ottawa.

None of the orbits determined with low dispersion at Ottawa and Allegheny is of stars with solar-type spectra, so that no measure can be thus obtained of the relative accuracy of this dispersion. Three solar-type binaries observed at the Lick Observatory with a single-prism spectrograph show fairly accurate results, especially *W Sagittarii*, with a plate error of ±0.90 km, and

PROBABLE ERRORS OF SINGLE OBSERVATIONS

Star	Spectral Type	Probable Error Average Plate	Dispersion	Observatory	Remarks
7 Camelopardalis	A 2 F	±2.4	One-prism	Ottawa	Lines good
α Draconis	VII a	3.4	One-prism	Ottawa	Lines good
θ Aquilae	VII a	4.0	One-prism	Ottawa	Changing elements?
B. D.−1°1004	B 3 A	5.2	One-prism	Ottawa	Broad line
α Coronae	VIII a b	5.4	One-prism	Ottawa	Lines broad and asymmetric
φ Persei	L	4.1	One-prism	Ottawa	Abnormal secondary effect
ε Herculis	VII a	6.4	One-prism	Ottawa	Abnormal secondary effect
ι Orionis	B, I b	6.8	One-prism	Ottawa	Lines diffuse and asymmetric
ψ Orionis	B, I b	6.8	One-prism	Ottawa	Lines very broad
τ Tauri	B, I b	10.8	One-prism	Ottawa	Lines very broad
β Orionis	VI c	2.0 3.2	Three-prism One-prism	Ottawa Ottawa	Lines good Lines good
η Virginis	VIII a	2.4	Three-prism	Ottawa	Lines fair
η Boötis	XIV a	0.50	Three-prism	Ottawa	Numerous well defined lines
π^4 *Orionis*	IV a	1.7	One-prism	Allegheny	Lines very good
ζ^1 *Lyrae*	A, I a 2	2.0	One-prism	Allegheny	Lines well defined
α Andromedae	VIII P	2.5	One-prism	Allegheny	Lines good
β Aurigae	VIII a	3.2	One-prism	Allegheny	Lines sharp and narrow
θ Aquilae	VII a	3.4	One-prism	Allegheny	Lines fair
δ Librae	VII a	4.7	One-prism	Allegheny	Ill-defined lines
α Coronae	VIII a b	4.9	One-prism	Allegheny	Broad lines
2 Lacertae	A, I b	5.3	One-prism	Allegheny	Fair lines
u Herculis	A, I b	8.0	One-prism	Allegheny	Broad lines
α Virginis	III b	9.9	One-prism	Allegheny	Very broad and diffuse
η Pegasi	XIV a	0.47	Three-prism	Lick	Good lines
α Aurigae	XIV a	0.50	Three-prism	Lick	Good lines
λ Andromedae	XV a	0.50	Three-prism	Lick	Good lines
β Herculis	XV a	0.52	Three-prism	Lick	Good lines
ι Pegasi	XII a	0.56	Three-prism	Lick	Good lines
α_2 *Geminorum*	VIII a	0.64	Three-prism	Lick	Good lines
α_1 *Geminorum*	VIII a	0.79	Three-prism	Lick	Good lines
ω Draconis	F 5 G	0.75	Three-prism	Lick	Diffuse lines
θ Draconis	XIII a	0.87	Three-prism	Lick	Diffuse lines
RT Aurigae	G	0.82	Three-prism	Lick	Diffuse lines
η Aquilae	XIV a c	0.85	Three-prism	Lick	Diffuse lines
W Sagittarii	F 5 G	0.90	One-prism	Lick	Fair lines
X Sagittarii	F 8 G	1.80	One-prism	Lick	Fair lines
Y Sagittarii	G	2.10	One-prism	Lick	Fair lines

which if three or four discrepant observations are omitted reduces to ±0.55 km. This is of the same order as the probable error of a single-prism plate of *Arcturus* as determined here (±0.70 km).

The probable errors of single observations of binary and constant-velocity solar-type stars with three-prism dispersion both at Lick and Ottawa seems to be very close to half a kilometer and in cases where it is greater it is apparently due to the poorer quality of the spectra for measurement. Three cases in which the probable error of a plate is less than half a kilometer are known to me. At Ottawa a series of 11 plates of *β Geminorum* gave a probable error of ±0.40 km per second. When observations are limited to certain hour angles and special precautions taken, as at Bonn by Küstner in determination of the solar parallax, 16 plates of *Arcturus* gave a probable error of a single plate of ±0.22 km; and at the Royal Observatory Cape of Good Hope, 22 plates of *β Geminorum* gave a probable error of ±0.34 and 55 plates of *α Boötis* of ±0.42 km. There is no doubt, though no values have been published, that the work at the Lick and Yerkes Observatories on solar-type constant-velocity stars is equally accurate.

It seems to me therefore that we may safely draw the following conclusions from the preceding discussion:

I. The accuracy of determination of the radial velocity of stars of solar type by means of spectrographs of different dispersions is not, as would be expected, inversely proportional to the dispersion, but in the cases under discussion only a small increase of probable error, 40 per cent, takes place when the dispersion is divided by three. As the relative exposures required are as about five to one, it is evident that stars more than a magnitude and a half fainter become available.

II. The probable error very rapidly increases with the increase in diffuseness of the lines in early-type stars, varying in low-dispersion spectrographs from about ±2 to ±11 km per second. Experience in work with these stars has convinced me that the whole of this error is not due to the accidental error of pointing, but that in many cases some physical cause in the star's atmosphere is responsible for a considerable part of the discrepancy.

III. The results of this and other investigations show that the probable error of a single ordinary observation of a good second-type star with the usual three-prism dispersion is in the neighborhood of 0.5 km per second. When greater than this, it indicates a spectrum with poorer lines, and when less, that special precautions and limitations were adopted in the making and measurement of the spectra. It is also shown that the probable error of determination with a thoroughly stable one-prism instrument of one-third the dispersion is for solar-type spectra measured on the spectro-comparator about ±0.70 km per second, and for spectra of earlier type varies from about ±2 to ±11 km per second.

IV. Generally speaking the major part of the errors in solar-type stars are due to systematic displacements of the lines as a whole owing to flexure, temperature changes, imperfect adjustments of the optical parts, faulty guiding, or other causes, and the accidental errors of pointing are responsible generally for only one-third or less of the total error. In the case of early-type stars the systematic displacements due to instrumental conditions will probably be approximately the same while the errors of pointing are correspondingly increased.

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Dominion Observatory
Ottawa, Canada
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THE COLLIMATION OF THE CORRECTING LENS

By J. S. PLASKETT

When the new single-prism spectrograph was brought into use it was noticed that the star spectra obtained were unusually weak in the violet. Although the absorption of the new prism of 51 mm aperture is somewhat greater than that of the one previously in use (35 mm aperture), this difference is by no means sufficient to account for the difference in intensity at the violet in the two cases.

In many star spectra the lines to the violet of $H\delta$ are important and the best measures obtained are frequently those of the K line. Moreover, greater accuracy should be obtained in this part of the spectrum on account of the greater linear scale. It was hence important to discover, if possible, the cause of and the remedy for this difficulty.

It was necessary, when the new spectrograph was attached to the telescope, to make some changes in the attachment of the correcting lens, and there was a possibility that the distance of the lens from the focus had become changed and the form of the color-curve altered sufficiently to throw the image in violet light considerably beyond the slit. However, a redetermination of the color-curve by Hartmann's method of extra-focal exposures showed that the minimum focus was at about λ 4325, that the focal points for light at λ 3930 and λ 4700 were each about three millimeters beyond that at λ 4325, and moreover that the position of the slit was the most favorable for obtaining uniform intensity of the photographed spectrum from $H\beta$ to K.

These two possible causes having thus been disposed of, the next tested was the guiding apparatus. The visible image produced by the combination of visual objective and correcting lens consists of a more or less condensed nucleus of blue-green light surrounded by an extended penumbra or halo of reddish light, and one guides by the blue-green nucleus while the slit is rendered partly visible by the penumbra. It is not possible, owing probably to chromatic aberration, to get both sharp at the same time, and there is usually considerable parallax, due partly to this cause and partly to the fact that the visual blue-green image is really a millimeter or more beyond the slit. It is possible, therefore, although the image may be kept apparently central, that its effective part is really to one side or other of the slit, producing diminished intensity of spectrum. However, a careful readjustment of the reflecting prism and guiding apparatus produced no perceptible improvement.

The following test was then made to determine whether guiding with the image apparently above or below the slit would have any effect. After the governors had been so adjusted that the star image drifted the width of the star window along the slit in 20 seconds, four spectra of the bright star *a Aquilae* were made, with an exposure on each of 60 seconds, 3 times drifting, side by side on the one plate. In the first of these the star image was kept bisected by the upper edge of the slit opening (as seen in the guiding telescope), in the second the image was kept central, in the third bisected by the lower edge, and in the fourth kept entirely below and just touching the lower edge of the slit. The result of this test is shown in the figure at *A* where *a*, *b*, *c*, *d* represent the four positions in guiding. It will be noticed at once that the point of maximum intensity moves toward the violet as the image is moved down. A little consideration renders it evident that this effect is due to a sort of dispersion of the image, that the violet part of it falls above the blue-green by more than the width of the slit. This cannot be detected by the eye on account of the very low visual intensity of the violet light. As a matter of fact the guiding must be done almost entirely by the image formed by light of wavelengths between λ 4600 and λ 4900. To the violet of this region it is too faint, and to the red the image is too far out of focus for any effect on the eye. It is evident from the figure that it is only when the image formed by this light seems to be entirely below the slit that the maximum intensity in the violet is obtained, and that in consequence the violet image is on the slit.

Such dispersion of the image may be assigned to one or more of three causes: (*a*) to faulty squaring of the objective; (*b*) to atmospheric dispersion; (*c*) to imperfect collimation of the correcting lens.

A test of the objective showed it to be in correct adjustment, and moreover throwing it considerably out of square had no appreciable effect on the appearance of the image, or on spectra made under the same conditions as before.

That it was not wholly due to atmospheric dispersion was at once proved by the same test, as described above on *α Aquilae*, applied to a star near the zenith, where a similar effect was produced.

That incorrect collimation of the corrector was the principal cause was finally shown when, after a change in collimation, the test gave a different arrangement of the intensity in the four spectra. The correcting lens, which was specially designed,[1] has an aperture of 102 mm (4 inches) and is placed 150 cm (59 inches) within the focus. It is mounted in a tube whose lower end is held in a flange which screws into the telescope adapter, while near the upper end a guide ring holds it central. It was carefully collimated with both the old and the new spectroscope by pointing the telescope to the zenith, removing the objective, and hanging a steel piano wire centrally through the objective cell, the correcting lens cell, the slit, and the collimator lens cell. It is possible that after collimation there may have been some movement of the spectroscope before it was finally rigidly fastened into position, and that this was sufficient to produce the observed effect, for, as will be seen later, a displacement of a millimeter is sufficient to produce marked changes in the distribution of intensity in the spectrum.

Some further tests showed the necessity for having the collimation of the correcting lens made adjustable. Consequently the upper end of the tube was made movable transversely to the slit by applying a screw of $\frac{1}{16}$ inch ($1\frac{1}{2}$ mm) pitch, with a divided head, to the guiding ring above mentioned. No longitudinal adjustment was provided, as any dispersion along the slit will not tend to throw any part of the image to one side or other of the slit opening.

A series of tests, similar to the one described above, was then made at different settings of the adjusting screw of the corrector tube, and it was soon discovered that the correcting lens was very much more sensitive to changes in collimation than had been imagined. Plate XIV shows tests at three settings, *B*, 1.6; *C*, 2.0; *D*, 2.4, of the adjusting screw, the distance between successive positions being 0.6 mm. The spectra are of the star *a Aquilae*, and the exposures, which are of 60 seconds each, are made with the star near the meridian and the telescope west of the pier. The slit was 0.051 mm wide and the image was guided so that *a*, *b*, *c*, *d* refer to the same positions as at *A*.

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[1] *Astrophysical Journal*, **28**, 139, 1908; *Report of Chief Astronomer*, 1908, p. 73.

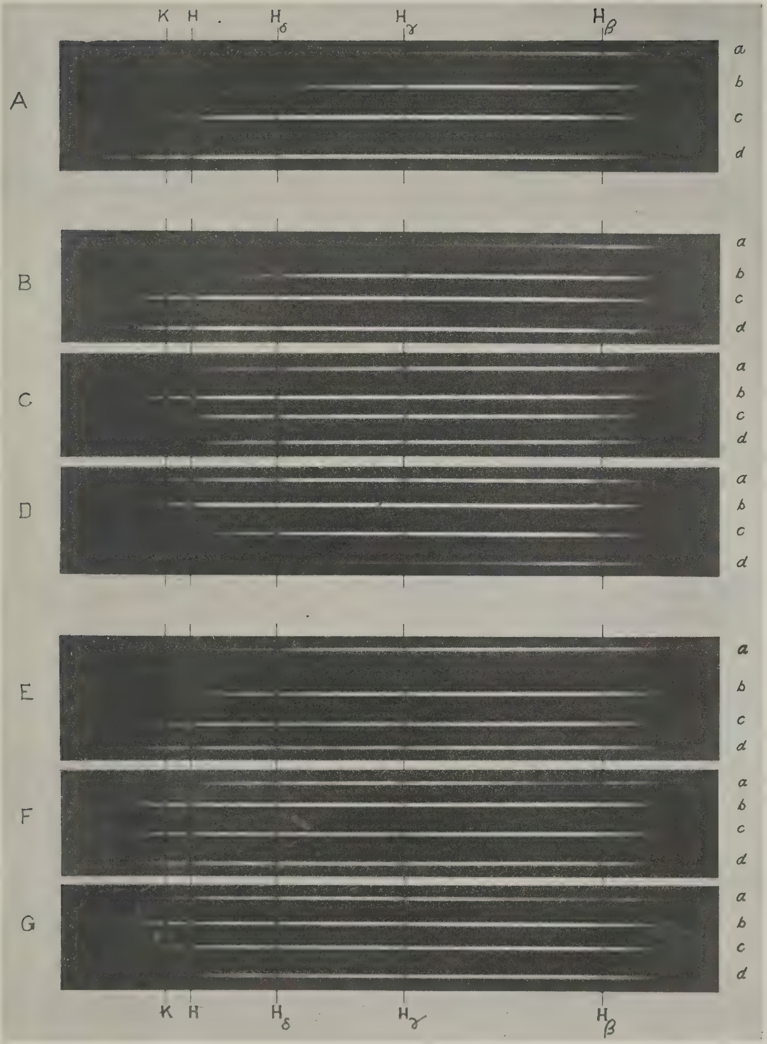
The criterion for determining the best position of the correcting lens must evidently be the intensity toward the violet end of the spectrum, for, owing to the greater photographic effect in the blue-green and the expanded image there due to the form of the color-curve of the corrector, a slightly asymmetric position of the image on the slit will have little effect on the intensity. It will be noticed then that *c* in *B*, *b* in *C*, and *a* in *D* are the spectra which have the greatest extension and intensity in the violet and are evidently therefore the positions where the violet part of the image was central on the slit. Hence in position *B*, setting 1.6, the violet image was apparently above; in position *D*, setting 2.4, apparently below the blue-green image on which guiding is necessarily performed; while in *C*, setting 2.0, they are superposed, as the greatest intensity in the violet is given when the image is kept central.

C, at setting 2.0 of the corrector screw, is therefore the position, not necessarily (as the star is not at the zenith) of exact collimation of the correcting lens, but the position where the dispersion produced by the atmosphere is just counteracted by the opposite dispersion caused by a slight departure of the corrector from exact collimation.

If we now look at the lower part of the figure, spectra made in a similar way of the same star near the meridian, but with the telescope east of the pier, where *E* is at setting 0.8, *F* 1.2, and *G* 1.6, we find that the best position *F* is at a setting of the correcting lens 0.8 rev., or 1.25 mm, distant from the best position west of the pier.

As the atmospheric dispersion acts in the same direction in each case, the change in collimation must be due to flexure of the tube

PLATE XIV



Tests of Collimation of Correcting Lens

of the telescope, which is sufficient to displace the correcting lens at the declination +8°.6 by 0.6 mm ($\frac{1}{40}$ of an inch), from the line joining the center of the objective and the slit. This is not unreasonably large when we consider that the correcting lens is 150 cm from the slit. A further test on *α Cygni*, which at the meridian is only half a degree from the zenith, showed the best position of the corrector to be at setting 1.6 midway between the others.

Consequently, assuming this change of collimation to be due to flexure, and that approximately this is proportional to the sine of the zenith distance, a table of the corrector settings for every ten degrees of zenith distance, from 0° to 70°, was computed and will be used, after some further tests at intermediate points and at large hour angles, for every spectrum taken.

The necessity of this is very clearly shown by the marked dispersion of the image indicated in *B*, *C*, *D* and *E*, *F*, *G* for a difference of position of the correcting lens of 0.6 mm. Indeed further tests showed that a change of collimation of only 0.25 mm can easily be recognized by the distribution of intensity in the spectra, and it is evident that, for the best results, we must insure very exact collimation. These tests show that a photographic correcting lens for visual objectives has no field, that it must be used along the axis. If the cone of light from the objective, which is 99 mm in diameter, falls even 0.25 mm to one side of the center of the correcting lens, the star image is dispersed in such a way that the image in violet light falls to the same side of the blue-green image. Estimating from the tests made, the dispersion or separation of the blue-green and violet images is approximately 0.05 mm for each 0.75 mm the corrector is moved.

These experiments have seemed worth recording, as showing how very carefully the corrector must be collimated, even to such an extent that the departure from collimation due to flexure of the telescope tube must be accurately compensated. It is convincingly demonstrated by the figure that a departure from collimation of only 0.6 mm (0.025 inch) will much increase, nearly double, the exposure time required for the violet end of the spectrum. Moreover, there is a further even more important consideration, that systematic displacement of the lines is less likely to occur under accurate collimation. Uniform illumination of the collimator lens by light of every wave-length (necessary to prevent possible systematic displacements) can be produced only when the images in light of all the wave-lengths used fall centrally on the slit.

It gives me much pleasure to express my appreciation of the interest shown by the Director, Dr. King, in this work.

Dominion Observatory
Ottawa, Canada
August 1910

ON THE REVERSAL OF SPECTRAL LINES[1]

By H. KONEN

1. In a repetition of Pflüger's[2] experiment on the absorption of luminous hydrogen, A. Ladenburg[3] observed certain phenomena of reversal when he used high dispersion, which he regarded as a proof that the quotient E/A, emission over absorption, is not constant within the line investigated. He further concluded that the radiation of hydrogen is not a temperature radiation.

In contrast, Pflüger[4] has shown that the explanation offered by Ladenburg is a possible one, but not a necessary one, and that the observed phenomena might equally well be explained for a constant E/A, if we take into account the fact that Ladenburg's emitting tube furnishes a spectrum by no means equivalent to the continuous spectrum of a black body, but that it has diffuse maxima at the positions of the hydrogen lines. He accordingly identified the phenomenon with the multiple reversal of spectral lines often observed and discussed by spectroscopic observers, and at the same time he has contributed toward an explanation of the phenomenon.

On first looking at Ladenburg's illustrations, I was reminded of the phenomena of reversal previously described by Konen and Hagenbach,[5] and I immediately communicated to Professor Kayser of Bonn a computation in complete agreement with the results of Pflüger. The latter then sent me his manuscript, and I accordingly laid aside my notes. It now appears to me, however, that they may have some interest in connection with those of Pflüger, inasmuch as they propose an entirely different way, and also permit a somewhat simpler and more rigorous manner of regarding the reversal phenomena of spectral lines than do the methods of representation hitherto given.

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[1] Translated from advanced proof sheets of an article appearing in the *Physikalische Zeitschrift*, **11**, 663, 1910.

[2] *Annalen der Physik* (4), **24**, 515, 1907.

[3] *Verhandlungen der deutschen physikalischen Gesellschaft*, **10**, 550, 1908; **12**, 54, 1910.

[4] *Ibid.*, **12**, 208, 1910.

[5] *Astrophysical Journal*, **19**, 110, 1904.

2. Let us designate by E_2 the emission of the source of light which is to serve as an absorbing medium, that of the emitting source by E_1, while A_2 and A_1 may signify the corresponding absorptions, and e the emissive power of the absolutely black body. The source of light I is then observed through the luminous gas II. Let it further be assumed that Kirchhoff's law holds good within the spectrum, i.e., $E_2/A_2=e''$, $E_1/A_1=e'$, in each instance for a definite temperature and wave-length. The resulting emission E then obviously is the value

$$E_2+E_1(1-A_2)=E\,,$$

and it may be represented by the curve $E=f(\lambda)$. A reversal will occur at each point where this curve has a minimum. Whether this reversal will be perceived in practice as such, i.e., as a relatively dark band on a brighter background or not, will depend upon the method of observation and on the relative intensity of this minimum. A decision on this point is, however, a secondary matter to be settled from case to case.

In the experiments on reversal which we are now considering, we are dealing throughout with spectral lines, that is, we may assume that the function E_2 has pronounced maxima within which the course of E_2 is an "ordinary" function.[1] The same thing is true of E_1. So we are able to follow the course of E with the aid of $dE/d\lambda$. It is evident, in order to have a reversal occur, that $dE/d\lambda$ must be at least once equal to zero in the region of an emission line of body II. Therefore we must have

$$\delta E/\delta\lambda=\delta E_2/\delta\lambda+\delta E_1/\delta\lambda-\frac{\delta}{\delta\lambda}\frac{(E_1E_2)}{e''}=0\,.$$

Within the narrow maxima corresponding to a line, e'' may be regarded as constant. The first condition therefore reads

$$\frac{\delta E_2}{\delta\lambda}\left(1-\frac{E_1}{e''}\right)+\frac{\delta E_1}{\delta\lambda}\left(1-\frac{E_2}{e''}\right)=0\,. \qquad (1)$$

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[1] The spectrum must be reproduced in order that E_2 may be investigated. This alone furnishes a justification for the assumption made.

3. It is not permissible that we should now regard every minimum in the course of E as a reversal. Minima and maxima in the course of E_1 and E_2 are to be excluded. It will therefore henceforth be assumed that neither E_1 nor E_2 possesses more than one maximum, and that they have no minimum. In other words, that the lines under consideration are simple and not compound. Further, let E_2 be a line and not a continuous spectrum. Finally, for simplicity, let us assume that the centers of the emission lines of E_1 and E_2 coincide. It is obvious that this involves no serious limitation. We may now state that $e''-E_2$ is always positive, never zero, and that the differential quotients $\delta E_1/\delta\lambda$ and $\delta E_2/\delta\lambda$ have the same sign. It therefore follows that the first member in equation (1) must have the opposite sign from the second for all points in which $\delta E_1/\delta\lambda$ and $\delta E_2/\delta\lambda$ do not disappear. This case always occurs at the center of the lines. We shall return to it soon.

If both differential quotients differ from zero, we shall obtain as the first condition for the occurrence of a reversal the well-known relation $E_1>E_2$, i.e., light-source I must have a higher temperature than source II.

4. The case remains where one or both of the differential quotients vanish. We have the relation $\delta E_1/\delta\lambda=0$, either when E_1 is a constant, therefore if I emits a continuous spectrum, or when E_1 has a maximum or a point of inflection. If I emits a continuous spectrum, then we have

$$\frac{\delta E_2}{\delta\lambda}(e''-E_1)=0\,. \tag{2}$$

This equation is always fulfilled if $\delta E_2/\delta\lambda=0$, or $e''-E_1=0$, or therefore at the center of the line of gas II, or if both bodies have the same temperature. If a reversal now occurs, $\delta E/\delta\lambda$ must be negative below the center and positive above the center. We therefore have for $\lambda<\lambda_0$, $\dfrac{\delta E_2}{\delta\lambda}(e''-E_1)<0$, for any selected points; therefore, since $\delta E_2/\delta\lambda$ is positive, $e''-E_1$ must be less than zero, therefore $E_1>e''$, and accordingly the temperature of I must be higher than that of body II. At the same time it results that equation (2) can be fulfilled only for the position $\lambda=\lambda_0$, since E_1

is assumed as constant. The result, therefore, is that against a continuous spectrum only a simple reversal can take place which has its maximum at the place of the maximum of emission, and that it occurs always and only when $E_1 > e''$. Therefore, if we observe a multiple reversal against a continuous background, we are to assume that E_2/A_2 varies, and that E_2 does not follow Kirchhoff's law, it being assumed that we are dealing with a simple line.

5. E_1 can have a maximum only when it consists of a single line. In the middle of this line $\delta E_1/\delta\lambda = 0$, so that equation (1) is satisfied, being identical. In order that a reversal should be visible at the center of E, the differential quotient $\delta E/\delta\lambda$ must be negative on one side of the center below λ_0, and positive above the center, within a finite range. We therefore obtain from equation (1)

$$\Delta = \frac{\delta E_2}{\delta\lambda}(e'' - E_1) + \frac{\delta E_1}{\delta\lambda}(e'' - E_2) < 0 . \tag{3}$$

The second number is now positive and similarly $\delta E_2/\delta\lambda$, whence $e'' - E_1$ must be less than zero. We therefore again obtain as a necessary, but this time not sufficient condition, that $E_1 > e''$, i.e., the temperature of I must be higher than that of II in order that a reversal at the center can be possible.

If E_1 has a point of inflection, but E_2 has not, then $\delta E_1/\delta\lambda = 0$, whence according to equation (1) $e'' - E_1 = 0$, inasmuch as $\delta E_2/\delta\lambda$ does not vanish. Below this point $E_1 < e''$, and above it $E_2 > e''$, in accordance with our assumption as to the lines in question. If a minimum or a reversal is to occur, then must $\delta E/\delta\lambda < 0$, hence

$$\Delta = \frac{\delta E_2}{\delta\lambda}(e'' - E_1) + \frac{\delta E_1}{\delta\lambda}(e'' - E_2) < 0 .$$

All the terms are here positive, and the equation is impossible. Points of inflection of E_1 therefore furnish no reversal. The same is true if E_2 has a point of inflection while E_1 has not. We therefore obtain the condition

$$\frac{\delta E_1}{\delta\lambda}(e'' - E_2) = 0 ,$$

which is impossible of fulfilment.

The last case to be considered would be that where both E_1 and E_2 have one point of inflection. Condition I is then fulfilled. Below this point, we must in case of a minimum have $\Delta < 0$, and above it $\Delta > 0$. Since all the members of Δ are positive, with the exception of $e'' - E_1$, this must be negative, and we therefore again obtain as a necessary but insufficient condition for the occurrence of a reversal, $E_1 > e''$, i.e., the temperature of E_1 must be higher than that of E_2.

6. The question arises what are the further conditions required to produce a reversal in the other cases. Inasmuch as $E_2 < e'' < E_1$, equation (1) may be written

$$+\frac{\delta F_2}{\delta\lambda}F_1 + \frac{\delta F_1}{\delta\lambda}F_2 = \frac{\delta}{\delta\lambda}(F_1 F_2) = 0$$

if we place

$$E_1 - e'' = F_1,\ e'' - E_2 = F_2\ .$$

A minimum therefore occurs if the product $F_1 \times F_2$ possesses a mimimun. F_1 is here a function which considerably increases from the edge of the line to its center, while F_2 decreases. If F_1 and F_2 are given, as in the experiment, the positions of reversal can therefore be reckoned when e'' is known. But here the question arises whether such cases are really thinkable. It is, however, the fact that quantities F_1 and F_2 can always be found, one of which continuously increases while the other continuously decreases, so that their product has several maxima and minima. We can convince ourselves of this by a drawing or by forming the quantities

$$\frac{\delta(F_1 F_2)}{\delta\lambda}$$

and

$$\frac{\delta^2(F_1 F_2)}{\delta\lambda}\ .$$

We therefore obtain

$$\frac{\delta F_1}{\delta\lambda} : \frac{\delta F_2}{\delta\lambda} = -F_1 : F_2$$

and by appropriate disposition of the second differential quotients

$$\frac{\delta^2(F_1 F_2)}{\delta\lambda^2}$$

make

$$\frac{\delta^2(F_1 F_2)}{\delta\lambda^2}$$

positive as well as negative for given positions. Hence, it follows that multiple reversals are always possible, even when Kirchhoff's law is valid, i.e., always when the product F_1F_2 has several maxima.[1]

7. We may now also determine in which case a reversal occurs in the center of the two lines, or in two points of inflection corresponding to the same value. Inasmuch as

$$\frac{\delta F_1}{\delta\lambda}=\frac{\delta E_1}{\delta\lambda}=0\,,\quad \frac{\delta F_2}{\delta\lambda}=-\frac{\delta E_2}{\delta\lambda}=0\,,$$

therefore

$$\frac{\delta^2(F_1F_2)}{\delta\lambda^2}$$

takes the value

$$F_1\frac{\delta^2F_2}{\delta\lambda^2}+F_2\frac{\delta^2F_2}{\delta\lambda^2}>0\,.$$

Now F_1 evidently has a maximum at the center of the line, whence

$$\frac{\delta^2F_1}{\delta\lambda^2}<0\,.$$

Conversely, F_2 has a minimum at the same place. We therefore get a dark center, without regard to the sign, of

$$F_1\frac{\delta^2F_2}{\delta\lambda^2}>F_2\frac{\delta^2F_2}{\delta\lambda^2}\,,$$

and a bright center, if the reverse holds true. This condition is obviously identical with equation (3). If E_1 and E_2, and hence also F_1 and F_2 have points of inflection, then

$$\frac{\delta^2F_1}{\delta\lambda^2}\,,\quad\frac{\delta^2F_2}{\delta\lambda^2}$$

also vanish, i.e., F_1F_2 also has a point of inflection. It therefore follows that no reversal can occur at a point where E_1 and E_2 have points of inflection.

8. The results of the whole discussion may be summarized in the following theorem.

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[1] I am indebted to the kindness of Professor Dehn of Münster for examples of what has been said, e.g.: $F_2=1/\lambda$, $F_1=y$, where y is defined by the equation $\frac{(y-\lambda)}{\sqrt{2}}=\frac{1}{\sqrt{2}}\sin\rho\left(\frac{y+\lambda}{2}\right)=0$. By an appropriate choice of ρ we may obtain as many maxima and minima as may be desired in a given interval of λ.

In order that reversals should occur, it is necessary that the emitting source of light should have a higher temperature than the absorbing source of light.

If the emitting body furnishes a continuous spectrum, then only a simple reversal is possible, and it occurs if $E_1 > e''$.

If the emitting body gives a spectrum with a pronounced maximum near the line of the absorbing body, a reversal will occur at the center of the line if $E_1 > e''$, and, disregarding signs, if

$$F_1\frac{\delta^2 F_2}{\delta\lambda^2} > F_2\frac{\delta^2 F_1}{\delta\lambda^2}\,.$$

Since $\delta F_2/\delta\lambda$ and $\delta F_1/\delta\lambda$ are equal to zero, this may also be written, with disregard for the sign,

$$|F_1\rho_1| > |F_2\rho_2|, \tag{4}$$

where ρ designates the radius of curvature at the center. This means that the occurrence of a reversal is favored the greater the difference $E_1 - e''$, and the flatter the curve E_1; or the sharper the emission line and the smaller the difference $e'' - E_2$; that is to say, the more the maximum of E_2 approaches the emission of the black body.

Away from the center of the line reversals are not possible at places where one or both of the curves have points of inflection. They may, however, occur at all places where the product $F_1 \cdot F_2$ has a minimum. Without respect to forms of E_1 and E_2, which are physically possible, this may be designated as always possible. An n-fold reversal (n black lines) will therefore be observed if

$$\frac{\delta(F_1F_2)}{\delta\lambda} \quad .\;.\;.\;.$$

disappears $(2n-1)$ times, and if n minima and $n-1$ maxima occur.

9. It is of interest to examine from the point of view of the above rules the forms of the energy-curves of simple lines derived theoretically. One example will suffice. We choose the function e^{-x^2} as used by Lord Rayleigh[1], which may for our purpose be written

$$E_1 = a_1 e^{-k_1^2(\lambda-\lambda_0)^2}$$

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[1] Kayser's *Handbuch der Spectroscopie*, Bd. 2, 343.

when λ_0 denotes the center of the line. We now have

$$F_1F_2=(E_1-e'')(e''-E_2)=\left(a_1e^{-k_1^2(\lambda-\lambda_0)^2}-e''\right)\left(e''-a_2e^{-k_2^2(\lambda-\lambda_0)^2}\right).$$

Therefore, in order that a reversal should be possible for the center of the line, $E_1>e''$, hence $a_1>e''$, and also $a_2<e''$. Further, inasmuch as E_1 corresponds to the higher temperature, its line must be the broader, hence $k_1^2<k_2^2$. It is evident that at the center of the two lines the condition

$$\frac{\delta(F_1F_2)}{\delta\lambda}=0$$

is fulfilled. The condition in equation (4) gives

$$|F_1\rho_1|>|F_2\rho_2|,$$

that is, since for $\lambda=\lambda_0$

$$\frac{\delta^2F_1}{\delta\lambda^2}=-2k_1^2a_1,$$

$$\frac{k_2^2a_2}{k_1^2a_1}>\frac{e''-a_2}{a_1-e''}.$$

In consequence of the condition

$$\frac{\delta(F_1F_2)}{\delta\lambda}=0,$$

and since neither $\delta E_1/\delta\lambda$ nor $\delta E_2/\delta\lambda$ can vanish, we have away from the center of the line

$$\frac{k_1^2E_1}{k_2^2E_2}=\frac{F_1}{F_2}.$$

This condition can evidently always be satisfied if

$$e''=\frac{(k_1^2+k_2^2)E_1E_2}{k_1^2E_1+k_2^2E_2}.$$

If we write this equation in the form

$$\frac{k_1^2}{E_2}+\frac{k_2^2}{E_1}=\frac{k_1^2+k_2^2}{e''},$$

and recall that both E_1 and E_2 increase from the edge toward the center of the line, and further that e'' is fixed, then we see that condition (1) can at most be fulfilled only once on each side of the line. In order to decide whether a minimum, that is, a reversal, can occur, we form the second derivative of F_1F_2. Regarding the relation

$$\frac{\delta(F_1F_2)}{\delta\lambda}=0,$$

this takes the value

$$4(\lambda-\lambda_0)^2[e''(k_1{}^4E_1-k_2{}^4E_2)-(k_1{}^2+k_2{}^2)E_1E_2]>0\,,$$

or, with the omission of the positive factor $4(\lambda-\lambda_0)^2$,

$$k_1{}^4E_1+k_1{}^4E_2>(k_1{}^2+k_2{}^2)^2E_1E_2\frac{1}{e''}\,.$$

If we introduce the value of e'' found above, we get

$$k_1{}^4E_1+k_2{}^4E_2>(k_1{}^2+k_2{}^2)(k_1{}^2E_1+k_2{}^2E_2)$$

or

$$0>+(E_1+E_2)k_1k_2\,,$$

which is not possible, since the right-hand member contains only positive quantities which cannot vanish.

This accordingly shows that two lines of the form e^{-x^2} cannot furnish a double reversal. Inasmuch as we may assume that the majority of the simple spectral lines have an approximately similar distribution of intensity, this explains at the same time why genuine double reversals are relatively so infrequent.

10. We may convince ourselves in a geometrical manner that the occurrence of double reversals is favored by an intensity-curve for the two lines in which $\delta E_1/\delta\lambda$ and $\delta E_2/\delta\lambda$ are subject to decided variations. Such cases will be observed particularly when non-homogeneous layers of different temperatures are located one behind another, as in an arc-lamp burning irregularly, or in a Geissler tube having different diameters, the discharge being stratified.

Finally, we may readily see what effect is produced if the intensity of E_1 is diminished to the p^{th} part, for instance by inserting a Nicol prism. If a reversal occurs without the Nicol prism at the position $\lambda=\lambda_1$, then at that point

$$\frac{\delta(F_1F_2)}{\delta\lambda}=0\,.$$

If E_1 is diminished to its p^{th} part, this furnishes the condition $E_1>pe''$. Consequently the new value of E_1 will lie nearer to the center of the line if the reversal is to persist. The reversal therefore moves with the position of the Nicol from the edge of the line toward the center.

Physical Institute of the University
Münster, i. W.

MINOR CONTRIBUTIONS AND NOTES

THE INTERNATIONAL UNION FOR CO-OPERATION IN SOLAR RESEARCH

The fourth meeting of the Union was held in four formal sessions on the three days August 31, September 1, and September 2, 1910, on Mount Wilson, California, under the auspices of the Solar Observatory of the Carnegie Institution. The general arrangements for the sessions were made by the Executive Committee of the Union, Messrs. Arthur Schuster, George E. Hale, and A. Riccò.

Most of those attending the meeting arrived in Pasadena on August 28. The Pasadena offices, laboratory, and shops of the Mt. Wilson Solar Observatory were inspected on the morning of August 29. On that afternoon the members and guests were entertained at a garden party by Mr. and Mrs. Hale. August 30 was devoted to the ascent of Mount Wilson, and to the inspection of the buildings and instruments of the observatory. At the regular sessions on the three following days no formal papers were presented, the time being fully occupied by committee reports and their discussion. An informal address was given by Mr. Hale at the opening of the session, in which, in addition to words of welcome, he explained some of the methods and results of the spectrographic, spectroheliographic, and magnetic study of the sun-spots at Mount Wilson. On the evening of August 31 an informal lecture on the solar constant and the recent methods for its determination was given by Mr. Charles G. Abbot of the Smithsonian Astrophysical Observatory. Mr. J. C. Kapteyn of Groningen, research associate of the Carnegie Institution, gave on the evening of September 1 an informal lecture on star streams among the stars of the *Orion* type.

Ample opportunity was given during the three days and four nights spent on the mountain for the visitors to examine the instruments of the observatory while in operation, and particularly to view celestial objects with the five-foot reflector, with the assistance of its constructor, Mr. G. W. Ritchey. The station of the Smithsonian Observatory, maintained on Mount Wilson during the summer season for spectrobolometric and other observations of the solar radiation, was also open for inspection daily.

The members of the Union and guests were entertained at a final banquet given by Mr. and Mrs. Hale on the evening of September 3 at Hotel Maryland, Pasadena.

Some of the more important resolutions adopted at the meeting are given herewith, but this is not to be regarded as in any sense an authoritative report of the meeting. The official record of the conference will be given in full in the next volume of the *Transactions* of the Union.

I. *Resolutions adopted as reported by the committee on standards of wave-length:*

1. In the region of the spectrum in which three independent measurements, by the interferometer method, of the lines of the iron arc are available, i.e., between λ 4282 and λ 6494, the arithmetical mean of the three measurements shall be adopted as definite international standards of the second order, provided that there is sufficient agreement between them.

2. The committee is given authority to publish these standards as soon as possible.[1]

3. For the part of the spectrum in the neighborhood of λ 5800, where the number and character of the iron lines is not satisfactory, the committee proposes the use of barium lines as additional standards.

4. The laboratories or observatories possessing first-rate concave gratings are invited to determine by interpolation, as soon as possible, standards of the third order in the spectrum of the iron arc, within the above range of the spectrum, λ 4282 to λ 6494.

5. The measurement of standards of the second order shall be extended to shorter and longer wave-lengths, and the arithmetical mean of three independent determinations shall be adopted as secondary standards.

6. Standards of the third order shall then be obtained in the manner indicated.

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[1] See p. 215 of this number of this *Journal.*

7. The above system of standards shall be called the international system, the unit on which it is based being called the international unit (I.A.) as defined by the Conference of 1907.

8. It is very desirable that in different laboratories possessing concave gratings of the first quality photographs of the arc, spark, and solar spectrum, and new measurements according to the international system, shall be made as soon as possible.

II. *Committee on the measurement of solar radiation:*

The report of the committee was drafted and presented by Charles G. Abbot of the Smithsonian Observatory at Washington.

III. *Committee for the organization of eclipse observations:*

A brief report for the committee was presented by Comte de la Baume Pluvinel, referring to the two eclipses which had occurred since the committee was appointed. Additional remarks on the eclipse expedition to Flint Island were made by W. W. Campbell of the Lick Observatory, who also exhibited photographs obtained by expeditions from the Lick Observatory at this and other eclipses.

IV. *Committee on the determination of solar rotation by means of the displacement of lines:*

The report was presented by Walter S. Adams of the Mount Wilson Solar Observatory.

The committee recommended that a larger number of lines be employed, both of the same elements and of different elements. Different parts of the spectrum were assigned to the various observers, and it was recommended that one portion of the spectrum be observed in common by all taking part in the work.

V. *Committee on work with the spectroheliograph:*

The report was presented by Edwin B. Frost of the Yerkes Observatory. The resolutions suggested by the committee were adopted.

Supplementary reports of work accomplished were presented from the observatories of Meudon, Catania, Tortosa, the Yerkes Observatory, and the Solar Physics Observatory in London, which included the work of the Kodaikánal Observatory.

After a general discussion at the final session it was

VOTED that the Union extend its activity so as to include general astrophysics.

VOTED that a committee be elected to consider and report on the question of the classification of stellar spectra.

This committee was subsequently elected by the members present, and was organized with Edward C. Pickering as chairman, and Frank Schlesinger of the Allegheny Observatory as secretary.

VOTED that the next meeting of the Union be held at Bonn in the summer of 1913, the exact date to be determined later.

The various committees of the Union were elected, with some additions and changes, and the two retiring members of the Executive Committee were re-elected.

The following list contains the names of most of the members of the Union and guests present at this meeting. F.

MEMBERS AND INVITED GUESTS PRESENT AT THE FOURTH CONFERENCE

J. S. Ames, Johns Hopkins University, Baltimore, Md.
Charles G. Abbot, Smithsonian Astrophysical Observatory, Washington, D.C.
Walter S. Adams, Mt. Wilson Solar Observatory
Harold D. Babcock, Mt. Wilson Solar Observatory
O. Backlund, Observatoire de Poulkovo, Poulkovo, Russia
E. E. Barnard, Yerkes Observatory, Williams Bay, Wis.
A. Belopolsky, Observatoire de Poulkovo, Poulkovo, Russia
Jean Bosler, Observatoire de Meudon, Meudon, France
F. P. Brackett, Pomona College, Claremont, Cal.
Miss Cora G. Burwell, Mt. Wilson Solar Observatory
W. W. Campbell, Lick Observatory, Mount Hamilton, Cal.
C. A. Chant, University of Toronto, Toronto, Canada
Henri Chrétien, Observatoire de Nice, Nice, France
P. R. Cirera, Observatorio del Ebro, Tortosa, Spain
W. W. Coblentz, Bureau of Standards, Washington, D.C.
A. L. Cortie, Stonyhurst College Observatory, Lancashire, England
A. Cotton, Ecole Normale Supérieure, Paris, France
H. Deslandres, Observatoire de Meudon, Meudon, France
N. Donitch, Observatoire de l'Université, St. Petersburg, Russia
Frank L. Drew, Mt. Wilson Solar Observatory
F. W. Dyson, Royal Observatory, Edinburgh, Scotland
Ferdinand Ellerman, Mt. Wilson Solar Observatory
P. Eversheim, University of Bonn, Bonn, Germany
Charles Fabry, Université de Marseille, Marseille, France
Edward A. Fath, Mt. Wilson Solar Observatory
Mrs. W. P. Fleming, Harvard College Observatory, Cambridge, Mass.
F. E. Fowle, Smithsonian Astrophysical Observatory, Washington, D.C.

A. Fowler, Imperial College of Science and Technology, South Kensington, London, England
Philip Fox, Dearborn Observatory, Evanston, Ill.
Edwin B. Frost, Yerkes Observatory, Williams Bay, Wis.
Henry G. Gale, University of Chicago, Chicago, Ill.
L. H. Gilmore, Throop Polytechnic Institute, Pasadena, Cal.
Miss Clementina D. Griffin, Mt. Wilson Solar Observatory
George E. Hale, Mt. Wilson Solar Observatory
M. Hamy, Observatoire de Paris, Paris, France
J. Hartmann, Königliche Sternwarte, Göttingen, Germany
K. Haussmann, Technische Hochschule, Aachen, Germany
J. v. Hepperger, Imperial Observatory, Vienna, Austria
E. H. Hills, 32 Prince's Garden, London, England
W. J. Humphreys, U.S. Weather Bureau, Washington, D.C.
P. Idrac, Observatoire de Meudon, Meudon, France
J. C. Kapteyn, Astronomical Laboratory, Groningen, Holland
H. Kayser, University of Bonn, Bonn, Germany
Arthur S. King, Mt. Wilson Solar Observatory
H. Konen, Physikalisches Institut, Münster, Germany
F. Küstner, Königliche Sternwarte, Bonn, Germany
C. O. Lampland, Lowell Observatory, Flagstaff, Arizona
Sir Joseph Larmor, Royal Society, London, England
Miss Jennie B. Lasby, Mt. Wilson Solar Observatory
A. O. Leuschner, University of California, Berkeley, Cal.
H. C. Lord, McMillin Observatory, Columbus, Ohio
A. G. McAdie, U.S. Weather Bureau, San Francisco, Cal.
J. D. Maddrill, International Latitude Observatory, Ukiah, Cal.
Walter M. Mitchell, Detroit Observatory, Ann Arbor, Mich.
H. F. Newall, University Observatory, Cambridge, England
F. G. Pease, Mt. Wilson Solar Observatory
H. B. Perkins, Throop Polytechnic Institute, Pasadena, Cal.
E. C. Pickering, Harvard College Observatory, Cambridge, Mass.
J. S. Plaskett, Dominion Observatory, Ottawa, Canada
Comte A. de la Baume Pluvinel, 7 Rue de la Baume, Paris, France
E. Pringsheim, University of Breslau, Breslau, Germany
P. Puiseux, Observatoire de Paris, Paris, France
A. Riccò, Osservatorio astrofisico, Catania, Sicily
G. W. Ritchey, Mt. Wilson Solar Observatory
A. L. Rotch, Blue Hill Observatory, Hyde Park, Mass.
Henry Norris Russell, Princeton University, Princeton, N.J.
J. R. Rydberg, University of Lund, Lund, Sweden
Charles E. St. John, Mt. Wilson Solar Observatory
Fernando Sanford, Leland Stanford Jr. University, Palo Alto, Cal.
Frank Schlesinger, Allegheny Observatory, Allegheny, Pa.

Arthur Schuster, Victoria Park, Manchester, England
K. Schwarzschild, Astrophysikalisches Observatorium, Potsdam, Germany
F. H. Seares, Mt. Wilson Solar Observatory
V. M. Slipher, Lowell Observatory, Flagstaff, Arizona
Frederick Slocum, Yerkes Observatory, Williams Bay, Wis.
Miss Ruth E. Smith, Mt. Wilson Solar Observatory
S. W. Stratton, Bureau of Standards, Washington, D.C.
H. Struve, Königliche Sternwarte, Berlin, Germany
S. D. Townley, Leland Stanford Jr. University, Palo Alto, Cal.
H. H. Turner, University Observatory, Oxford, England
Miss Louise Ware, Mt. Wilson Solar Observatory
Miss Phoebe Waterman, Mt. Wilson Solar Observatory
F. R. Watson, University of Illinois, Champaign, Ill.
H. C. Wilson, Goodsell Observatory, Northfield, Minn.
A. Wolfer, Sternwarte des eidgenossischen Polytechnikums, Zurich, Switzerland

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Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

Accuracy in the proof is gained by having manuscripts type-written, provided the author carefully examines the sheets and eliminates any errors introduced by the stenographer. It is suggested that the author should retain a carbon or tissue copy of the manuscript, as it is generally necessary to keep the original manuscript at the editorial office until the article is printed.

All drawings should be carefully made with India ink on stiff paper, usually each on a separate sheet, on about double the scale of the engraving desired. Lettering of diagrams will be done in type around the margins of the cut where feasible. Otherwise printed letters should be put in lightly with pencil, to be later impressed with type at the editorial office, or should be pasted on the drawing where required.

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THE DISPERSION OF METALS IN THE INFRA-RED SPECTRUM

By L. R. INGERSOLL

INTRODUCTION

This paper presents the results of an experimental study[1] of the optical constants of a number of metals, particularly in the region of the infra-red spectrum.

Considerable interest has been manifested in the optical properties of metals during the last two or three decades, as is evidenced by the large amount of work in this field. With the exception of the striking experiments of Kundt[2] and others on thin prismatic metal films, these properties have generally been determined by the less direct but more accurate method of studying the character of polarization of the reflected light when plane-polarized light is incident on a clean and highly polished surface of the metal at a large angle of incidence.

Prominent among the many investigations in this field stands the work of P. Drude[3] who made a very careful study of the optical constants of a large number of metals, chiefly for the wave-length of sodium light. R. S. Minor,[4] following the photographic method due to W. Voigt,[5] measured the dispersion of several metals for the greater part the visible spectrum and extended the work into the ultra-violet with interesting results. Quite recently A. Q. Tool[1] has measured the dispersion of several metals for the visible spectrum, as has also A. L. Bernoulli.[2] The latter's method, however, has been somewhat criticized by Voigt.[3]

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[1] This work has been materially aided by grants from the Rumford fund of the American Academy of Arts and Sciences.

[2] *Wiedemann's Annalen*, **34**, 469, 1888.

[3] *Ibid.*, **39**, 481, 1890; **42**, 186, 1891; **64**, 159, 1898.

[4] *Annalen der Physik*, **10**, 581, 1903.

[5] *Physikalische Zeitschrift*, **2**, 303, 1901.

In the region of the infra-red, on the other hand, practically nothing has heretofore been done. It is true that H. Knoblauch[4] showed some thirty years ago that effects of elliptical polarization could be obtained by reflecting "heat rays" from a metal surface and that he was even able to conclude that the principal angle of incidence is greater in this region than in the visible spectrum; but he made no attempt at a study of the effect for particular wave-lengths, and his lack of agreement in the case of observations in the visible spectrum with now accepted values shows that his metal surfaces must have been far from the "normal" state.

A study of this problem was undertaken as being part of the general field of work in which the writer has been engaged for several years, viz., the extension into the infra-red spectrum of our knowledge of various optical phenomena which depend on polarization effects, e.g., magnetic rotation, optical constants, etc. In the present case measurements have been made as far into the infra-red as $\lambda = 2.25\ \mu$ and these, when combined with the results of other observers, give the dispersion of a number of the more common metals from the ultra-violet to this wave-length of the infra-red spectrum.

METHOD

As is well known, two constants are required to characterize a metal optically—n the refractive index, and k the absorption index. When plane-polarized light is allowed to fall on a polished metal surface it is in general reduced to a state of elliptical polarization because of the relative phase change introduced between the two rectangular components vibrating in and perpendicular to the plane of incidence. For a certain angle of incidence $\bar{\phi}$ (principal incidence), this phase change is 90°, and if the plane-polarized incident beam has a certain azimuth $\bar{\psi}$ (principal azimuth), a circularly polarized beam will result. The constants n and k[1] are connected with these angles by the familiar approximation equations

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[1] *Physical Review*, **31**, 1, 1910.

[2] *Annalen der Physik*, **29**, 585, 1909; **33**, 209, 1910.

[3] *Ibid.*, **29**, 956, 1909.

[4] *Festschr. d. naturf. Ges. zu Halle*, 1879; *Nova Acta d. kaiserl. Leop. Carol. deutsch. Ak. d. Naturf.*, **50**, 487, 1887.

$$k = \tan 2\bar{\psi}\,; \quad n = \frac{\sin \bar{\phi} \tan \bar{\phi}}{\sqrt{1+k^2}} \tag{1}$$

The usual method of determining these constants has been through the analysis of the reflected elliptical vibration by measuring the phase-change which must be introduced to reduce it again to a state of plane polarization, as may be done with some such instrument as a Babinet compensator. In the present case it was believed that the principle of the compensator would be difficult of application in the infra-red, so a somewhat different scheme was developed as will be described.

In principle the method is extremely simple. A circularly polarized beam of light is to be distinguished from elliptical by the fact that components of the former taken in any azimuth will all be equal. Such components may be taken by measuring the intensity (or rather amplitude) of the beam after passage through any polarizing agent, e.g., a nicol. When, for any particular wave-length, the azimuth of the plane-polarized incident beam as well as its angle of incidence has been adjusted until all such components are found equal, that is, until the reflected beam is circularly polarized, then these angles are the principal azimuth and incidence respectively, and the optical constants are computed from them by the foregoing relations.

The use of a nicol in this connection, however, would entail certain difficulties which may be overcome if it be replaced by a double-image prism. For this prism gives in its two transmitted beams two rectangular components of the incident vibration, and when these components are equal—regardless of how the prism is rotated about the axis of this beam—then the beam is circularly polarized, granted that it is not entirely unpolarized. The necessity for rotating the double-image prism relative to the beam is readily seen, since it is evident that an elliptical or even plane-polarized vibration would possess equal components for certain azimuths, but only a circular one, equal components for all azimuths.

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[1] The constant n has the same meaning as for transparent substances. The constant k has the following significance: The amplitude of a wave after traveling one wave-length, λ' measured in the metal, is reduced in the ratio $1 : e^{-2\pi k}$, or, more generally, for any distance d in the metal the amplitude is reduced by the factor $\frac{1}{e^{-\frac{2\pi dk}{\lambda'}}}$ For the same wave-length measured in air, this factor would be $\frac{1}{e^{-\frac{2\pi dnk}{\lambda}}}$. The product nk is sometimes called the extinction coefficient.

APPARATUS

The general arrangement of apparatus is shown in the diagram of Fig. 1 and the photograph, Fig, 2, the lettering being arranged to correspond in the two cases. Light from a water-jacketed Nernst glower G_1, after being rendered parallel by the concave mirror M_1, was plane polarized by P_1 and then fell on S, the metallic surface to be studied. The angle of incidence on S could be varied at will, since the glower, mirror, and polarizer were mounted on an arm which rotated around the axis xx'. The beam reflected from S would in general be elliptically polarized, but was reduced on passing through the double-image prism P_2 (P_3 being removed) to two rectangular components with a small angle of divergence between them. Because of this angle of divergence there were formed on the slit Sl, on reflection from the concave mirror M_2, two images of the glower vertically separated from each other by about one centimeter. Consequently on passage through the spectrometer two spectra were formed and these fell on the two strips of a special bolometer which had strips vertically over one another and so connected that when they were illuminated by beams of equal intensity no deflection of the galvanometer resulted.

The principle of operation in brief was so to determine the angle of incidence and azimuth of the plane-polarized beam falling on S that a truly circular vibration should result. The angle of incidence could be changed, as already noted, by turning the polarizing part of the apparatus about the axis xx', while the azimuth was altered by rotating the polarizer in its mounting. The circularly polarized beam was recognized as such by the equal intensity of the two beams from the analyzer; that is, by the fact that the galvanometer showed no deflection, the spectrometer being set for some chosen wave-length. But, as before stated, to be sure the vibration is circular it must be possible to rotate the analyzer relative to the beam. This would apparently be a very simple thing to do until it is considered that this would result in displacing the two images of the glower, i.e., the foci of the two beams, off the slit. To avoid this and still accomplish the purpose, the entire polarizing part of the apparatus, including the metal surface (but excluding the analyzer), was arranged so that it could be rotated about a horizontal axis yy'. In the photograph it is shown as turned about twenty degrees to the right from its vertical or zero position.

The spectrobolometric part of the apparatus was that already used by the writer in a study of magnetic rotation, and for a detailed description of the spectrometer, bolometer, and galvanometer the reader is referred to this paper.[1] Suffice it to say here that the spectrometer might be used with either of two large glass prisms of angles 25° and 45° and of such dispersive powers that the latter gave a spectrum of some two or three times the breadth of the former; the use of these two prisms will be discussed in connection with sources of error. It may be noted that the use of a prism of glass rather than of rock salt was permissible, since the limit of the spectral range to which it is possible to work is fixed by the absorption of the calcite of the polarizing apparatus, which is about the same as that of glass.

The bolometer strips were each $\frac{1}{2}$ mm×8 mm and, as already noted, so arranged that when both were exposed to beams of equal intensity the galvanometer showed no deflection. The galvanometer itself was of the Thomson type with a sensibility of about 5×10^{-10} amperes per mm for a scale one meter distant. The magnetic shield which surrounded it gave a shielding effect of about six thousand fold.

Polarizing agents.—For this purpose double-image prisms were exclusively used. The reason for using one as an analyzer has already been discussed, but it is not evident without explanation why one would also serve as polarizer, since here only a single beam

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[1] *Phil. Mag.* (6), **18**, 74–102, July 1909.

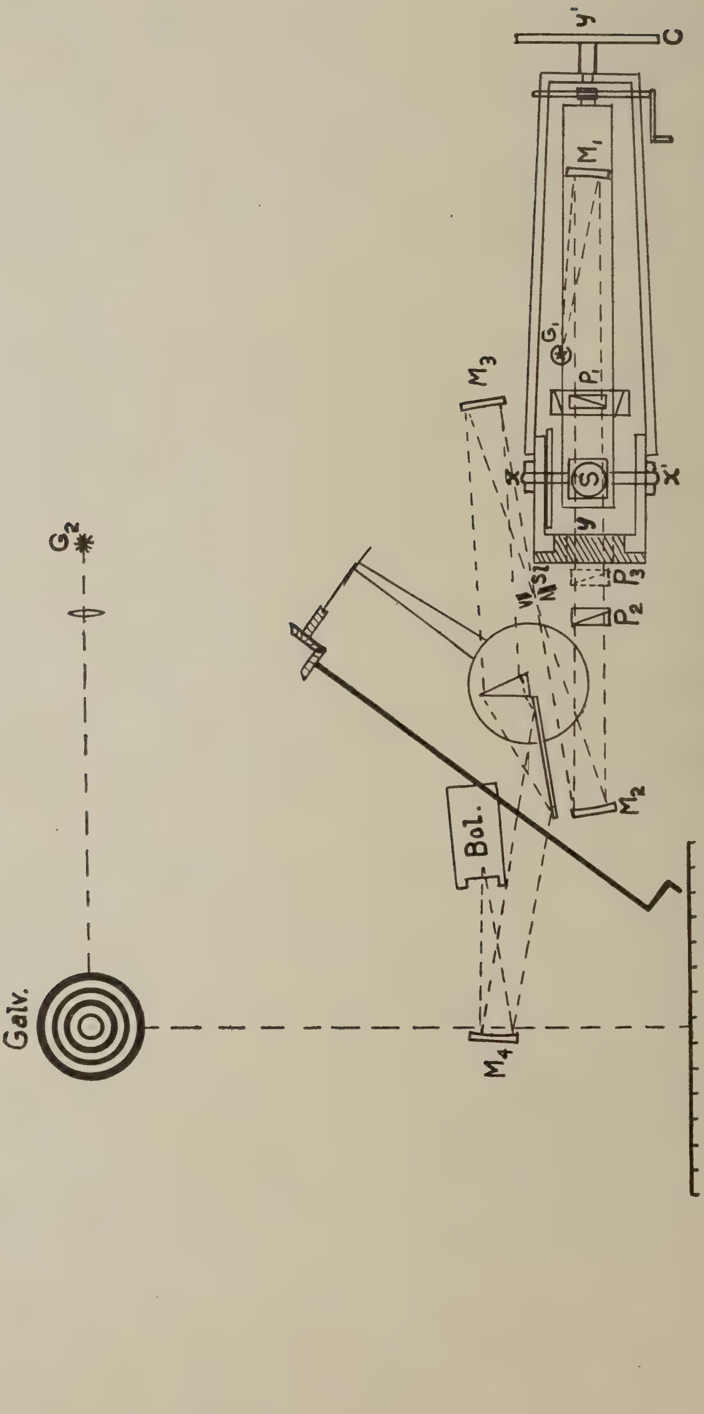

Fig. 1

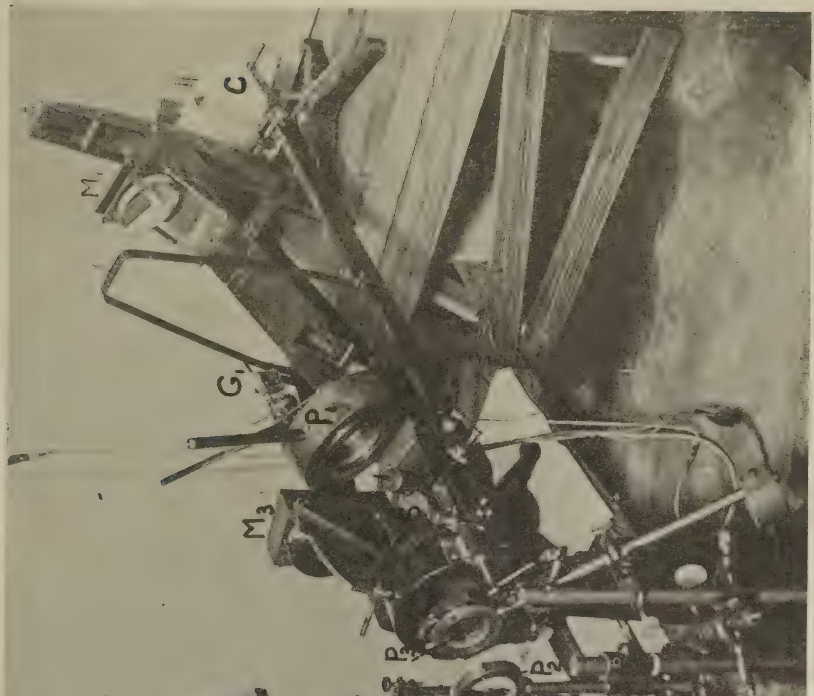

FIG. 2

of plane-polarized light was wanted. But the single beam was readily secured from the double-image polarizer, as one of the two beams was deviated so as to miss the slit entirely and hence be of no effect. The practical reasons for using a double-image prism instead of a nicol in this connection were the smaller thickness of glass and calcite necessary, and the ease of securing a larger aperture.

The polarizer P_1 was of the Rochon type, of calcite achromatized with glass, giving a separation of the two beams of about 1°. It was of about 40 mm aperture. The analyzer P_2 was a large Wollaston prism of 48 mm aperture of such angle as to give a separation of $2^\circ\!.25$ between the two rays. A third prism P_3 was also used in the latter part of the work in a way which will be mentioned in a discussion of errors. The three prisms were splendid specimens of optical work, having been specially made by Messrs. Steeg and Reuter.

Other instrumental details may briefly be mentioned. Mirrors M_3 and M_4 were of 10 cm aperture and 30 cm focal length each; M_1 and M_2 of 8 cm aperture and of 23 cm and 30 cm focal length respectively. It was found necessary to surround the glower G_1 with a jacket to secure constancy of radiation, and the jacket itself had to be water-cooled to keep down its own radiation. The polarizer P_1 was mounted in a large conical brass bearing carrying a divided circle with vernier reading to $2'$. This was accordingly the limit of accuracy for reading azimuths. The glower, mirror, and polarizer were carried on a large arm which could be rotated about the axis xx' by rack and pinion and could also be rigidly locked at any angle. The main circle about this axis was divided to half-degrees and carried a vernier reading to $1'$, whence the angle of incidence was determinate to $0'\!.5$. The metallic surface S was mounted on an adjustable table pivoted about this same axis. A rough circle C at the end y' sufficed to give the angle about the axis yy'.

PROCESS OF MAKING OBSERVATIONS

That one may understand the conditions which make for accuracy as well as for error in the results, the process of measuring optical constants must be described in some detail. Let it be supposed that the apparatus, including the spectrometer, had been carefully aligned and adjusted and that the polarizer was turned so that its principal plane made an angle of 45° with each of the two planes of the analyzer. Then the vibration transmitted by the polarizer should have had the same component in each of the analyzer sections, hence if the galvanometer did not stand at zero (that is, show equal intensity of radiation on each bolometer strip) it would be because of unequal absorption of the two beams on their subsequent passage through the spectrometer. This was corrected for by screening off a small fraction of one or the other image as it fell on the slit, by a little device[1] which moved directly across and in front of the slit on a screw. This was turned until the deflection was zero and then one might be assured that, when in the analysis of any sort of vibration, the deflection reduced to zero, the two components of the vibration were equal. This process may be referred to as that of "setting the bolometer zero."

Having made this preliminary adjustment with the spectrometer set for any chosen wave-length, the metallic surface was put in place and an angle of incidence chosen which was thought to be near the principal incidence. The azimuth of the polarizer was then varied till the galvanometer regained its zero position, indicating the equality of the two reflected components.

In general, however, this would not mean a truly circular vibration but rather an elliptical one, so oriented that it gave equal components.[2] To detect this the polarizing arrangement including the metallic surface was rotated 45° about the axis of the beam yy' first to the right and then to the left, and an elliptical vibration would be shown by the corresponding galvanometer deflection, which would be first to the one side and then to the other, respectively. Having noted these deflections, a second setting of the angle of incidence and its corresponding azimuth was made, and usually a third setting could be chosen so well as to produce an almost exactly circular vibration. In that case the azimuth and incidence would be the principal azimuth and incidence respectively. A second determination of the azimuth in another quadrant of the polarizer should yield the same result. If it did not, it meant that the zero point for measurements of the rotation about the axis yy' was at fault. This was located by a stop which had to be occasionally adjusted. In general a careful determination of the principal angles for any one wave-length could be made in fifteen or twenty minutes.

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[1] See *Phil. Mag.* (6), **18**, 81, July 1909.

[2] For a further discussion of this point see a paper by Mr. Littleton and the writer in *Physical Review*, November 1910, on the application of this method in a visual polarimeter.

ERRORS

The following sources of error have been studied:

Use of approximation equations.—When a method of measuring optical constants—e.g., Babinet compensator—affords a direct determination of the phase difference between the two components of vibration for any angle of incidence, rigorous equations for computing the constants are available. On the other hand when, as in many cases including the present, only the principal angles are directly measured, the rigorous equations are not directly applicable and the approximation relations (1) are those in general use. These may result in an error of several per cent in the constants, but fortunately this can be almost entirely eliminated by the use of a sort of correction factor,[1] a simple function of the square of the cotangent of the principal incidence, which is applied as follows:

If we call the values of the constants computed by (1)

$$k' = \tan 2\bar{\psi}, \quad \text{and } n' = \frac{\sin \bar{\phi} \tan \bar{\phi}}{\sqrt{1+k^2}},$$

then values n and k which are much more nearly correct are given by

$$k = k'(1-\cot^2\bar{\phi}), \quad \text{and } n = n'(1+\tfrac{1}{2}\cot^2\bar{\phi}) \tag{2}$$

These equations (2) have been tested by applying them in connection with (1) to a number of the principal angles listed by Drude and Minor. The results afforded were in general in good agreement with their tabulated values of the constants.

Condition of surfaces.—The various precautions to avoid films, which were taken in the preparation and cleaning of the surfaces, will be considered in a discussion of the results, but the degree of success in obtaining perfect surfaces may be mentioned here. When the dispersion curves obtained for the various metals were extrapolated from $\lambda=0.65\,\mu$ back to the wave-length of sodium light the values they afforded of the constants were, with one or two exceptions, in sufficiently good agreement with the accepted values for these metals to warrant considering the surfaces in the normal condition. In the case of steel, however, all the surfaces gave values of the principal incidence from 20′ to 1° too small. This pointed very clearly to surface imperfections or films, and while they could not be entirely eliminated they were readily allowed for by adding a small correction to the principal incidence as measured. The results for the several steel surfaces were in much better agreement for the longer wave-lengths than for the shorter, hence this correction was reduced from about 20′ at $\lambda=0.65\,\mu$ to only 8′ at $\lambda=2\,\mu$. This is much smaller than the correction made by Minor to his measurements on cobalt.

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[1] P. Drude, *Annalen der Physik*, **36**, 546, 1889.

Low dispersion.—Since an error might be expected to arise from the low dispersion of the 25° prism, a large number of the measurements were repeated using the 45° prism of heavy glass giving a spectrum of perhaps three times the dispersion. No appreciable discrepancy between the two sets of results could be detected. The wave-length scale for both prisms was determined by double dispersion with the aid of a second prism of rock salt. It was also checked by locating the absorption bands in sunlight.

Azimuth determinations.—The consistency of successive incidence and azimuth settings was remarkable. Under favorable conditions indeed it was apparently limited only by the accuracy with which the circles could be read. That is, a very small *change* (much smaller, for certain reasons than could even be detected on the circles) could be measured, and on this account some experiments will shortly be carried out with this apparatus with the view of detecting any possible change in the constants which may be brought about by giving the metallic surface an electric charge, i.e., by adding to its number of electrons. As it is believed that its sensibility in the detection of such an effect can be made from ten to one hundred times greater than that possible with visual methods, the experiment seems worth trying.

In the matter of absolute determinations of the constants, however, the possible accuracy is very much less than would be concluded from the above discussion, for successive measurements with intervening changes in the adjustment of the apparatus do not show any such consistency. Rather curiously perhaps, the error seems to be confined almost entirely to principal azimuth determinations and for only the longer wave-lengths. Some specific results will illustrate this point: thus for nickel at $\lambda = 1.75\ \mu$ three principal incidence measurements made under as many different instrumental arrangements and adjustments gave 83° 20′, 83° 26′, and 83° 25′, while four corresponding principal azimuth determinations were 32° 33′, 34° 3′, 33° 27′, and 32° 51′. On the other hand, for $\lambda = 0.65\ \mu$ the principal incidences as found were 77° 30′, 77° 48′, and 77° 47′, while the azimuths came 32° 28′, 32° 28′, 32° 46,′ and 32° 34′.

These results, which are very fair samples of those obtained for any metal, show at once that the error is confined to the azimuth determinations and is of a serious nature only for the longer wave-lengths. The limits of error, however, are not quite as great as these figures would cause one to conclude, for, in the case of the latter two of the four azimuths listed for each wave-length, an important modification was adopted in the method of setting the bolometer zero. This involved the introduction of a third double-image prism P_3 with its planes set at 45° to those of the analyzer P_2. This was mounted so that it could be temporarily turned into position in front of P_2 and removed as soon as the bolometer zero had been set. Before the introduction of this third prism the azimuth determinations had been found to depend in some measure on a number of apparently extraneous conditions such as the shape and size of the aperture limiting the beam, the condition of the Nernst glower, etc. But by its use these effects were almost entirely removed, hence the last two azimuth measurements have greater weight than the others. But even at best the accurate determination of principal azimuths for wave-lengths about 1.75 μ or longer is extremely difficult, since for these longer waves conditions are complicated by a dispersive effect arising from the polarizing agents. Until an extensive study can be undertaken of these various effects it must suffice to accept the most probable values and to estimate the limits of error involved.

One other source of error in the azimuth measurements has been considered. Because of the length of glower and image necessary to get sufficient energy the radiation was incident on the metal surface over a range of angle of about 1°, but careful tests of this source of error indicate that it is negligible in comparison with that just mentioned.

Visual measurements with a Babinet compensator, using sodium light, were made on the steel, nickel, cobalt, and silicon surfaces. The principal azimuths agreed in most cases very well with the points taken from the extrapolated infra-red curve, but the principal incidences came a few minutes low. This might perhaps be interpreted as meaning that the method yields slightly too high results for the principal incidence, although the measurements on silicon would not bear out this conclusion; but in any event, since the tendency of any surface impurities is to reduce the principal incidence, this error is in a measure self-eliminating.

The effect of these sources of error on the final results has been carefully considered and an estimate of the probable limits of error is included with each set of results. In the case of steel, nickel, and cobalt it will be observed that the limits of error are not much larger, save in the case of the longer waves, than might reasonably be expected for visual observations. For copper and silver, however, the limits are large, for the azimuths are so nearly 45° that the effects of small errors are greatly magnified.

RESULTS

Six different metals were tested in all, viz., steel, nickel, cobalt, silver, copper, and silicon. This includes the four used by Minor and in addition nickel and silicon. Although the latter has small claim to being considered a metal in this connection, the results are not without interest.

Surfaces.—As might be anticipated, one of the greatest difficulties encountered was the securing of suitably polished surfaces. It may be observed that the present requirements in the way of a plane and polished surface were considerably more rigorous and difficult of fulfilment than if the work had been confined to the visible spectrum. For visual observations can be made on a surface of only a few square millimeters in area and of only the roughest approximation to planeness, while in the present infra-red work the accuracy of the measurements was largely dependent on the area of surface available, and this surface had to be plane to a few wave-lengths of light.

After discouraging experiences with some of the best optical workers, some very fair surfaces were prepared in this laboratory with a deal of difficulty; later, two or three very excellent surfaces were polished by Zeiss. A detailed description of the specimens tested will be given in connection with the specific results, but some general considerations applying to all may be mentioned here.

Even after a comparatively perfect surface of any particular metal had been obtained, the values of the constants it afforded still depended to some extent on the manner in which it had been cleaned. It is usually considered allowable to clean such a surface with absorbent cotton and alcohol, but it was found that even with the purest alcohol obtainable and with the least possible rubbing with cotton, indications of surface films were still sometimes obtained, and in the case of steel the best results were obtained just after polishing with rouge on dry chamois skin and without further cleaning.

Purity.—From what has been said it will be granted that the prime factor in such work as this is the condition of the surface, and this overshadows in importance even the question of the purity of the metal itself. Every effort was made, however, to secure specimens as pure as possible. The copper and silicon were each of tested purity 99.8 per cent or better. The silver, which was deposited chemically, was probably not as pure as this, while the steel specimens were of high-grade tool steels, but of unknown purity. The cobalt and nickel were from sheet material from Kahlbaum. While they were the best obtainable in this form, chemical tests indicated the presence of perhaps 2 per cent of cobalt in the nickel, and probably there was at least as much nickel in the cobalt. It is very likely, however, that they were of the same order of purity as the specimens of these metals tested by Drude, Minor, and others.

Determinations to the number of many hundred have been made of the principal angles of the various metals, and for upward of twenty wave-lengths, but after it was found that the dispersion-curves were quite regular in form, it was thought best to concentrate the efforts on measurements for only a few wave-lengths, with the view of determining the constants for these points as accurately as possible. Nine points in the spectrum, ranging from $\lambda=0.65\,\mu$ to $2.25\,\mu$, were accordingly chosen and repeated determinations made for these wave-lengths. Later this number was reduced to only five, viz., $\lambda=0.65\,\mu$; $0.87\,\mu$; $1.25\,\mu$; $1.75\,\mu$; $2.25\,\mu$. The final and presumably most accurate measurements were made for these wave-lengths only. Also since it was found that the use of the 25° prism, in spite of its low dispersion, led to no appreciable error, it was used almost exclusively, thereby furnishing more energy in the spectrum and hence allowing more accurate measurements.

The results of the present work listed in the tables have been arrived at as follows: All the worthy determinations of the principal angles for any given metal were plotted with wave-length. The best average curves were then drawn and the angles given in the first two columns were taken from these curves; the constants were computed from these angles in the way already described.

To give a general survey of the whole spectral region which has been studied to date in the case of metals, some of the results of Minor and others have been included in the tables and curves. Following the practice of Drude, the results are expressed in general to only two decimals, as the discrepancies between different observers extend into even the first decimal place. For purposes of comparison with the theoretical reflecting powers as computed from these constants, the measurements of Hagen and Rubens[1] on reflecting powers of metals are also listed.

Steel.—Five surfaces of tool steel were experimented upon. Four of these ranging in size from 2 cm×2 cm to 3 cm×7 cm were polished in this laboratory. The method used on the larger pieces was to polish dry on graded emery paper, finishing with wet rouge on a pitch tool in a polishing machine. Others were ground with graded carborundum on a wet lead plate and finished with rouge on pitch. A surface by Zeiss 2 cm×4 cm was also available through the kindness of Professor Minor. This was the one used by him in his measurements on steel, but unfortunately it was not in as perfect condition as originally. The various surfaces gave results in fairly good agreement, especially for the longer wave-lengths, but the principal incidences were all slightly too low for the shorter wave-lengths (in comparison with the values of Drude and others), necessitating a small correction as explained in a consideration of the sources of error.

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[1] *Annalen der Physik*, **8**, 1, 1902; see also *Phil. Mag.* (6), **7**, 162, 1904.

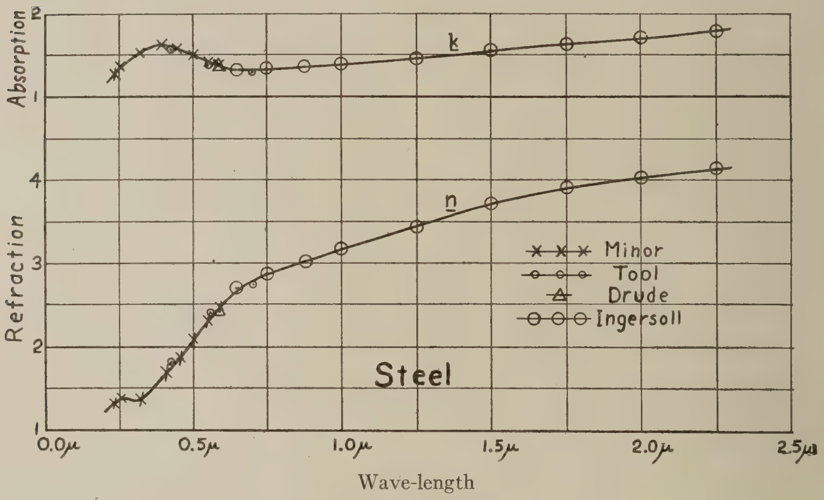

FIG. 3.—Curves showing the dispersion of steel

The results are contained in Table I and are shown graphically in the curves of Fig. 3. The best estimate it is possible to make of the limits of error places them at perhaps ±0.07 at $\lambda=0.65\ \mu$, increasing gradually to ±0.20 at 2.25 μ. This is for the refractive index *n*; the corresponding limits for *k* are ±0.03 and ±0.10. In addition to Hagen and Rubens' reflecting powers a measurement by Trowbridge on the reflecting power of iron for 2.00 μ is included. It may be observed that in this as well as in succeeding tables it has been frequently necessary to interpolate the Hagen and Rubens determinations to secure data for the particular wave-lengths here used.

TABLE I

STEEL

λ	$\bar{\phi}$	$\bar{\psi}$	n	k	nk	Authority	R (Computed)	R (Hagen and Rubens)
0.226 μ	66° 51′	28° 17′	1.30	1.26	1.64	Minor	34.8%	....
.257	68 35	28 45	1.38	1.35	1.86	Minor	39.6	33.0%
.325	69 57	30 9	1.37	1.53	2.09	Minor	44.8	40.3
.400	73 39	30 16	1.68	1.62	2.72	Minor	53.9	49.6
.420	74 9	29 48	1.79	1.57	2.82	Tool	54.5	51.9
.450	74 48	29 36	1.88	1.57	2.93	Minor	55.4	54.4
.500	75 47	29 2	2.09	1.50	3.14	Minor	56.9	54.8
.550	76 35	28 18	2.31	1.43	3.30	Minor	57.7	54.9
.560	76 39	27 24	2.40	1.34	3.21	Tool	56.1	55.0
.589	77 9	27 45	2.48	1.38	3.43	Minor	58.4	55.3
.589	77 3	27 49	2.41	1.38	3.40	Drude	58.5	55.3
.650	77 48	27 9	2.70	1.33	3.59	Ingersoll	59.2	56.8
.700	77 36	26 36	2.73	1.27	3.47	Tool	58.0	57.6
.750	78 28	27 8	2.86	1.33	3.80	Ingersoll	61.2	58.0
.870	79 9	27 16	3.02	1.35	4.08	Ingersoll	63.2	59.4
1.00	79 48	27 33	3.19	1.39	4.43	Ingersoll	65.5	63.1
1.25	80 55	28 15	3.45	1.47	5.08	Ingersoll	69.7	68.8
1.50	81 48	28 51	3.71	1.55	5.75	Ingersoll	73.1	70.8
1.75	82 26	29 28	3.88	1.63	6.32	Ingersoll	75.6	....
2.00	82 55	30 1	4.02	1.71	6.88	Ingersoll	77.7	76.7 78.7*
2.25	83 22	30 36	4.14	1.79	7.41	Ingersoll	79.7	....

* See paper by A. Trowbridge, *Wiedemann's Annalen*, **65**, 595, 1898.

Cobalt.—The results are shown in Table II and Fig. 4. They were obtained with a surface 5 cm in diameter polished by Zeiss. The quality of the surface was very good indeed, as it contained only one or two small imperfections and showed a brilliant mirror-like polish without scratches. The estimated limits of error are, for n, from ±0.03 at $\lambda = 0.650\ \mu$ to ±0.26 at 2.25 μ. The corresponding limits for k are ±0.02 and ±0.06.

TABLE II
COBALT

λ	$\bar{\phi}$	$\bar{\psi}$	n	k	nk	Authority	R (Computed)
0.231 μ	64° 31′	29° 39′	1.10	1.30	1.43	Minor	31.8 %
.275	70 22	29 59	1.41	1.52	2.14	Minor	45.7
.347	72 18	30 20	1.54	1.61	2.47	Minor	51.1
.395	74 15	31 20	1.63	1.79	2.91	Minor	57.7
.440	75 18	31 45	1.70	1.87	3.19	Tool	61.1
.450	76 8	31 52	1.79	1.91	3.42	Minor	63.3
.500	77 5	31 53	1.93	1.93	3.72	Minor	65.5
.550	77 40	31 43	2.05	1.91	3.90	Minor	66.6
.560	77 30	31 12	2.09	1.82	3.81	Tool	65.3
.589	78 5	31 40	2.12	1.90	4.04	Drude	67.5
.650	79 0	31 25	2.35	1.87	4.40	Ingersoll	69.2
.680	79 12	30 45	2.50	1.77	4.44	Tool	68.7
.750	80 2	30 48	2.71	1.79	4.85	Ingersoll	71.0
.870	81 4	30 0	3.18	1.69	5.37	Ingersoll	72.5
1.00	81 45	29 6	3.63	1.58	5.73	Ingersoll	73.3
1.25	82 44	27 30	4.50	1.40	6.30	Ingersoll	74.3
1.50	83 21	26 18	5.22	1.29	6.73	Ingersoll	75.1
1.75	83 33	25 39	5.53	1.23	6.81	Ingersoll	75.1
2.00	83 41	25 39	5.65	1.23	6.95	Ingersoll	75.7
2.25	83 48	26 5	5.65	1.27	7.18	Ingersoll	76.5

Nickel.—The best nickel surface used was one polished by Zeiss, quite similar to the cobalt surface. The limits of error run from ±0.04 at $\lambda = 0.650\,\mu$ to ±0.26 at $2.25\,\mu$, for n; and from ±0.06 to ±0.15 for k.

TABLE III
NICKEL

λ	$\bar{\phi}$	$\bar{\psi}$	n	k	nk	Authority	R (Computed)	R (Hagen and Rubens)
0.420 μ	72° 20′	31° 42′	1.41	1.79	2.53	Tool	53.7%	56.6%
.436	75 23	31 3	1.74	1.89	3.28	Bernoulli	61.9	58.1
.492	75 40	31 20	1.74	1.93	3.37	Bernoulli	63.1	60.7
.500	74 20	32 15	1.54	1.93	2.97	Tool	59.7	60.8
.578	76 0	31 40	1.75	1.99	3.48	Bernoulli	64.4	63.9
.589	76 1	31 41	1.79	1.86	3.33	Drude	62.0	64.4
.615	76 48	31 49	1.85	2.01	3.73	Bernoulli	66.3	65.1
.650	77 35	32 35	1.91	2.06	3.93	Ingersoll	67.8	67.2
.700	77 50	32 3	2.02	1.96	3.98	Tool	67.6	68.8
.750	78 45	32 6	2.19	1.99	4.36	Ingersoll	69.6	69.1
.870	79 44	31 51	2.45	1.96	4.80	Ingersoll	71.7	71.0
1.00	80 33	32 2	2.63	2.00	5.26	Ingersoll	74.1	73.0
1.25	81 47	32 31	2.92	2.11	6.15	Ingersoll	78.0	77.3
1.50	82 42	32 53	3.21	2.18	7.00	Ingersoll	80.6	79.9
1.75	83 22	33 10	3.45	2.25	7.76	Ingersoll	82.7	....
2.00	83 55	33 24	3.70	2.31	8.54	Ingersoll	84.4	83.5
2.25	84 21	33 30	3.95	2.33	9.20	Ingersoll	85.3	....

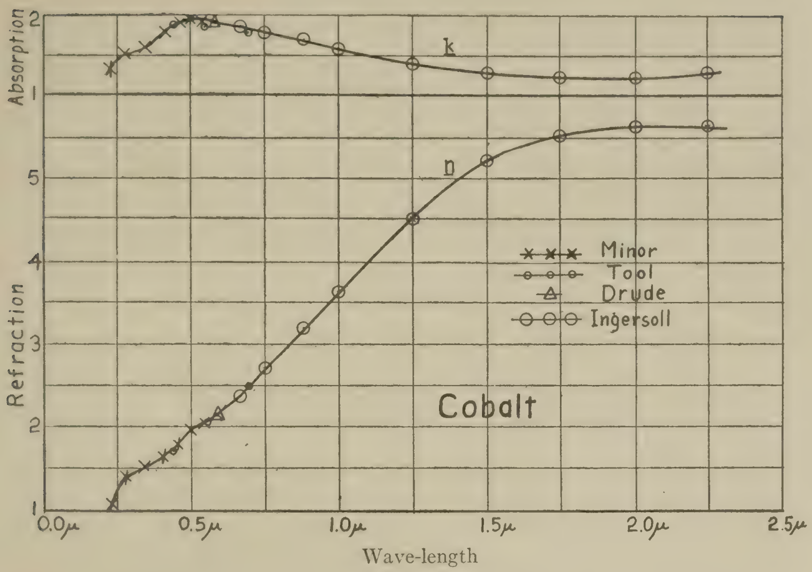

FIG. 4.—Curves showing the dispersion of cobalt

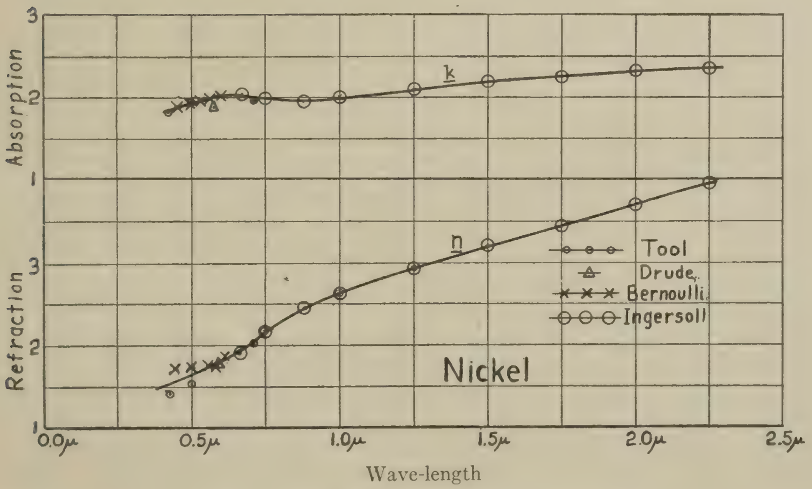

FIG. 5.—Curves showing the dispersion of nickel

Silver.—Because of the great effect of a small error in azimuth (this being so nearly 45°), this was perhaps the hardest metal to test. A considerable area of plane surface was required, and, everything considered, it was thought best to use for this purpose a chemically silvered glass plate. For while it is known that silver in this form will not yield results absolutely identical with those obtained with surfaces on the massive metal, such errors are small compared with the necessary experimental errors in this case.

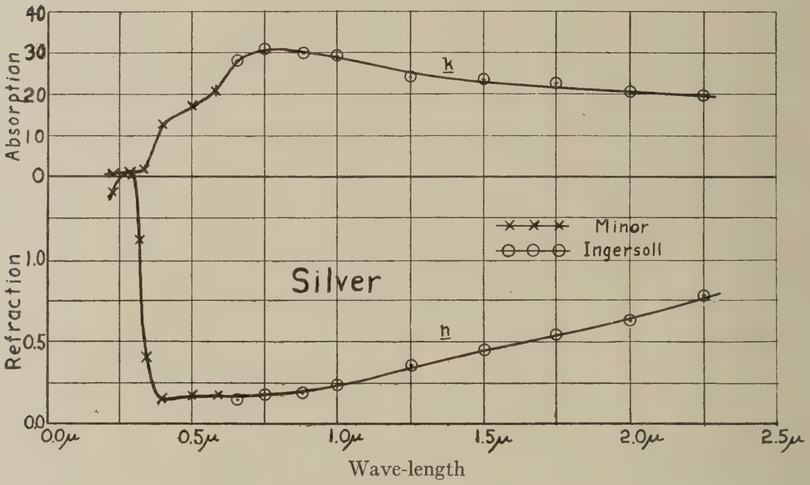

FIG. 6.—Curves showing the dispersion of silver

Accordingly a piece of plate glass, 6 cm × 10 cm, of tested planeness, was silvered by the Brashear process and polished with rouge on chamois. The measurements were made immediately after polishing.

For purposes of comparison the values of *nk* obtained by Hagen and Rubens[1] are included, the figures for these particular wave-lengths having been necessarily interpolated in many cases. The agreement is fairly good and probably well within the limits of error which are very large in this case, especially for the longer wave-lengths, because, as noted above, of the great effect of small errors in the azimuth determinations. They run from ±0.03 at $\lambda=0.650\,\mu$ to ±0.40 at $2.25\,\mu$ for n; and for k from ±4.0 to probably ±10.0 (i.e., 50 per cent).

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[1] *Annalen der Physik* (4), 8, 432, 1902.

TABLE IV

SILVER

λ	$\bar{\phi}$	$\bar{\psi}$	n	k	nk	Authority	R (Computed)	Hagen and Rubens' Observed Values	
								nk	R
0.226 μ	62°41′	22°16′	1.41	0.75	1.11	Minor	18.4%	....	....
.293	63 14	18 56	1.57	0.62	0.97	Minor	16.7	0.95	15.0
.316	52 28	15 38	1.13	0.38	0.43	Minor	4.2	0.45	4.2
.332	52 1	37 2	0.41	1.61	0.65	Minor	32.5	0.55	35.5
.395	66 36	43 6	0.155	12.32	1.91	Minor	87.1	1.94	83.4
.500	72 31	43 29	0.169	17.14	2.94	Minor	93.2	3.21	91.3
.589	75 35	43 47	0.177	20.55	3.64	Minor	95.0	4.12	92.7
.650	77 35	44 2	0.154	28.2	4.34	Ingersoll	96.9	4.77	95.7
.750	79 26	44 6	0.168	30.7	5.16	Ingersoll	97.4	5.85	96.6
.870	80 54	44 4	0.203	29.9	6.07	Ingersoll	97.8	6.76	97.2
1.00	82 0	44 2	0.24	29.0	6.96	Ingersoll	98.1	8.0	97.5
1.25	83 40	43 50	0.37	24.1	8.88	Ingersoll	98.2	10.7	97.7
1.50	84 42	43 48	0.45	23.7	10.7	Ingersoll	98.4	12.4	97.9
1.75	85 24	43 44	0.55	22.4	12.3	Ingersoll	98.5	....	....
2.00	85 55	43 36	0.68	20.3	13.7	Ingersoll	98.5	....	97.8
2.25	86 18	43 34	0.77	19.9	15.4	Ingersoll	98.6	....	....

Copper.—The copper surface was polished by Zeiss on a 5 cm plate of electrolytic copper of great purity. While probably the best which could be produced, at least on this specimen, the polish was far from good. To improve this and to remove the small amount of oxidation which had formed since it had been finished, it became necessary to "buff" the surface to produce a polish which would yield optical constants in even fair agreement with accepted values. This fact, together with the large azimuth afforded by copper and its consequent attendant errors, renders the results obtained with this metal the least satisfactory of any. The limits of error are from 30 per cent to 50 per cent of the results themselves, i.e., for n from ±0.15 to ±0.35, and for k from ±2.5 to ±5.0. Both for silver and copper the figures for wave-length greater than say $1.5\,\mu$ can be considered only as indicative of the general trend of the curves.

TABLE V

COPPER

λ	$\overline{\phi}$	$\overline{\psi}$	n	k	nk	Authority	R (Computed)	R (Hagen and Rubens)
0.231 μ	65°57′	26°14′	1.39	1.05	1.45	Minor	29.0%	....
.347	65 6	28 16	1.19	1.23	1.47	Minor	31.5	26.4%
.450	69 41	32 39	1.13	1.90	2.15	Minor	50.5	37.0
.460	70 36	32 45	1.18	1.93	2.28	Tool	52.6	38.3
.500	70 44	33 46	1.10	2.13	2.34	Minor	55.5	43.7
.589	71 34	39 5	0.62	4.26	2.63	Minor	74.1	64.0
.650	74 16	41 30	0.44	7.4	3.26	Ingersoll	86.2	80.0
.660	76 12	40 24	0.65	5.77	3.78	Tool	84.6	80.7
.870	78 40	42 30	0.35	11.0	3.85	Ingersoll	91.5	89.0
1.25	82 14	42 50	0.56	13.0	7.28	Ingersoll	95.8	93.6
1.75	84 4	42 30	0.83	11.4	9.46	Ingersoll	96.4	....
2.25	85 13	42 30	1.03	11.4	11.7	Ingersoll	97.1	96.0

Silicon.—The silicon was in the form of a plate of perhaps 3 sq. cm area, and was polished by Mr. J. T. Littleton, Jr., of this laboratory from a crystal of 99.75 per cent purity furnished by the Carborundum Company of Niagara Falls.

TABLE VI

SILICON

λ	$\overline{\phi}$	$\overline{\psi}$	n	k	nk	R (Computed)
0.589 μ	76°36′	2°42′	4.18	0.090	0.377	37.7%
0.870	75 11	2 29	3.77	0.081	0.305	33.9
1.25	74 51	2 28	3.67	0.080	0.293	32.8
1.75	74 31	2 29	3.59	0.080	0.287	31.7
2.25	74 16	2 25	3.53	0.078	0.275	31.3

Discussion of the results.—The most obvious thing to be noted from the tables and curves is the fact that for the first five metals the refractive indices all show an increase with increasing wavelength. In the case of steel, nickel, and cobalt this rise of the dispersion-curve is merely a continuation of the tendency shown by visual observations, but in the case of copper and silver it could not be predicted. That this increase is not as rapid, however, as would be judged from such visual observations, is shown by the curves for steel and particularly for cobalt, where the refractive index has apparently reached a maximum at wave-length 2.00 μ.

The absorption index curves, on the other hand, show no such general tendency. A gradual rise on the side of the infra-red in the case of steel and nickel and a corresponding lowering for the other three metals may be noted, but in general the change is less after the visible spectrum has been passed and the infra-red reached. This means that the product nk, the extinction coefficient, increases in general with the wave-length.

In silicon, on the other hand, the refractive index shows a small decrease as the wave-length increases, in the same sense and of much the same order of magnitude as exhibited by dielectrics. The reflecting powers, too, show a decrease on the side of the infra-red, just as in the case of dielectrics, and these facts should go far to remove what small claim silicon may have to be considered a metal, optically speaking.[1]

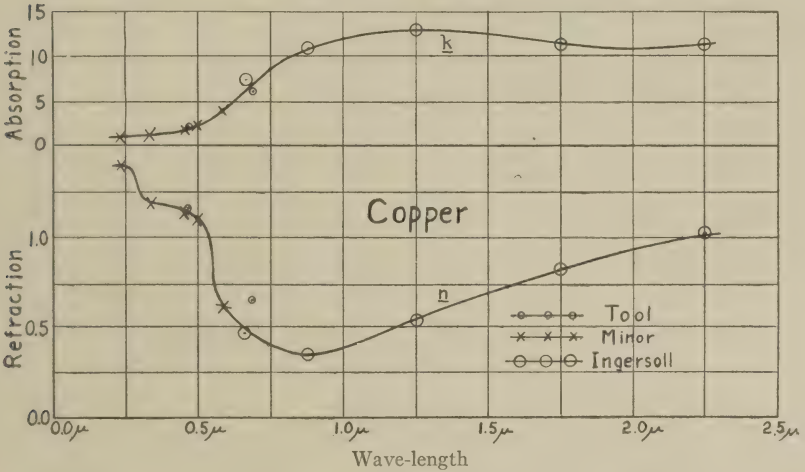

FIG. 7.—Curves showing the dispersion of copper

Comparison with theory.—In attempting to compare the results with theory the same difficulty is encountered that is generally met with in any attempt at reconciling in a quantitative way the predictions of the electromagnetic theory with experimental facts. To bring about an agreement the equations are complicated by terms which take into account, in the case of metals, the presence of conducting and bound electrons, and involve coefficients which it is well-nigh impossible to determine in many cases.

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[1] In other words, silicon has a much larger proportion of non-conducting, or "bound," electrons than most other metallic substances.

But while the agreement of the metals with the simple Maxwell theory may be very poor for the region of the visible spectrum, Hagen and Rubens[1] find that for wave-lengths longer than about 8.85 μ the agreement of optical and electrical properties is very nearly such as demanded by this theory. It becomes of interest then to discuss the question of whether there is a gradual transition from the state of affairs exhibited by metals in the visible spectrum to that shown well out in the infra-red, or whether this takes place largely in only a certain range of spectrum.

It has been the writer's contention[2] that the visible and very short infra-red spectrum is in the case of metals a region somewhat analogous to an anomalous dispersion region in the case of dielectric substances, and once wave-length about 1 μ is passed we should expect to find much better agreement of theory and experiment. This conclusion was based on some experimental results[3] obtained on the Kerr effect, or magnetic rotation on reflection from surfaces of the magnetic metals in a strong field. These showed, just as found by other observers, that while in the visible spectrum the dispersion of this rotation is anomalous, i.e., the rotation increases with the wave-length, when the effect is studied in the infra-red it is found to reach a maximum at $\lambda = 1\ \mu$ and thereafter to decrease much as in the case of dielectric substances.

In the present work this idea meets both agreement and disagreement. Maxwell's original theory demands that, if σ is the electrical conductivity and T the period of the light-wave, then the relation should hold that $n^2k = \sigma T$. This relation is not even approximately verified for the visible spectrum and the agreement is only a little better at $\lambda = 2\ \mu$, so we must evidently go much farther into the infra-red to get rid of the disturbing influence of the bound electrons, at least in so far as they affect this relation. On the other hand, in the case of reflecting powers we find a much better agreement between theory and experiment for the longer waves than for the shorter. A comparison of R as computed from the measured constants and R as observed by Hagen and Rubens shows this very well.

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[1] E. Hagen and H. Rubens, *Phil. Mag.* (6), **7**, 157, 1904; *Annalen der Physik*, **11**, 873, 1903; also see *Sitzungsberichte der Preuss. Akad. der Wiss.*, Berlin, **16**, 478, 1909.

[2] *Phil. Mag.* (6), **11**, 70, 1906.

[3] *Ibid.*, **11**, 62, 1906.

Kerr effect.—Voigt[1] deduces the expression for the Kerr rotation at normal incidence

$$x=\frac{|Q|\cos\left(q+\frac{1}{2}\pi-2\bar{\psi}\right)}{\sin\bar{\phi}\tan\bar{\phi}}, \tag{3}$$

where Q is a complex which does not contain n, and q is a small angle measuring the effect of the magnetic field on k. Now it is not possible at present to evaluate these two quantities for longer wave-lengths than the visible, although the writer hopes shortly to undertake some experiments which should determine directly the values of q, but it may be interesting to compare the results of this equation with experiment, on the assumption that the numerator is constant, that is, independent of the wave-length. Accordingly the following table has been made out. The figures for the Kerr rotation are an average of the writer's results,[2] reduced, as is $C/\sin\bar{\phi}\tan\bar{\phi}$, to unity at $\lambda=1\ \mu$.

TABLE VII

Wave-Length		0.50 μ	1.00 μ	2.00 μ
Steel	Kerr rotation	0.62	1.00	0.69
	$C/\sin\bar{\varphi}\tan\bar{\varphi}$	1.43	1.00	.68
Cobalt	Kerr rotation	0.83	1.00	.70
	$C/\sin\bar{\varphi}\tan\bar{\varphi}$	1.62	1.00	.76

As the Kerr rotation for nickel reduces to zero and changes sign at wave-length 1.5 μ, such a table would mean nothing for this metal. For the other two metals it is evident that equation (3) with the above assumption represents the facts fairly well for the longer waves, but not at all for the shorter. If q had been assumed constant and the variations in $\bar{\psi}$ taken into account, the agreement would not be quite so good. On the whole, however, this may be looked upon as an argument for the general contention that, once clear of the visible spectrum, we may expect to find a rapid improvement in the behavior of metals as regards their obedience to electromagnetic laws. That this improvement continues, though probably more slowly, out to much longer infra-red wave-lengths, the results of Hagen and Rubens' experiments show beyond a doubt.

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[1] W. Voigt, *Magneto- und Elektrooptik*, p. 317. Leipzig, 1908.

[2] *Phil. Mag.*, **11**, 64–67, 1906.

SUMMARY

1. The optical constants of steel, cobalt, nickel, silver, copper, and silicon have been measured by a new method over the range of spectrum from $\lambda = 0.65\,\mu$ to $2.25\,\mu$.

2. The first five metals show an increase of refractive index n with increasing wave-length. The absorption index k decreases in some cases and increases in others, but in general does not show nearly as much change beyond wave-length $1\,\mu$ as for shorter waves. The extinction coefficient nk shows a gradual increase with the wave-length.

3. Silicon shows a slight decrease of n with increasing wave-length, much as do dielectric substances. The reflecting power also decreases, hence silicon has small right to be considered a metal optically speaking.

4. The reflecting powers computed from these constants are in good agreement with the observed values of Hagen and Rubens, especially for the longer wave-lengths. Also on certain assumptions, Voigt's equation connecting the optical constants and the Kerr rotation is fulfilled for the longer waves though not for the shorter, and these facts go to bear out the writer's contention that, once outside the visible spectrum, we may expect a much better agreement between theory and experiment in the case of the optical properties of metals.

Physical Laboratory
University of Wisconsin
November 1910

THE ABSORPTION SPECTRUM OF BROMINE AT HIGH TEMPERATURES

By E. J. EVANS

The absorption spectrum of bromine has been studied by Miller, Robiquet, Roscoe and Thorpe, Moser, Dale, and Hasselberg.[1] Later, Galitzin and Wilip[2] made an exhaustive study of the absorption spectrum at different temperatures and pressures. The most important results of their work, from the point of view of the present research, were those obtained at high temperatures, and low pressures. In some of their experiments, the quartz tube containing the vapor at a pressure of 13 mm was heated to a temperature of 1060° C., and the absorption spectrum photographed. It was found that the high-temperature spectrum was very different from the low-temperature spectrum. Some of the lines were broadened, and others were sharpened. Some lines which were very sharp and clear at low temperatures became indistinct and vanished at high temperatures, and vice versa. Also the bandlike character of the spectrum was lost at high temperatures.

Researches on the absorption spectrum of iodine have shown that the absorption lines diminish in intensity with increase of temperature, and finally disappear. The disappearance of the absorption lines at high temperatures has also been observed in the present experiments on bromine vapor, and the temperature of disappearance of the lines was found to increase as the pressure of the vapor was increased. The primary object of the research was to determine whether there is any connection between the absorption spectrum, and the state of the vapor with regard to dissociation. The temperatures of disappearance of the absorption spectrum for definite pressures were observed, and the degree of dissociation for each temperature and pressure evaluated by means of the following equation due to Nernst:

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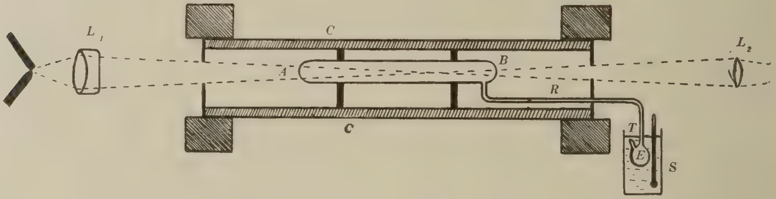
[1] Kayser's *Handbuch der Spectroscopie*, Bd. III, 320–21.

[2] *Mémoires de l'Académie des Sciences de St.-Pétersbourg*, 17, 1–112, 1906.

$$\log\frac{1-x^2}{x^2P}=-\frac{Q}{4.571\,T}-1.75\log T+\Sigma\nu c.$$

In the above equation x represents the degree of dissociation, P the pressure in atmospheres, $-Q$ the heat of dissociation of the vapor, T the absolute temperature, and $\Sigma\nu c$ a constant. The values of Q and $\Sigma\nu c$ were calculated by Brill[1] from the experiments of Perman and Atkinson[2] on the vapor density of bromine, and found to be −55,300 and −1.7 respectively.



EXPERIMENTAL ARRANGEMENT

The apparatus employed in the present research was very similar to that used by the author in his investigation of the absorption spectrum of iodine at high temperatures,[3] and it is necessary to give only a brief description. The bromine vapor was heated in the quartz tube AB, which was placed inside the carbon-tube electric furnace CC. By adjusting the current passing through the carbon tube CC, the temperature of the vapor could be varied from 100° C. to 1350° C. The tube AB was connected with the bulb E by means of the quartz tube R. A small quantity of pure liquid bromine was introduced into the bulb by means of the small side tube T. The bulb was kept at a temperature of −40° C., and the side tube T connected to a Fleuss pump for evacuation. When the pressure was about 1 mm, the tube T was sealed off in the oxyhydrogen flame. To observe the absorption spectrum, the light from the positive pole of the electric arc, after passing through the large condensing lens L_1 and the vapor contained in the tube AB, was brought to a focus on the slit of a concave grating by means of the lens L_2. The grating had a radius of 1 meter and 15,000 lines to the inch. The first-order spectrum was always observed.

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[1] *Zeitschrift Phys. Chem.*, **57**, 721, 1907.

[2] *Proc. Roy. Soc.*, **66**, 10, 489, 1899.

[3] *Astrophysical Journal*, **32**, 1–16, 1910.

MEASUREMENT OF PRESSURE AND TEMPERATURE

The bulb containing the liquid bromine was placed in a beaker containing alcohol, and temperatures below 0° C. were obtained by dropping solid CO_2 into the liquid. The liquid was stirred, and the temperature kept constant within a degree by adding a little CO_2 from time to time. The beaker containing the alcohol was placed inside a larger vessel, and the space between them was filled with the badly conducting substance known as Kieselguhr. The bath S was always made the coldest part of the apparatus. The pressure of the bromine vapor in the tube AB was then the same as the vapor-pressure of the bromine at the temperature of the bath. Since the vapor-pressures of bromine at different temperatures were known, it was only necessary to know the temperature of the bath S to determine the pressure of the bromine vapor in the quartz tube AB. The following table taken from Galitzin and Wilip's paper (*loc. cit.*) was employed to determine the vapor-pressure of the bromine:

Temperature	Pressure	Temperature	Pressure
	mm		mm
−25° C.	8	5° C.	86
−20	15	10	110
−15	24	15	138
−10	35	20	173
− 5	49	25	215
0	66	30	267

The temperature of the vapor was measured by a *Pt-Rh* thermocouple, which was connected to a galvanometer reading directly in degrees Centigrade. The couple was placed inside a quartz tube to protect it from furnace gases, and the junction was situated beyond the end A of the tube AB.

MODE OF WORK AND EXPERIMENTAL RESULTS

The experiments were carried out with the object of determining as accurately as possible the temperature at which the absorption lines completely vanished, and no detailed observations of the changes in the appearance of particular lines at different temperatures were made. To illustrate the method of experimenting, the observations on bromine at pressures of 8.6 cm and 16 cm will be discussed. The bath temperature was kept constant at 5° C. (pressure of vapor 8.6 cm), and the temperature of the furnace was gradually raised. At first, dark bands were seen in the red, yellow, and green, and there was a general absorption in the blue and violet. The absorption in the blue and violet diminished as the temperature was raised, and when a temperature of about 1100° C. was reached there was scarcely any absorption in those regions, and the intensity of the bands in the red, yellow, and green was greatly diminished. Every trace of absorption disappeared when the temperature of the vapor had been raised to 1220° C., but on cooling, the absorption lines again made their appearance. The temperature of the bath was then raised from 5° C. to 17° C. (pressure of vapor 16 cm), and the temperature of the furnace kept at 1000° C. As the bath temperature increased, the pressure of the bromine vapor in the tube *AB* increased, and visual observations of the absorption spectrum showed that the intensity of the dark bands had also increased. The temperature at which the absorption lines vanished was found to be 1320° C. The accurate fixing of the temperature of disappearance was always difficult, as the absorption lines were very faint over a wide range of temperature. However, by keeping the eyepiece in motion during the visual observations of the spectrum, very faint absorption lines could be detected. The temperatures of disappearance of the absorption spectrum for different pressures will be found in the accompanying table.

Temperature of Bath	Temperature of Disappearance of Spectrum	Temperature of Bath	Temperature of Disappearance of Spectrum
−20° C.	830° C.	−0.8° C.	1136° C.
−15	950	+5.4	1220
−10	1030	17.8	1320
− 5	1090	28	Absorption spectrum had not disappeared at 1320° C.

When the temperature of the bath was 28° C. the ends of the quartz tube became opaque, and the tube collapsed in the furnace before the absorption lines had disappeared.

EXPERIMENTS ON A CONSTANT MASS OF BROMINE VAPOR

In the previous experiments the pressure of the vapor in the quartz tube was kept constant, and the absorption spectrum observed as the temperature increased. It was therefore considered desirable to investigate the effect of high temperature on the absorption spectrum when the quantity of bromine in the tube was kept constant. The bulb containing the liquid bromine was immersed in a bath of alcohol and solid CO_2, which was kept at a temperature of −40° C., and the apparatus was evacuated to a pressure of 1 mm. The temperature of the bath was then allowed to rise to −25° C., and was kept constant at that value for 5 minutes. The side tube R near the end B of the quartz tube AB was sealed off in the oxyhydrogen flame. The tube containing the vapor was then fixed inside the furnace, and the absorption spectrum examined as the temperature of the furnace increased. At 600° C., absorption lines were seen in the red, yellow, and green, but they were much fainter when the temperature had reached 960° C., and had completely vanished at 1040° C. Photographs of the spectrum were taken at the ordinary temperature of the room, and at 550°, 870°, and 1080° C. The absorption lines, which were very faint, could be distinguished in the first three photographs, but not in the photograph taken when the temperature of the furnace was 1080° C. The pressure of the vapor in the tube, when the latter was at the room temperature, was 8 mm, and it was therefore possible to calculate the pressure at 1040° C., which was the temperature of disappearance of the absorption spectrum. The volume of the quartz tube was not much altered by heating it to a high temperature, and if it be assumed that the molecular state of the vapor was unaltered, and represented by Br_2, the pressure was simply proportional to the absolute temperature. The calculated value of the pressure was 36 mm. Actually, the pressure was greater, because a percentage of the molecules had been dissociated. The constant pressure experiments have shown that the absorption spectrum disappeared at 1030° C., when the pressure of the bromine vapor was 35 mm. There is fair agreement between the results, considering the difficulty of accurately estimating the pressure at which the bromine vapor was sealed in the quartz tube AB.

QUANTITY OF BROMINE DETECTABLE BY ITS ABSORPTION SPECTRUM

The bulb containing the liquid bromine was placed in the bath containing alcohol and solid CO_2, and the absorption spectrum observed at different temperatures. At $-25°$ C. faint absorption lines were visible, but when a temperature of about $-33°$ C. was reached all traces of absorption had disappeared. The values of the vapor-pressure of bromine at temperatures below $-25°$ C. do not seem to have been determined, and as the knowledge of the amount of bromine detectable by the grating spectroscope was important in the discussion of the experimental results, another method was adopted. The method employed had the advantage of determining the amount detectable at a high temperature, and also the vapor was known to be made up of a definite mixture of diatomic and monatomic molecules. The mode of procedure consisted in keeping the temperature of the bath constant at $-20°$ C., and raising the temperature of the furnace until the absorption lines had completely disappeared. In this way the temperature of disappearance was found to be 830° C.

The dissociation of bromine is represented by the following equation:

$$Br_2 \leftrightarrows Br + Br.$$

Let x represent the fraction of diatomic molecules dissociated, P the total pressure, and p_1 the pressure of the undissociated molecules. Then it can be shown that

$$p_1 = \frac{1-x}{1+x} P.$$

P, the total pressure of the vapor, was the same as the vapor-pressure of bromine at a temperature of $-20°$ C., viz., 15 mm. The fraction of dissociation x was calculated from Nernst's thermodynamic equation, and found to be 0.076. p_1 was calculated by means of the above equation, and found to be 12.9 mm.

It is therefore seen from the results of the above experiment that a pressure of 12.9 mm of the diatomic molecules can be detected. Here the assumption is made that the monatomic molecules do not contribute anything to the absorption.

The amount of diatomic bromine in the tube when the absorption lines were no longer recognizable can now be determined.

Let M=mass of bromine vapor in a tube of volume V.

Let m=molecular wt. of bromine vapor, p the pressure in dynes, and θ the absolute temperature.

Then

$$M=\frac{Vpm}{R\theta}$$

where

$$R=8.26\times 10^7 .$$

The tube employed was 19.2 cm long and 0.512 cm in radius.

$$\therefore V=15.8 \text{ cc}$$

$$p=0.017 \text{ atmosphere}=0.017\times 10^6 \text{ dynes}$$

$$m=160$$

$$\theta=1103^\circ \text{ Abs.}$$

$$M=4.7\times 10^{-4} \text{ gr.}$$

The experiment indicated that 4.7×10^{-4} gr. of bromine vapor in the diatomic state could be detected at 830° C.

DISCUSSION OF RESULTS

To facilitate the discussion of the experimental results the following table has been constructed:

Pressure of Vapor	Temperature of Disappearance of Spectrum	Fraction of Dissociation	Fraction of Dissociation Observable by Spectroscope	Remarks
mm				
24	950° C.	0.22	0.25	
35	1030	.37	.39	
49	1090	.49	.51	
63	1136	.58	.58	
88	1220	.74	.68	
160	1320	.84	.79	
246	Spectrum had not disappeared at 1320° C.	...	...	The tube became opaque and collapsed at about 1400° C.

The third column gives the fraction of the diatomic molecules which had been dissociated at the temperature of disappearance of the absorption lines. This fraction was determined by inserting in Nernst's equation the appropriate values of the temperature and pressure. The numbers in the fourth column were deduced on the assumption that 4.73×10^{-4} grams of Br_2 molecules were necessary for the detection of the absorption lines. The pressure of the diatomic molecules present at each temperature of disappearance could readily be determined by the equation $p=\frac{MR\theta}{Vm}$, and x, the fraction of dissociation possible, evaluated by means of the relation $p=\frac{1-x}{1+x}P$, where P is the total pressure of the bromine vapor.

An error of 20° C. in the estimation of the temperature of disappearance of the absorption lines will make an appreciable difference in the calculated value of the fraction of dissociation. The following example is given as an illustration. If the absorption lines of bromine vapor at a pressure of 8.6 cm disappeared at a temperature of 1180° C., the calculated value of the fraction of dissociation would be 0.65, but if the temperature of disappearance had been 1200° C., the calculated value would have been 0.7. It is important to note that the temperatures of disappearance of the absorption spectrum for given pressures will vary with the lengths and diameters of the tubes employed. If longer tubes were employed in the above experiments, the temperatures of disappearance of the absorption lines would be correspondingly raised.

The tabulated results show that the temperature of disappearance of the absorption lines is higher the greater the pressure, and that the disappearance of the absorption is closely connected with the dissociation of the vapor. The disappearance of the absorption lines at high temperatures may be explained on the assumption that the monatomic bromine molecules give no absorption spectrum (except perhaps in very thick layers) within the range λ 3500 to λ 6800. In other words, the absorption spectrum of bromine is assigned to the diatomic molecules, and the vanishing of the absorption lines at higher temperatures is ascribed to the production of monatomic molecules, which give no absorption within the above range. The increase in the temperature of disappearance of the absorption lines with increase of pressure is a natural consequence of the above hypothesis, for increase of pressure at constant temperature is accompanied by a diminution in the fraction of diatomic molecules dissociated.

Heated bromine vapor is known to give an emission spectrum, the lines of which coincide in position with those of the absorption spectrum. During the progress of the present work an attempt was made to observe this emission spectrum visually, but without success. In the author's paper on iodine it was pointed out that the apparatus employed in these researches was not suitable for viewing the emission spectrum, and the inability to observe the emission lines may possibly be due to the above cause.

To account for the disappearance of the absorption lines of iodine vapor, the following hypothesis has been advanced. The absorption spectrum was ascribed to the diatomic molecules, and the disappearance of the absorption lines was considered to take place when the emission due to dissociation and recombination of the iodine molecules was exactly balanced by the absorption of the iodine molecules still in the undissociated state.

The above hypothesis may possibly explain the disappearance of the absorption spectrum in the case of bromine, but it appears difficult to test it quantitatively.

SUMMARY OF RESULTS

a) The absorption spectrum of bromine disappears at high temperatures, and the temperature of disappearance is increased by increase of pressure.

b) The disappearance of the absorption lines is closely connected with the dissociation of the diatomic molecules and can be explained on the hypothesis that the monatomic molecules produced by dissociation give no absorption spectrum within the limits λ 3500 to λ 6800.

The author is indebted to Professor Rutherford and to Professor Schuster for the great interest they have taken in the work, and for placing the necessary apparatus at his disposal.

Physical Laboratory
Manchester University
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MEASURES ON NINETEEN NEW SPECTROSCOPIC BINARIES

By OLIVER J. LEE

The binary character of fifteen of the stars in this article was announced in a paper by Frost and Lee read at the Cambridge meeting of the Astronomical and Astrophysical Society of America on August 17, 1910. Nothing has hitherto been published regarding the variable velocities of ι *Cassiopeiae*, γ *Ophiuchi*, and α *Pegasi*.

The dispersion of one prism was employed on all plates here considered, except in the case of η *Coronae*, for which two prisms were used for the plates designated by IIB. The usual initials have been given in specifying the observers by whom the plates were taken, viz., A=Adams; B=Barrett; F=Frost; L=Lee; M=S. A. Mitchell; S=Sullivan, who regularly assisted in the guiding.

25 σ Andromedae ($\alpha=0^{\mathrm{h}}\ 13^{\mathrm{m}}$; $\delta=+36°\ 14'$; Mag.$=4.5$)

Plate	Date	G.M.T.	Taken by	Center		Viol. Comp.		Red Comp.		Quality
				No. Lines	Velocity	No. Lines	Velocity	No. Lines	Velocity	
					km		km		km	
IB 1198	1907 Oct. 11	$18^{\mathrm{h}}20^{\mathrm{m}}$	B	5	−26	5	−59	3	+49	g.
1719	1908 Sept. 7	17 37	B	8	+ 5					g.
1728	Sept. 8	16 14	L	5	− 6	7	−58	5	+29	v. g.
2194	1909 Nov. 25	15 16	L	7	−22					g.

g.=goood; w.=weak; v.=very.

The spectrum is of type A2. The violet component was the stronger in each case where two components were measured. Metallic lines are numerous and give fairly accordant velocities.

35 Hev. ι *Cassiopeiae* ($\alpha=2^{\mathrm{h}}\ 21^{\mathrm{m}}$; $\delta=+66°\ 57'$; Mag.$=4.6$)

Plate	Date	G.M.T.	Taken by	No. Lines	Velocity	Quality
					km	
IB 2111.....	1909 Aug. 23	$18^{\mathrm{h}}40^{\mathrm{m}}$	F, L	12	− 7	g.
2217.....	Dec. 22	12 15	L, B	5	+10	w.
2230.....	1910 Jan. 3	12 11	B, L	8	−21	g.
2247.....	Jan. 14	12 02	L, B	9	− 4	v. g.

This star is No. 1262 in Burnham's *General Catalogue* and is a triple star. The magnitudes are: A 4.8, B 7.0, C 8.2. The relative motion is slow and the proper motion small. The spectrum is of type A5 and has many fair lines.

86 ρ Tauri ($\alpha=4^h\ 28^m$; $\delta=+14°\ 38'$; Mag.=4.8)

Plate	Date	G.M.T.	Taken by	Center		Viol. Comp.		Red Comp.		Quality
				No. Lines	Velocity	No. Lines	Velocity	No. Lines	Velocity	
					km		km		km	
IB 1802	1908 Oct. 16	$18^h 55^m$	B, L	11	+28	2	− 6	3	+76	w.
1837	Nov. 8	19 57	L	8	+24	9	−22	7	+52	v. g.
1885	Dec. 4	18 34	L			7	−32	10	+63	v. g.

This star is No. 1067 of Boss's *Preliminary General Catalogue* and is a member of the "Moving Cluster in *Taurus*" (*Astronomical Journal*, **26**, 31, 1908) for which he predicts a velocity of +41.0 km. The binary character of the star was discovered by inspection of the second plate. On No. 1885 the double lines stand out so clearly that measures of the centers were not made. The red component is the stronger on the last two plates. The spectrum is of type A5.

18 ν Geminorum ($\alpha=6^h\ 23^m$; $\delta=+20°\ 17'$; Mag.=4.1)

Plate	Date	G.M.T.	Taken by	No. Lines	Velocity	No. Lines	Velocity H and K	Quality
					km		km	
IB 213....	1903 Dec. 1	$22^h 52^m$	F	4	+24	2	− 6	g.
224....	Dec. 25	22 40	A	4	+33	2	+ 1	g.
705....	1906 Feb. 26	17 49	F	3	+ 6	1	+22	g.
1371....	1908 Jan. 24	17 27	B	3	+15	1	+12	g.
1947....	1909 Jan. 1	18 37	B, L	3	+20			g.

In this spectrum, of type B5, only the hydrogen lines, and one or two helium lines with H and K, are measurable. Double measures were made on No. 224 with the result +36, +30 km for the broad lines, and+4, −2 km for the calcium lines H and K. No. 1947 was too weak in the K region for measurement. The difference in velocity of the sharp *Ca* lines from the others is distinct and should be further investigated.

42 Camelopardalis ($\alpha = 6^{h}\ 41^{m}$; $\delta = +67^{\circ}\ 41'$; Mag. = 5.0)

PLATE	DATE	G.M.T.	TAKEN BY	CENTER		VIOL. COMP.		RED COMP.		QUALITY
				No. Lines	Velocity	No. Lines	Velocity	No. Lines	Velocity	
					km		km		km	
IB 230..	1903 Dec. 26	$23^{h}11^{m}$	F	4	+ 3					g.
243..	Dec. 31	23 27	F	7	0					g.
266..	1904 Jan. 23	19 41	A	9	−17					v. g.
618..	1905 Nov. 10	22 43	B	6	+11	3	−30	3	+53	w.
961..	1907 Jan. 25	22 3	B	3	−12	4	−46	5	+27	g.

The spectrum of this star is of type B3. Its lines are fairly distinct. The calcium lines H and K were measured on plates Nos. 243 and 266, the only two that are strong enough in the violet end. The means of accordant measures, reduced to the sun, give +17 km and +17 km, respectively, for the two plates. This suggests that the velocity for these calcium lines may not be variable.

83 φ Geminorum ($\alpha = 7^{h}\ 47^{m}$; $\delta = +27^{\circ}\ 1'$; Mag. = 5.0)

PLATE	DATE	G.M.T.	TAKEN BY	CENTER		VIOL. COMP.		RED COMP.		QUALITY
				No. Lines	Velocity	No. Lines	Velocity	No. Lines	Velocity	
					km		km		km	
IB 1823	1908 Nov. 2	$23^{h}14^{m}$	B			5	−48	6	+ 49	g.
1840	Nov. 8	23 28	L			8	+ 7	3	+103	g.
1888	Dec. 4	22 18	L	6	+16					g.

The spectral type A2 of *φ Geminorum* permits of the use of a number of fairly defined metallic lines. The first two plates show such definite components that no measures were made of the centers of the lines. The appearance of the lines on the last plate suggests that they are unresolved doubles.

43 γ Cancri ($\alpha = 8^{h}\ 38^{m}$; $\delta = +21^{\circ}\ 50'$; Mag. = 4.7)

Plate	Date	G.M.T.	Taken by	No. Lines	Velocity	Quality
					km	
IB 1000.....	1907 Feb. 22	$20^{h}47^{m}$	B	7	+16	v. g.
1887.....	1908 Dec. 4	21 4	L	8	+35	v. g.
1931.....	Dec. 26	20 3	B	9	+25	v. g.
2253.....	1910 Jan. 14	17 19	B	11	+20	v. g.
2326.....	May 9	15 56	L	6	+25	g.

Some accuracy may be claimed for measures of this spectrum, classified as A. No. 1000 shows components which referred to the sun give −44 km as mean of two lines and +42 as mean of three. It is probable that a higher dispersion would reveal double lines.

22 *θ Hydrae* ($\alpha = 9^h\ 9^m$; $\delta = +2°\ 44'$; Mag. = 3.8)

Plate	Date	G.M.T.	Taken by	No. Lines	Velocity	Quality
					km	
IB 1870.....	1908 Nov. 16	23^h46^m	B	5	+11	g.
1932.....	Dec. 26	20 54	B	7	+ 9	v. g.
1949.....	1909 Jan. 1	20 25	L	5	− 7	v. g.

The magnesium line λ 4481 is the best in this spectrum, of type A, and most weight was given to it. The best lines on these plates seem more or less distorted, but no consistent nor definite components have been found. Possibly they are close doubles with components of unequal intensity. The spectral lines are unusual in appearance and suggest a critical stage in the physical history of the star.

77 *σ Leonis* ($\alpha = 11^h\ 16^m$; $\delta = +6°\ 35'$; Mag. = 4.1)

Plate	Date	G.M.T.	Taken by	No. Lines	Velocity	Quality
					km	
IB 317.....	1904 Apr. 16	16^h45^m	A	11	−11	g.
679.....	1906 Jan. 29	20 2	B	3	−13	w.
716.....	Mar. 30	17 52	F	8	−17	g.
731.....	Apr. 10	15 7	F, B	6	− 9	g.
1030.....	1907 Apr. 20	18 20	Fox	8	−12	g.
1272.....	Dec. 4	23 29	B	8	+ 4	g.
1305.....	Dec. 30	22 5	S	8	+ 4	v. g.
1309.....	1908 Jan. 6	21 49	L	8	− 2	v. g.
1315.....	Jan. 7	20 15	B	8	− 5	v. g.
1455.....	Feb. 16	19 57	L	6	−10	g.
2297.....	1910 Mar. 7	16 53	L	7	−15	v. g.

Estimates made on the earlier plates several years ago led Mr. Frost to suspect a variation in the radial velocity of this star. He also observed a suggestion of a bright line fringe to the hydrogen lines β, γ, and δ. The value −17 km for No. 716 is the mean of two independent measures, which gave −16.5 and −18.1 km. Several plates have complex lines which were not resolvable into components in measuring. The spectrum is of type A.

23 Comae Berenices ($\alpha = 12^h\ 30^m$; $\delta = +23^\circ\ 11'$; Mag. $= 4.8$)

PLATE	DATE	G.M.T.	TAKEN BY	CENTER		VIOL. COMP.		RED COMP.		QUALITY
				No. Lines	Velocity	No. Lines	Velocity	No. Lines	Velocity	
					km		km		km	
IB 1934	1908 Dec. 26	$23^h 24^m$	B	7	− 1	2	−43	2	+36	v. g.
2243	1910 Jan. 5	21 30	L	6	−23	2	−78	3	+22	g.
2306	Mar. 14	19 20	B	14	− 7			1	+46	v. g.
2328	May 9	16 45	L	6	−14					w.
2362	June 6	14 34	L	9	−16	4	−46	4	+15	v. g.

The spectrum of this star is given as A. A number of metallic lines were used in measurement. On plates Nos. 1934 and 2243 the intenser component of the magnesium line λ 4481 is on the side toward the violet, while No. 2362 has the stronger on the side toward the red. No. 2306 has traces of close double lines.

2 η Coronae ($\alpha = 15^h\ 19^m$; $\delta = +30^\circ\ 39'$; Mag. $= 5.6$)

Plate	Date	G.M.T.	Taken by	No. Lines	Velocity	Quality
					km	
IIB 99....	1907 Mar. 29	$21^h 46^m$	B	20	−12.3	v. g.
IB 1021....	Apr. 13	20 54	Fox	13	− 8.0	v. g.
IIB 123....	May 27	15 22	F	21	−10.9	v. g.
IB 1544....	1908 Mar. 20	22 14	B	13	− 3.0	v. g.
1563....	Apr. 11	21 21	L	13	− 4.3	v. g.
1611....	May 2	19 33	B	14	− 6.2	v. g.
2383....	1910 June 20	15 7	L	7	−12.3	g.
2427....	July 18	14 48	L	9	−20.4	g.

This spectrum, of type G, has numerous good lines. The star is the well-known visual double 7251 in Burnham's *General Catalogue.* The components are 5.6 and 6.1 in magnitude. The orbit of the visual binary is fairly well determined, the period being about 42 years; hence an immediate determination of the orbit of the binary component is very desirable. A dispersion equal to that of the Bruce spectrograph with two prisms is preferable for this spectrum.

In 1902 *ι Serpentis* was found to be a close double by Hussey, who gives the distance as 0″.21. It is Hu 580 or 7360 in Burnham's *General Catalogue.* The components are of equal magnitude but the following data necessitate a binary nature for one or the other

21 *ι Serpentis* ($\alpha=15^{\mathrm{h}}\,37^{\mathrm{m}}$; $\delta=+20^\circ\,0'$; Mag.$=4.5$)

Plate	Date	G.M.T.	Taken by	Center		Viol. Comp.		Red Comp.		Quality
				No. Lines	Velocity	No. Lines	Velocity	No. Lines	Velocity	
					km		km		km	
IB 758	1906 May 11	$17^{\mathrm{h}}49^{\mathrm{m}}$	B	6	−45				+40	g.
1624	1908 May 11	18 25	L	6	−22					g.
2245	1910 Jan. 5	23 52	L			8	−30	6	+60	g.
2312	Mar. 18	20 52	L	6	− 6					g.
2319	Mar. 25	20 30	L			8	−19	5	+69	g.

of the visual components. The value +40 km for the red component of No. 758 is an estimate merely of some very faint, hazy lines.

γ Coronae ($\alpha=15^{\mathrm{h}}\,39^{\mathrm{m}}$; $\delta=+26^\circ\,37'$; Mag.$=3.9$)

Plate	Date	G.M.T.	Taken by	Center		Viol. Comp.		Red Comp.		Quality
				No. Lines	Velocity	No. Lines	Velocity	No. Lines	Velocity	
					km		km		km	
IB 291	1904 Mar. 8	$20^{\mathrm{h}}58^{\mathrm{m}}$	F	7	+13	4	−20			v. g.
1019	1907 Apr. 13	18 21	Fox	10	−14					v. g.
1044	Apr. 26	20 52	F	10	−15					v. g.
1577	1908 Apr. 13	18 28	B,L	7	− 8	5	−53	3	+29	g.
2321	1910 May 6	16 45	B	8	−17					v. w.
2428	July 18	15 52	L	4	−26	6	−80	6	+26	v. g.
2441	July 25	14 6	L	10	−23					v. g.

This is the close visual double star No. 7368 of Burnham's *General Catalogue*. See (*Evolution of the Stellar Systems*) gives its period as 73 years and its semi-major axis as 0″736, the plane of its orbit lying almost in the line of sight. The fainter component is of the 7th magnitude. The spectrum of the brighter component is classified as A, although the metallic lines are numerous and permit of considerable accuracy in measurement. When the spectroscopic components are visible they are of equal intensity. As in the case of *η Coronae*, an early determination of the orbit is desirable.

The spectrum of *π Serpentis* is classified as A2. Numerous metallic lines were used in measuring velocities. Peculiarly, the violet component seemed the more intense on all the plates. The

44 π Serpentis ($\alpha=15^{\rm h}\ 58^{\rm m}$; $\delta=+23^\circ\ 5'$; Mag.=4.8)

PLATE	DATE	G.M.T.	TAKEN BY	VIOL. COMP.		RED COMP.		QUALITY
				No. Lines	Velocity	No. Lines	Velocity	
					km		km	
IB 1995....	1909 Mar. 1	$21^{\rm h}15^{\rm m}$	L	12	−41	3	+35	v. g.
2270....	1910 Jan. 21	23 40	L	10	−15	2	+69	g.
2299....	Mar. 7	19 47	B	10	−62	10	+29	v. g.
2365....	June 6	17 22	L	8	−48	6	+38	v. g.

components were so definite that no attention was given to measurements of the centers of the lines.

62 γ Ophiuchi ($\alpha=17^{\rm h}\ 43^{\rm m}$; $\delta=+2^\circ\ 45'$; Mag.=3.7)

Plate	Date	G.M.T.	Taken by	No. Lines	Velocity	Quality
					km	
IB 721.....	1906 Mar. 30	$22^{\rm h}40^{\rm m}$	B	4	+ 2	v. g.
1095.....	1907 June 29	19 19	Fox	5	+11	g.
1098.....	July 1	16 0	B	8	−19	g.
2314.....	1910 Mar. 18	22 51	L	6	+35	v. g.
2358.....	May 30	19 59	B	8	+ 4	g.
2410.....	July 1	16 7	B	7	− 9	g.
2416.....	July 4	17 8	B	8	− 4	g.

The binary character of this star was early suspected by Mr. Frost. No. 721 shows five lines resolved into measurable components, the resulting velocities for which are −51 km and +92 km. The spectrum is of type A.

27 φ Sagittarii ($\alpha=18^{\rm h}\ 39^{\rm m}$; $\delta=-27^\circ\ 6'$; Mag.=3.3)

Plate	Date	G.M.T.	Taken by	No. Lines	Velocity	Quality
					km	
IB 38.....	1903 June 6	$20^{\rm h}19^{\rm m}$	A	1	+20	v. w.
362.....	1904 June 11	18 48	F, B	9	− 2	v. g.
1680.....	1908 Aug. 17	15 49	F, B	8	+39	g.
1685.....	Aug. 21	13 54	L	8	+32	v. g.
1690.....	Aug. 24	14 6	B	7	+26	v. g.
1699.....	Aug. 25	14 31	L	7	+32	v. g.
1708.....	Aug. 28	14 49	L	7	+36	g.
2021.....	1909 May 17	20 26	B	10	+17	v. g.
2436.....	1910 July 19	16 38	L	9	+34	v. g.

Mr. Frost on the basis of his preliminary measures announced ϕ *Sagittarii* as a spectroscopic binary at the meeting of the American Astronomical and Astrophysical Society at Put-in-Bay in 1908. This meeting was reported in *Science* for December 11, 1908. The individual measures have not before been published. The spectrum is of type B5A.

13 Vulpeculae ($\alpha = 19^{h}\ 49^{m}$; $\delta = +23^{\circ}\ 49'$; Mag. = 4.5)

Plate	Date	G.M.T.	Taken by	No. Lines	Velocity km	Quality
IB 761.....	1906 May 11	$20^{h}58^{m}$	F	8	−27	v. g.
2015.....	1909 May 10	21 38	B	9	−35	v. g.
2028.....	May 24	19 5	B	3	−15	w.
2054.....	June 18	17 16	M	8	−30	v. g.
2348.....	1910 May 23	21 10	B	9	−36	v. g.
2374.....	June 10	19 46	L	8	−36	g.
2406.....	June 27	18 25	L, M	8	−15	g.
2430.....	July 18	18 20	L, B	10	−26	v. g.

The spectrum of this star is of type A with a number of fairly sharp metallic lines. On No. 2054 the best two lines measured each have a faint component. The mean, referred to the sun, gives +119 km. No. 2406 also shows components at −51 km from four lines and +12 km from three lines.

16 Lacertae ($\alpha = 22^{h}\ 52^{m}$; $\delta = +41^{\circ}\ 4'$; Mag. = 5.5)

Plate	Date	G.M.T.	Taken by	No. Lines	Velocity km	Quality
IB 2110.....	1909 Aug. 23	$17^{h}1^{m}$	F	9	−43	g.
2178.....	Nov. 8	16 42	L	10	+12	g.

The spectrum is B3 and has many fairly good lines. On plate No. 2178 two lines show well two components, giving velocities of −12 km and +45 km. The *Ca* lines H and K are sharp, and accordant measures of the two give −15 km and −17 km, respectively, for the two plates. This indicates that the star may belong to the class of binaries having a constant velocity for *Ca* lines.

α Pegasi ($\alpha=23^h\ 0^m$; $\delta=+14°\ 40'$; Mag.=2.6)

Plate	Date	G. M. T.	Taken by	Center		Viol. Comp.		Red Comp.		Quality
				No. Lines	Velocity	No. Lines	Velocity	No. Lines	Velocity	
					km		km		km	
IB 2420	1910 July 4	$21^h 39^m$	L			9	−24	4	+61	v. g.
2425	July 8	21 38	L	7	+13					g.
2460	Aug. 1	17 56	L	5	− 3					g.
2469	Aug. 5	20 18	L	7	+ 9	2	−24	2	+60	v. g.
2474	Aug. 8	17 51	L	7	+28	1	−24	2	+80	v. g.
2480	Aug. 12	17 40	B	5	− 1					v. g.
2488	Aug. 19	16 5	B	5	− 3	2	−30	2	+54	v. g.
2491	Aug. 25	16 30	B	4	− 2					v. g.
2499	Aug. 27	16 48	L	6	− 7					v. g.
2501	Sept. 12	15 54	L	6	− 2					v. g.

α Pegasi has a spectrum of type A. Metallic lines are faintly present, but few are of any value for measurement. The K line of calcium has a peculiarly distorted double appearance, without showing measurable components, however, except on the first plate. Most of the spectrograms of this star were made on "Seed 23" plates.

Yerkes Observatory
September 30, 1910

THE ABSORPTION IN THE RED OF THE ACETATE, NITRATE, AND SULPHATE OF COBALT

By FRANK L. COOPER

INTRODUCTION

In a previous number of this *Journal*,[1] the writer published the results of an investigation on "The Absorption of Certain Salts in Aqueous and Non-aqueous Solutions." In this work the absorption spectra of several salts of cobalt, copper, iron, and neodymium were examined, and from the evidence obtained, the following conclusions were reached:

1. The absorption of light by the solutions studied was due to at least two different kinds of absorbers, one of which was the molecule of the salt, and another the atom of the metal itself.

2. The absorption spectra of these salts in solution depend to a great extent upon the solvent, which probably forms some simple hydrate.

It was observed that in the case of the solutions of cobalt bromide, cobalt chloride, and cobalt iodide several narrow absorption bands in the red region of the spectrum were characteristic of each salt; while the red was almost entirely transmitted by salts having oxygen as a part of the radical. In the more concentrated solutions of cobalt nitrate and cobalt sulphate, however, there appeared to be an absorption band which approached from the infra-red and absorbed a little of the light in the extreme red. For this reason, the present work was undertaken, using solutions of greater concentrations than those which were used in the previous work, to ascertain whether there were absorption bands which approached the visible spectrum from the infra-red, and whether these were also characteristic of the salt solution used.

APPARATUS, MATERIAL, ETC.

The spectrograph employed for this work was the same instrument which was used in the previous investigation. It consisted of a plane Rowland grating of about 14,000 lines to the inch which was mounted in a light-tight box. The absorption spectrum of each solution was also carefully examined by means of the small spectroscope. Both of these instruments were described in the previous paper. There were two cells used, so that the length of the absorbing layer was either 2 cm or 4 cm. The photographic plates were "spectrum panchromatic plates" obtained from Wratten and Wainwright of Croydon, England, and were sensitive as far as λ 8000 and very sensitive to about λ 7600. The plates were developed with dianol developer obtained from the Lumière Company.

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[1] *Astrophysical Journal*, 31, 4, 1910.

The source of light was a 220 volt direct-current Nernst lamp carrying a current of 0.5 ampere, a variable resistance being also placed in the circuit to maintain a constant current. For comparison, the spectrum of the iron arc with Norway iron electrodes was used.

The solutions were made up in the same manner as those used in the previous work. A chosen volume of mother-solution of a colored salt was measured out from a burette into a measuring flask of known capacity. The portion of the solution in the flask was then diluted by the addition of pure water until the volume of the resulting homogeneous liquid was exactly equal to the fixed capacity of the flask. The concentrations will always be expressed as multiples of normal. The term normal will be used to mean gram-molecular normal.

Through the courtesy of Professor J. S. Ames of the Johns Hopkins University, it was possible to measure the plates with the dividing engine belonging to the physical laboratory of that institution. This engine is fully described by Humphreys in the *Astrophysical Journal*, **6**, 180, 1897.

METHOD OF INVESTIGATION

The light from the Nernst lamps was allowed to pass through the cell containing the absorbing solution, and focused upon the slit of the spectrograph. After reflection from the grating, it was brought to a focus at the camera box.

Spectrograms were made both when aqueous and also amyl alcohol solutions of the salts were used as the absorbing media. The results were also verified by visual observations.

RESULTS

Aqueous solutions.—Solutions of the acetate, nitrate, and sulphate of cobalt were studied, the concentration of the solutions increasing from two-normal. In every case, it was found that with increasing concentration of the solution the absorption band in the green always approached the red end of the spectrum, as had been observed in the previous work. Moreover, in each case there was always an absorption band which slowly widened, coming from the infra-red, and gradually absorbed the lower part of the red region of the spectrum. These red absorption bands moved much more slowly, however, than did the band in the green, especially in the case of the more dilute solutions. The position of these absorption bands differs according to the salt solution used as the absorbing medium, but the exact position of their center could not be determined photographically, as they are in the infra-red. In the case of each of the solutions studied, the red absorption bands widen very slowly if the solution is dilute, but quite rapidly with solutions of greater concentration. Similar results were previously found in the case of solutions of cobalt bromide, cobalt chloride, and cobalt iodide. Therefore it is quite probable that these absorption bands are very similar to the red absorption bands of the bromide, chloride, and iodide of cobalt, and that the reason that they differ in position for different salts is because the absorber which is responsible for their existence is the molecule of the salt. On the other hand, the absorption band in the green was always found to begin in about the same position in the spectrum, about λ 5200, which is what we would expect if the absorber which produced this band was the atom of the metal itself. The width of this absorption band increased most rapidly, with increase of concentration, in the case of cobalt acetate, and least rapidly when a solution of cobalt nitrate was used as the absorbing medium. This shows that the absorber which is responsible for the green band cannot be the cobalt cation. Arranged in order of increasing dissociation, the salts would be acetate, sulphate, and nitrate. If the absorber responsible for the green band had been the cobalt cation, then this band should have been broadest in the case of the solutions in which there was the most dissociation, i.e., the solution of cobalt nitrate.

Below is given the approximate position of the limit of transmission for the green absorption band, in the case of a three-normal solution of the different salts studied:

Cobalt nitrate,	λ 5725
Cobalt sulphate,	λ 5850
Cobalt acetate,	λ 6150

The limits of transmission in the red corresponding to the above were approximately

Cobalt nitrate,	λ 7450
Cobalt sulphate,	λ 7340
Cobalt acetate,	λ 7270

Alcoholic solutions.—The absorption spectra of cobalt acetate, cobalt nitrate, and cobalt sulphate, when dissolved in amyl alcohol, were also studied. All of these, like the aqueous solutions of these salts, were found to have absorption bands both in the green and the infra-red. It should be stated, however, that some of the salts were not absolutely anhydrous. The positions of these bands were not the same as those in the case of the water solutions, as was found in the case of the ethyl and methyl alcohol solutions. This therefore confirms the conclusion that, in the solution, there is probably formed some simple solvate.

Below is given the limit of transmission in the red, as the concentration of the amyl alcohol solution of cobalt nitrate was increased by equal amounts as far as saturation:

λ 7380
λ 7360
λ 7329
λ 7312
λ 7279
λ 7212
λ 7109
λ 7014

SUMMARY

The results obtained with the solutions of cobalt acetate, cobalt nitrate, and cobalt sulphate confirm the conclusions reached from the previous work, viz.:

1. The absorption of light by the solutions studied was due to at least two different kinds of absorbers, one of which was the molecule and another the atom.

2. The absorption spectra of these salts in solution depend to a great extent upon the solvent, which probably forms some simple solvate.

Sheffield Scientific School
Yale University
November 1910

GIOVANNI VIRGINIO SCHIAPARELLI

MINOR CONTRIBUTIONS AND NOTES

GIOVANNI VIRGINIO SCHIAPARELLI

G. V. Schiaparelli was born in Savigliano (province of Cuneo in Piedmont) on March 14, 1835. Beginning his studies in Savigliano, he continued them in Turin, where he followed the courses in civil engineering, taking the degree in 1854. In the two following years Schiaparelli taught mathematics, and studied modern languages and astronomy. He felt greatly attracted to this science and was not diverted from it, although favorable opportunities in another kind of work were opened to him by someone who had discovered in the youth of twenty years an extraordinary talent, strong purpose, noble ideas, and faith in their possible execution, which led later on to great achievements. In 1857 the government of Piedmont sent him to study astronomy in Berlin and Pulkowa under Encke, O. Struve, and Winnecke. There he spent three years in researches in theoretical and practical astronomy, as well as in various other branches of science, as mathematics, geography, and physics. In 1860 he was called as second astronomer to the Observatory of Brera in Milan, and two years later succeeded Francesco Carlini as director. He remained in charge until 1900, when, after forty years of service, he asked to be retired. Thereafter he lived privately in Milan, until his death on July 4, 1910.

From this bare description of the life of Schiaparelli we see that two periods are prominent: one, of twenty-five years, from 1835 to 1860; the other, of fifty years, from 1860 to 1910. In the first, the man grows, studies, and prepares himself for life; in the second, he works up to his last breath, without a rest, and always for science. The history of the first forty years of this second period has been recorded by thirty-six Italian astronomers in the quarto memoir of 86 pages issued in 1900 on the occasion of his retirement from the observatory, and bearing the title, *All' Astronomo G. V. Schiaparelli—Omaggio—30 Giugno 1860—30 Giugno*

1900. The activity of the great astronomer during the forty years is well described in that publication, which gives for each year a list of his numerous contributions to science (in all, 236 numbers). The esteem in which he was held by his contemporaries is sufficiently indicated by the enumeration there given of the forty-eight learned societies which made him their associate and by the honorary degrees conferred upon him.

It is hard to believe that the lifetime of one man was sufficient to give the world all the various scientific productions of Schiaparelli. These are perfected in each detail, as condensed in thought as Latin classics, elegant in form, clear as to formulae and tables, neat as to drawings, easy and smooth in style. A single year of his activity would be enough to make an astronomer famous; and not only an astronomer, but also a geodesist, meteorologist, historian, or mathematician.

Entering as astronomer in Milan and having old instruments, the equatorial sector of Sisson and the meridian circle of Starke, each with a telescope of four inches, he discovers with the first the sixty-ninth asteroid, *Esperia*, and observes with the second the stars of the zone +6° and −2° of declination, before the Astronomische Gesellschaft had been organized for the observations of all zones of Argelander. He does not stop at his observations, but reduces and computes them, reaching results that increase our astronomical patrimony. In this way we have now the translation of the classic memoir of Encke on the determination of an elliptical orbit with three observations, and the star-catalogue for the equinox of 1870 observed and reduced by him and his successor, Giovanni Celoria (*Publications of Brera*, No. XLI).

The young astronomer was already well known in 1862, in spite of his great modesty and retired life. The clever men who at this time ruled, not only over little Piedmont, but also over new Italy, recognized the worth of the man and provided him with a better instrument, the well-known equatorial of Merz, 8 inches in aperture. Schiaparelli tries the instrument on *Mars* in 1877, and lays out great programs of work, all of which he accomplishes. Hence we have today in the first five memoirs on *Mars* printed in the *Lincei* and in his measures of double stars in the *Publications of*

Brera, No. XXXIII, a permanent memorial, increasing the field of knowledge in physical astronomy and in the astronomy of position. While accumulating observations at every moment of the clear nights, never resting; while he is preparing the material for future works, almost fearing the want of stones for his great edifice in the field of practical astronomy, he writes the famous letters to Father Secchi, and concludes his theory on the shooting stars which made his name famous. And he writes, almost as a relaxation from his persistent observing: *I precursori di Copernico nell' Antichità*, and *Le sfere omocentriche di Eudosso, di Callippo e di Aristotele.* He divines and proves the cosmical origin of meteors and their connection with the comets, giving a splendid demonstration of the genesis of meteoric swarms from the disaggregation of the comets. To the ancient Greek school he ascribed the first honor of the heliocentric hypothesis. He also corrects the view as to the homocentric spheres which had not been understood by the historians, whose knowledge of mathematics and language was not sufficient to explain those ideas, so that in ridicule they made of them the famous crystalline spheres.

Despite this great intellectual activity, Schiaparelli keeps in touch with the world and advances with it, finding joy in his family life and in the education of his sons. He maintains close association with his colleagues, with the academies, and with the famous men of his time, such as Secchi, Struve, Dembowski, Donati, De Gasparis, and Baeyer. With the last named, and with other members of the International Geodetic Association (of which he became a member in 1864) he co-operates for the advancement of the geodetic survey. His help and advice are freely accorded to all. W. Tempel is given a position in his observatory and then sent to Arcetri; Father Denza receives his effective co-operation in his propaganda for cosmic meteorology; the Observatory of Padua obtains the equatorial of Dembowski; Antonio Favaro has Schiaparelli as consulting co-operator in the national edition of the *Opere di Galileo Galilei;* C. A. Nallino, with the help of the Brera Observatory, is sent to the Escurial to copy in that library the only existing Arabic text of Al-Battānī, which copy is then printed in No. XL of the *Publications of Brera.*

Examples like these could be multiplied in great number. His latest service was given to the new Italian Astronomical Society, especially when the worthy astronomer of Teramo became its president. It is with emotion that we read his last work (the life of Ignazio Porro) in the *Review* of the society, in the number for July of this year. In the last days of May, while sending it to Vincenzo Cerulli, he foresees his end, and writes to him:

> A me preme di mostrare col fatto che, malgrado il mio stato di crescente infermità, io conservo sempre il desiderio di far cosa grata a Lei ed utile alla Rivista. Caro amico, questa orribile primavera mi ha assassinato: i miei acciacchi crescono: ai vecchi se ne aggiungono di nuovi: presto verrà per me il tempo di dire: Fuimus Troës. . . . .

To the skilful direction of Schiaparelli we owe collective works worthy of the greatest observatory, endowed with much more means than the one of Brera: for instance, the *Effemeridi astronomiche di Milano*, which were abandoned for the sake of other publications such as the *Pubblicazioni del Reale Osservatorio di Brera in Milano*, which began in 1873. With this great and continuous program are connected many special researches which, presented modestly before academies, or in scientific and popular reviews, treat of new facts deduced from his observations. Such are the rotation of *Mercury* and *Venus* in times equal to their sidereal revolution, the appearances of the rings of *Saturn*, the ellipticity of *Uranus;* or he writes to oppose wrong interpretations of phenomena which have not actually happened, as the change in color of *Sirius*, concerning which he sent to the Academy of Rovereto in 1896–1897 the two classic papers entitled "Rubra canicula."

Let us go back to his observatory in 1878 and we shall see that Schiaparelli, already made famous by his researches with the 8-inch telescope, asked the Italian government for the means for purchasing a more powerful instrument. His wish was hardly expressed before he obtained the approval by King Humbert of the act under which the Senate and the Chamber proposed to purchase and place in the Observatory of Brera a refractor of 18 inches aperture. The new instrument arrives in Milan in 1882, but difficulty in the construction of the dome delays the use of it until 1886, fourteen years before Schiaparelli leaves the observatory. The history of fourteen years of observations with the great Merz-Repsold refractor is recorded in two memoirs of the *Lincei* on *Mars*, the last of which has just now been published in 1910, and in the volume on double stars which is publication No. XLVI of Brera, for the year 1909. For many of the most interesting binaries the measures are repeated year after year, so that we have a valuable and almost continuous record, made by a single observer, through a considerable length of time, of the motion of those systems. The close of the introduction to this work reveals once more the man. There he remembers the help given to him for this publication with brotherly benevolence by his friend and colleague, G. Celoria, and calls Otto Struve and Ercole Dembowski his masters in the subject of double stars.

Schiaparelli's broad interest in humanity is illustrated by the following quotation from one of his letters, written in 1900:

> Science, which is now only in its beginning, has already brought many material benefits to humanity, but much greater will be the measure of all the good that it will be able to procure, especially for moral progress.

The life of the indefatigable man is declining, his sight is weakening, more and more, and he is obliged to discontinue observing, but still he does not rest after his retirement from official life. His broad culture is again emphasized when he writes in the *Manuali Hoepli* the history of the Hebrew astronomy ("L'astronomia nell' antico Testamento"), discussing the original texts written in that language. From the Arabians, Greeks, and Hebrews he goes to the Babylonians and writes for the review *Scientia* (1908) the two studies: "I primordi dell' astronomia presso i Babilonesi" and "I progressi dell' astronomia presso i Babilonesi." Remarkable is the conclusion of this last article, which condenses in few words the astronomical history of twenty-five centuries, from 600 B.C. to the present day. He writes:

> Concluding briefly, we will say that the true merit of the Babylonians was to establish in empirical form, with continuous observations and with the art of computing, the first basis of scientific astronomy. Starting from this point, the Grecians created geometrical astronomy, that is, the description of the arrangement and forms of the celestial movements. This found its cul-

mination and perfection in Copernicus and Kepler; after which Newton, starting from Galileo's principles of mechanics, showed how to deduce all the laws of these movements from the one physical cause, gravitation. This mechanical astronomy seems to have reached its highest point in regard to principles; but in the application there is a long way to go yet, since we have to study not only the planetary system, but also all the stellar systems: a formidable problem, the first lines of which are only now beginning to be laid down.

To this third stadium recently a fourth one has been connected—astrophysics. . . . .

Schiaparelli felt intuitively the great importance of this branch of astronomy, and although it was impossible for him, owing to his advanced age, to take an active part in its development, still he followed with the greatest interest the rapid progress of the new science. In his last days he spoke with enthusiasm of the new and powerful developments in his beloved science, especially appreciating the work of American astronomers, and admiring not only the great facilities they secured, but also the tenacity of their scientific purpose in making these powerful means effective.

Schiaparelli appreciated highly the visit made by Mr. Hale at his house in Via Fatebenefratelli, just about a year ago.

Honors and prizes were never sought by Schiaparelli, but he nevertheless received many of them. But so far as he was concerned, they would be unknown to others, since he thought he did not deserve them, holding it to be his duty to do all in his power without expecting any other reward. "Omni enim habenti dabitur" (Matt. 25:29). He had numerous civil, national, and foreign honors. He was made senator of the Kingdom of Italy in 1889, but he was not sworn in until nine years later, since he could not reconcile himself to the idea that he should have to make laws for men, while he was accustomed to say: "I know only very little those of the skies!" Three gold medals (Italian, English, and German) and two Lalande prizes from the Institut de France, in 1868, 1872, 1876, and in 1890, attest to the esteem in which he was held by his contemporaries. Another evidence of this was shown in the anxiety felt by the whole intellectual world during his short sickness, indicated by the Italian and foreign newspapers of the first days of last July. On the morning of July 4 his soul left his frail body: "Spiritus ubi vult spirat, et vocem ejus audis" (John 3:8). We hear and shall continue to hear his voice in his works.

He died on a day of brilliant sunshine, at a short distance from the Observatory of Brera, which he had made famous. He seemed to sleep in sweet resignation among his sons, relatives, and friends, in his bright and quiet room. The funeral services were simple, as requested by the deceased, who had no taste for ostentation, but they were solemn for the company of distinguished men of Milan and elsewhere, and for people of all classes. Now his mortal remains lie in a secluded place in the cemetery of Milan, but his spirit is sufficient to keep the mind of the learned world awake: he lives forever.

A.

ON THE TEMPERATURES OF STARS

The good feeling with which Professor Wilsing writes relating to my criticisms of the work of Wilsing and Scheiner, and the important corrections which he has determined, which go so far toward reconciling certain discrepancies, have pleased and interested me greatly. I am emboldened to venture a little farther, however, taking as my text the following quotation from Professor Wilsing: "From Mr. Abbot's remark, 'It seems misleading to compute temperatures from a spectral range of only $0.2\ \mu$,' it would seem that he has not considered the form of the equations of condition from which the temperatures are determined."

I had less in mind the mathematics of the matter than the march of the sun's energy-curve. Suppose that one observer should use photographic methods and determine by observations at five wave-lengths the form of the sun's energy-curve outside the atmosphere from $\lambda=0.3\ \mu$ to $\lambda=0.5\ \mu$, and another should use visual methods and determine it from $\lambda=0.5\ \mu$ to $\lambda=0.7\ \mu$. Let us suppose further that the results of each observer should agree with the mean results of our bolometric work on Mount Wilson and Mount Whitney, which I take roughly as follows from a curve to be given in a book on *The Sun* which I hope will be soon forthcoming:

	μ 0.30	μ 0.35	μ 0.40	μ 0.45	μ 0.50	μ 0.55	μ 0.60	μ 0.65	μ 0.70
Wave-length.....									
Intensity........	11.2	113	167	216	210	195	177	157	130

The supposed two investigators might proceed to determine the solar temperature by the formula

$$\log\frac{E_1}{E_2}=-5\log\frac{\lambda_1}{\lambda_2}-\frac{c\log e}{T}\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)$$

according to the method of least squares.

If I make no error their results would be (for $c=14,200$) $T=3932°$ and $T=6900°$, respectively. If the first observer should reject the observation at 0.30 μ and the second should reject the observation at 0.70 μ the results would be $T=5142°$ and $T=7056°$, respectively.

Keeping all the observations, the results differ by about 3000°, and keeping four of each set, they differ by about 2000°. In either case the divergence seems very large.

Professor Wilsing will perhaps rejoin that the first observer should have known better than to take his observations in a region full of Fraunhofer lines, and so, of course, he should. But if the sun had been a star of the third or fourth type, would not the second observer or even one observing from $\lambda=0.45\ \mu$ to $\lambda=0.65\ \mu$ encounter the same difficulty? On the other hand, do we know that the energy-spectra of stars of the first type are free from peculiarities of a similar tendency, though not of a similar cause? In other words, can we be sure that the region of spectrum $\lambda=0.45\ \mu$ to $\lambda=0.65\ \mu$ may be assumed typical of their general energy-distribution, so that it may serve to indicate their approximate temperatures? Indeed, may not the stars of the second type itself differ so much in the effects of their selective absorption and general scattering that in many of them the energy-distribution from $\lambda=0.45\ \mu$ to $\lambda=0.65\ \mu$ gives a false view of their temperatures?

It is with considerations like these in mind that I am led to believe that we shall do well to mistranslate the title of the valuable work of Wilsing and Scheiner, and to read "Determination of the

Spectral Distribution of the Visible Radiation" rather than "Temperature Determination."

C. G. Abbot

Smithsonian Station
Mount Wilson, Cal.
October 21, 1910

NOTE ON BELL'S PAPER ON STAR COLORS

I have read Mr. Louis Bell's paper entitled "Star Colors: A Study in Physiological Optics," in your issue of April last (**31**, 234–257, 1910).

It is true that in the first half of the last century, several double-star observers were fantastic in assigning colors to double stars; W. Struve and Smyth being perhaps the most so. The greater number of modern observers, including Burnham, Hussey, and Aitken, are most sparing in noting the colors of double stars. Thus the excess of fancy to which Mr. Bell justly objects is not a fault of today. Still, I venture to believe that the astronomer should record the colors of the stars he observes, but only on the modest "Chandler" scale varying from white (steely bluish) through yellow and orange to red, adding "b" for bluish and "B" for quite blue. It is probable enough that whenever "b" or "B" has been used by myself a subjective effect is present. I have never swept upon nor seen a decidedly blue, purple, or green isolated star. White (or what many people call steely blue), yellow, orange, and red, or mixtures thereof, are the only colors so found. To Mr. Bell's laboratory experiments we can add a celestial one which was produced by the near approach of the ruddy *Mars* to the yellow binary γ *Virginis* in March 1903. *Mars* was then 4 or 5 on Chandler's scale, γ *Virginis* was decidedly bluish (see *Cape Annals*, **2**, p. xc).

But all the cases presented by double stars will not fit in with a purely subjective theory. In the great cluster around x *Carinae* (the finest coarse cluster in the sky) there is one exceedingly red star and several yellowish stars, but there are no bluish or blue stars whatever. In the great cluster Δ 597 *Scorpii*, there are some yellow stars, but no bluish or blue stars. Thus, although a contrast of color is present in both cases, it leads to nothing. In the pair *α Centauri*, the brighter star is quite yellow, the fainter star is, however, yellower, and this is only a typical case. The pair *α Circini* has been described as follows:

	Bright Star	Faint Star
J. Herschel..........	White	Yellow
Russell.............	Faint Yellow	Orange
Innes...............	2 (yellowish)	7½ (decidedly reddish)

The 6″ faint companion to χ_3 *Fornacis* is an orange-red star. Again, many stars quite yellow have companions which are only slightly bluish, while some nearly white stars have very blue companions; thus the 5″ pair *h* 3891 has a white primary with a tinge of yellow, while the smaller star is decidedly blue.

R. T. A. INNES

TRANSVAAL OBSERVATORY
September 13, 1910

REVIEWS

The Scientific Papers of Sir William Huggins. Edited by SIR WILLIAM AND LADY HUGGINS. London: William Wesley & Sons, 1909. Quarto. Pp. xii+539, with photogravure portraits and many illustrations. 30*s*.

The publication of this splendid volume is justly a ground of general congratulation: to the distinguished author and his efficient coadjutor, that they have been permitted[1] to arrange in an appropriate, permanent manner these records of the devotion of a lifetime to science; to present and future workers in the field, that these pioneer researches of astrophysics are available for the consultation and study which their fundamental quality and historical importance will always require. The subtitle, "Publications of Sir William Huggins's Observatory, Vol. II," relates the work properly to its preceding volume, published ten years ago, *An Atlas of Representative Stellar Spectra from λ 4870 to λ 3300, Together with a Discussion of the Evolutional Order of the Stars, and the Interpretation of Their Spectra.*[2] That volume laid especial emphasis upon the reproductions of the spectra, many of them classical, and their discussion; this collects the published papers on the work done at the observatory in the half-century since its foundation in 1856. The papers are arranged by subjects, in eleven sections, generally following chronological order under each section. All the papers are reprinted as they appeared, in full, without alteration or omission. In some cases, where it appeared desirable to the editors, short notes are added. These, it is hardly necessary to say, lend an increased value to the original statements. The topics of the sections are as follows: I, Observatory and Instruments, 32 pp.; II, Spectra of the Fixed Stars, 68 pp.; III, Spectra of Nebulae, 90 pp.; IV, Motion in the Line of Sight, 41 pp.; V, New or Temporary Stars, 25 pp.; VI, Spectra of Comets, 39 pp.; VII, Sun and Corona, 54 pp.; VIII, Moon, Planets, and Aurora, 38 pp.; IX, Chemical Spectra, 64 pp.; X, Miscellaneous, 20 pp.; XI, Lectures and Addresses, 60 pp.

With this great range of research in new fields, it is not easy to select the chapters of greatest appeal and importance. The line of the reader's own work or interest will be to some extent controlling. It is evident that there is fundamental matter enough to cover most of the field of astrophysics. The very quality of the book makes it especially difficult to review, for the fascination of reading and re-reading some of these classical expositions of the progress of investigations in these unexplored regions of knowledge precludes the use of the reviewer's pen.

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[1] Written before the lamented decease of Sir William.

[2] Also published by Wesley. Folio, cloth. Price £1:5*s*., net.

However, the chapters on the application of spectroscopy to the nebulae, to the fixed stars, and to the sun, and that on motion in the line of sight, record the most significant of the Huggins' discoveries. There is a temptation to quote liberally from them, but that would involve far too much space: one would scarcely know how to stop. The volume should be read by all who are at work in this field, and in its borderlands physicists and chemists will find much to attract and interest them. The charm of the lectures and addresses will appeal to all caring for the history of science, but not desiring to follow all the details of the original papers.

There is a fine spirit of patience and calmness throughout the work of the author. He does not rush into print to anticipate his contemporaries. Thus twelve years intervene between the paper "On the Spectra of Some of the Fixed Stars" (in collaboration with Dr. W. A. Miller), 1864, and the first paper "On the Photographic Spectra of Stars," 1876. Part II of this last does not appear until 1880. He has meanwhile been working on the spectra of nebulae and other objects, of course; but his method is inductive, built upon personal observations slowly, painstakingly but not painfully, gathered with what now would be considered wholly inadequate instruments, in an unfavorable climate. We of the present day would consider it almost hopeless to derive positive information from the visual study of the feeble, flickering band of spectrum produced by a small spectroscope attached to an eight-inch refractor.

Sir William Huggins was always alert for new developments by other workers in his special science and its cognate branches, physics and chemistry; and his application of such new findings was always keen. His position in regard to his contemporaries was frank and generous, and rarely controversial. He frequently felt it necessary, however, to test by the most precise possible measurements the deductions of one of his fellow-pioneers, whose genius at times leaped the barriers of direct demonstration and found confirmation of theories in data of insufficient accuracy.

The volume is illustrated with the diagrams and maps accompanying the original papers, and is greatly embellished by excellent photogravures of the two authors.

The printing is excellent, as would be expected. The volume unfortunately could not well be made to match the style of Vol. I, which had to be larger to include on a sufficient scale the many reproductions of plates of spectra. But the series of publications of the Tulse Hill Observatory ends with these two noble books, which record in conveniently accessible, permanent form the original statements of some of the greatest advances in astronomy of the nineteenth century.

The teacher whose time may be too fully occupied with classroom work, or whose equipment may seem too meager, for personal work in research will derive from the repeated study of these volumes a new inspiration for his efforts, which cannot fail to impress his students afresh with the marvel of the revelations of the spectroscope in its astronomical applications.

E. B. F.

Acht Vorlesungen über theoretische Physik. Von MAX PLANCK. Leipzig: S. Hirzel, 1910. 8vo, pp. 127, figs. 5. M. 3.60; bound, M. 4.20.

Die Einheit des physikalischen Weltbildes. Von MAX PLANCK. Leipzig: S. Hirzel, 1909. 8vo, pp. 38. M. 1.25.

The publication of these lectures by one of the most stimulating and successful of the professors at Berlin makes them available to a far wider circle than that which was able to hear them—at Columbia University in case of the eight first named, and at Leyden in case of the last.

They are written in the clear and logical style of their author, and in print they add much to the agreeable impression received by their hearers. The topics of the Columbia lectures are as follows: (1) Reversibility and Irreversibility; (2) Thermodynamic Conditions of Equilibrium in Dilute Solutions; (3) Atomic Theory of Matter; (4) Characteristic Equation of a Monatomic Gas; (5) Electrodynamic Theory of Heat Radiation; (6) Statistical Theory of Heat Radiation; (7) Principle of Least Action; (8) Principle of Relativity.

The Leyden lecture deals with the unity which the modern physicist regards as basic in his conception of the universe. The author traces some of the lines along which this physical conception may be expected to develop in future. He concludes with an affirmation of the objectivity of the laws of nature.

The lectures are handsomely printed, the first set in Latin type, the pamphlet in that attractive variation of the Latin occasionally used by the Germans for semi-popular works.

E. B. F.

The Natural History and Scientific Book Circular No. 145. London: William Wesley & Sons, 1910. Pp. 120. 1*s.*

This catalogue of manuscripts, books, and pamphlets on astronomy is of permanent value. The 3641 numbers are classified and arranged in sections of a few pages each, astrophysics being represented by more than 600 titles, while stellar astronomy has nearly as many. About 750 deal with the solar system and 900 with spherical, practical, and theoretical astronomy. The remainder consist of general works and periodical publications and an unusually complete and interesting selection of the older writers up to the middle of the eighteenth century. The latter section appeals to the epicure on the lookout for a find; the seeker of a particular book may prefer a quick lunch on a single long list of authors alphabetically arranged.

While the plan of the catalogue necessitates some duplication of titles—for such books as the *Selenographia* of Hevelius must obviously appear in at least two places—it makes the book worthy of a modest space on the shelf by the side of Houzeau's *Vade Mecum.* It is neatly bound and is sold for a shilling.

R. W. WILLSON

Tables du Bulletin de la Société belge d'astronomie (1895–1909). By AUGUSTE COLLARD. Brussels: G. Van Oest & Cie., 1910. 8vo, pp. 123. Paper, Fr. 3.

This is a general index by subjects and by authors to Vols. I–XIV of the *Bulletin*—all that were published before the *Bulletin* was merged with *Ciel et Terre* in January of this year. It is the fourth of a series of twenty general indices of Belgian publications to be published by the *Association des archivistes et bibliothécaires belges.* As there are sixteen indices yet to appear, presumably following the same general plan, it may be of use to point out what seems to be a serious fault. The subject index does not give volume and page directly, but refers to the author index for them, and thus doubles the work of consulting it without any saving in space whatever. There is other evidence that the subject index has been less carefully treated than the author index. The appearance of the catch-words Manora, Paris, Rio Janeiro, Yale, etc., leads one to expect to find other observatories listed under Belgique, Harvard, Lick, Yerkes, but in vain; they, with others, are to be found only under "Observatoire." Now, if under the latter catch-word were listed *all* titles properly coming under this head, no great harm might be done. This is unfortunately not the case; see, for instance, Greenwich, Grenade, Meudon, and Uccle.

S. B. B.

Geographical Essays. By WILLIAM MORRIS DAVIS. Boston: Ginn & Co., 1909. Pp. 777, with 130 diagrams. $2.75.

At the close of Professor Davis' course of lectures at the University of Berlin last winter, a German professor was asked what sort of impression Professor Davis had made upon the Germans. He replied with great enthusiasm, "Prachtvoll!" The same term might well be applied to the impression produced by this collection of essays. The essays are especially adapted to the use of advanced students and teachers of physical geography, but their style is so clear, concise, and interesting that they furnish good reading for anyone, whether especially interested in geography or not.

The book is divided into two parts: Part I, "Educational Essays"; Part II, "Physiographic Essays." In the first part Professor Davis appears as a pioneer defining his subject in "An Inductive Study of the Content of Geography"; pleading for the introduction of physical geography into school, college, and university, in five essays; and dealing with the methods of teaching geography in four more.

In Part II are found some of the results of Professor Davis' researches. The three essays on "The Geographical Cycle" read like an interesting story. Such general subjects as plains, rivers and valleys, river-terraces, and glacial action are treated, and several essays are devoted to the geographical investigation of special regions ranging from "The Outline of Cape Cod" to "The Mountain Ranges of the Great Basin."

These essays are not new, but have been collected from twenty different sources and are now rendered more conveniently accessible in this one volume.

FREDERICK SLOCUM

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention is given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

Accuracy in the proof is gained by having manuscripts typewritten, provided the author carefully examines the sheets and eliminates any errors introduced by the stenographer. It is suggested that the author should retain a carbon or tissue copy of the manuscript, as it is generally necessary to keep the original manuscript at the editorial office until the article is printed.

All drawings should be carefully made with India ink on stiff paper, usually each on a separate sheet, on about double the scale of the engraving desired. Lettering of diagrams will be done in type around the margins of the cut where feasible. Otherwise printed letters should be put in lightly with pencil, to be later impressed with type at the editorial office, or should be pasted on the drawing where required.

Where an unusual number of illustrations may be required for an article, special arrangements are made whereby the expense is shared by the author or by the institution he represents.

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THE
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THE MAGNETIC SEPARATION OF ABSORPTION LINES IN CONNECTION WITH SUN-SPOT SPECTRA[1]

By P. ZEEMAN and B. WINAWER

1. As a consequence of the intimate connection between emission and absorption, there exists, closely corresponding to the magnetic separation of emission lines, a magnetic division of absorption lines. The dark lines which appear in a continuous spectrum, if a beam of white light traverses an absorbing flame, are divided and polarized under the influence of magnetic forces in exactly the same way as the emission lines. This correspondence between emission and absorption was shown to exist in some of the earliest experiments on the subject by one of the present authors. Our knowledge of emission spectra under magnetic influence has since been extended considerably. The experimental study of the inverse effect, i.e., the magnetic division of absorption lines, has however advanced less.

After the first experiments of the first-named of the authors of this paper, the change of absorption lines in a magnetic field was studied by König[2] and Cotton;[3] Righi[4] made an elaborate study of the subject, to which we have to return later on. It contains the only investigation of the magnetic effect in a direction inclined to the lines of force. Closely connected with our subject are finally some observations by Lodge and Davies[1] on the influence of a magnetic field on flames emitting "reversed" lines.

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[1] Paper presented in three parts to the Royal Academy of Sciences of Amsterdam. From the Proceedings of the Meetings of Jan. 29, Feb. 26, May 28, and June 25, 1910.

[2] *Annalen der Physik*, **62**, 240, 1897.

[3] *Eclairage électrique*, 5 et 26 mars 1898.

[4] "Sul fenomeno di Zeeman nel caso generale d'un raggio luminosa comunque inclinato sulla direzione della forza magnetica," *Mem. di. Bologna*, 17 Dicembre 1899.

The consideration of the inverse effect forms the basis of Voigt's magneto-optical theories;[2] and it is considered also by Lorentz[3] in his investigation of the magnetic separation in a direction inclined to the line of force.

Theory indicates different points, which may be tested by experiment. The inverse effect has become of supreme interest in solar physics, since Hale's[4] discovery that the dark lines of the sun-spot spectrum exhibit the characteristic phenomena of magnetic separation.

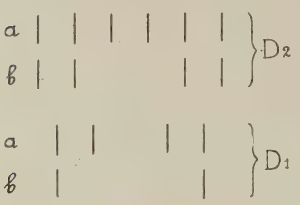

FIG. 1

The experiments we intend to describe in the present communication relate to the division of the sodium lines D_1 and D_2. Some of our results may already be found in the work of the authors cited, but in order to present the subject in a connected form it seemed necessary not to exclude these.

The facts now ascertained in combination with former results appear to be of some value in explaining peculiarities observed in sun-spot spectra. Some instances will be given later on.

2. Type and relative amount of the magnetic division of the sodium emission lines, D_1 and D_2, are given in Fig. 1.

a represents the observations when the line of sight is at right angles to the magnetic field, *b* when it is parallel to the field.

In a weak magnetic field, D_2 exhibits the triplet type at right angles to the field; the doublet type, if the light is examined parallel to the lines of force. D_1 seems to exhibit a doublet in both principal directions.

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[1] *Proc. Roy. Soc.*, **61**, 413, 1897.

[2] *Magneto- und Elektrooptik*, chap. iv and the papers there cited.

[3] *Proceedings of the Royal Academy of Sciences of Amsterdam*, **12**, 321, 1909.

[4] "On the Probable Existence of a Magnetic Field in Sun-Spots," *Contributions from the Mount Wilson Solar Observatory*, No. 30; *Astrophysical Journal*, **28**, 315, 1908.

The Fraunhofer lines in the spectra of sun-spots investigated by Hale are either broadened, or changed to doublets (often incompletely resolved quartets), or to triplets. The resolutions exhibited by sodium vapor are therefore the very types of special importance to astrophysics; this and also the facility of producing sodium vapor in the magnetic field induced us to commence our experiments with this substance.

3. The explanation of the inverse effect is easily understood by means of the well-known law of resonance. If there are in a flame under the influence of a magnetic field three periods of free vibrations, then we may expect that from incident white light, vibrations of just these three periods will be taken away. The absorption is a selective one, with the peculiarity that the selection refers not only to the period but also to the direction of vibration. Consider for example the central component of a triplet which in the emission spectrum is due to vibrations parallel to the field. From incident white light, only vibrations corresponding as to period, as well as to direction of vibration with the middle component, are absorbed. Vibrations perpendicular to the field, though of the period of the unmodified line, pass unimpeded. On the contrary, white light of periods coinciding with those of the outer components is deprived of its vertical constituents only.

It will be clear from these very simple considerations what we may expect to observe with white light under the conditions of the experiment. The arrangement was the following: White light of the incandescent positive pole of an arc lamp traverses a sodium flame, placed between the poles of a Du Bois electromagnet. This light is analyzed by means of a stigmatic spectroscope having a large Rowland grating. The observations are made in the first order.

If the observation is made at right angles to the lines of force, we see in the continuous spectrum four dark components in the case of D_1, six dark components in the case of D_2, as represented for both lines under *a* in the diagrammatical Fig. 1. In order to observe all these components the field must be strong and the vapor-density adapted to the field. The groups of lines indicated by *b* are seen, if the light is examined axially. All these components, if narrow, are seen only diffuse and not black. From the considerations above given the reason will be clear at once: each of the components absorbs only *half* the incident natural light. With very diluted vapor no absorption at all or only very weak traces of absorption are seen.

4. The introduction of a nicol in the beam, before or after the field, entirely changes the phenomenon. The absorption lines can then be seen very narrow and black. Let the observation be made at right angles to the horizontal field, then, if the nicol is placed with its plane of vibration vertical, D_1 exhibits its two, D_2 its four outer components. After a rotation of the nicol through 90° both D_1 and D_2 give only the two horizontally vibrating components.

Let a beam of natural white light traverse axially the magnetized vapor placed between the perforated poles of an electromagnet. Then by means of a quarter-wave plate and a nicol we may quench either the right-handed or the left-handed circularly polarized component.

A combination of a quarter-wave plate and a nicol, converting incident light into right-handed circularly polarized light, may be called a right-handed circular analyzer. The absorption line corresponding to a right-handed circularly polarized component is seen with both increased clearness and darkness by examining it with a right-handed circular analyzer. We introduce this simple matter here because there has been occasionally some confusion on this subject.

5. The behavior of horizontal and vertical vibrations may be studied simultaneously by using a calc-spar rhomb, according to the suggestion of Cornu and König. By means of it we can obtain two oppositely polarized images of a horizontal slit of suitable width, placed near the magnetic field. Right-handed and left-handed circular vibrations can be separated on the same plan by the introduction of a Fresnel rhomb between the calc-spar and the slit of the spectroscope.

It is, however, of considerable interest to examine also the behavior of the lines in natural light. A separate examination after the removal of the polarizers might be made. The vapor-density ought to be the same in both experiments. It seems difficult to realize this, in practice.

The desired end is secured more simply and surely, and with only half the labor, by adopting the width of the horizontal slit and the thickness of the calc-spar in such a manner that the two images given by the calc-spar partially overlap. We now obtain three strips; the central one exhibits the phenomena as seen without polarizing apparatus (see Fig. 2). The upper and lower strips show the influence of polarized light on the phenomenon.

The observations given in this communication have been made by the method described. By its use all particulars of the phenomenon are simultaneously exhibited; we also succeeded in photographing the essential points. Examples of our photographs are given on the accompanying plates.

6. If the absorption lines are not narrow or if the magnetic field is weak, the components of a magnetically divided line will partially overlap. This partial superposition is the cause of some peculiarities, especially manifest in the inverse effect and probably also apparent in sun-spot spectra. The nature of these peculiarities may be illustrated by a few examples. We will consider the case of the magnetic triplet and the magnetic doublet.

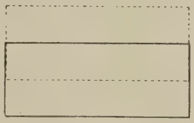

FIG. 2

In Fig. 3 the curves show the distribution of intensity of the three components of a triplet, if the light is examined at right angles to the lines of force. If natural light traverses a source of light placed in a magnetic field, two black bands are seen, corresponding to the wave-length for which vertical as well as horizontal vibrations are absorbed. These black bands are surrounded by less dark parts, which absorb only one of the principal vibrations, the other proceeding unimpeded (see paragraphs 3 and 4).

If a nicol, with its plane of vibration vertical, is now introduced, two black bands are again seen. The darkest part of these components corresponds to the maximum of the curves relating to vertical vibrations. As a general rule the distance of the components exceeds that of the lines first considered.

7. Parallel to the lines of force a partial, not too small, overlapping of the components produces a black line limited by two less dark parts. This case is illustrated diagrammatically in Fig. 4.

The two components may be separated by a circular analyzer. These considerations may be applied to the magnetic division in sun-spot spectra; as a general rule we may expect that the separation of lines in spot spectra becomes more distinct and of larger amount by the use of analyzers.

The introduction of a nicol in the beam may also reveal lines invisible without analyzer. Several peculiarities observed in the distribution of intensity in spot lines remind one of the superposition phenomena now specified;[1] see 19 below.

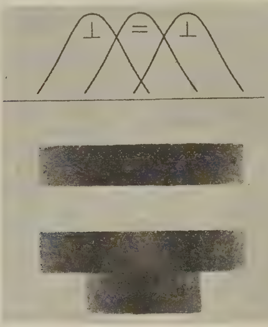

FIG. 3

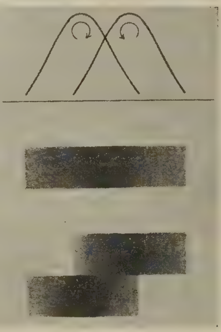

FIG. 4

8. Superposition effects of nearly, though not exactly, the same nature occur if lines with the same direction of vibration are superposed and if the continuous source of light emits unpolarized light. In the more complicated divisions the superposition now specified occurs also. It is just possible that the superposition of the outer components of the sextet, type D_2, produces only dark, that of the inner and the next outer components, black lines in the continuous spectrum. It is easily seen that also in the case of the quartet, type D_1, black lines may be produced. The darkest parts may be seen somewhat nearer to the middle of the complete figure than the outer components of the quartet. It seems unnecessary to illustrate

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[1] A figure equivalent to the one now given concerning the influence of superposition of magnetically divided components was already drawn for *emission* lines in Zeeman's "Doublets and Triplets in the Spectrum Produced by External Magnetic Forces," *Phil. Mag.*, **44**, 55, July 1897.

this by figures. Examples of the actions specified will be given presently.

9. Our observations and spectrograms also relate to the two principal directions (parallel and at right angles to the lines of force), and to directions inclined to the field.

In the present, first, communication, observations are discussed, relating to five different angles between the field and the direction of propagation of the beam (Voigt's ϕ, Lorentz' θ). These values are: 90°, 0°, 60°, 45°, 36°–39°. The results of the work relating to these angles have been recorded on nearly one hundred spectrograms.

10. Observations perpendicular to the field. In the upper of the three strips which are present in the field of view (see 5), the light vibrates vertically; in the lower one, horizontally, whereas the middle part relates to natural light.

Under the influence of the magnetic field we therefore see the vertically vibrating components as narrow black lines. The quartet of the D_1 line, the sextet of the D_2 line may be seen very clearly by this method. A small disturbance is produced by the narrow reversed lines due to the electric arc-light. The intensity of these lines depends upon somewhat variable circumstances of the arc itself. In some cases these lines are almost invisible, in other cases more prominent. They are to be seen on some of our reproductions; with our present subject they have nothing to do.

As regards the central strip we refer to the remark previously made, that the image of the separation must become rather indefinite, and weak (paragraph 3), because the absorption is only partial. The partial superposition of components gives, at least in the case of diluted vapor, the most conspicuous lines (6 and 7). In the case of the quartet, for example, one sometimes sees, instead of four, only two components, situated between the inner and outer ones. We made experiments with different vapor-densities. The observed phenomena may be classified under three phases:

1) The vapor is *very dilute.* The components are clearly visible in the upper and lower strips. In the central strip the absorption is either hardly perceptible (Plate XVI, Fig. 1) or the components of the quartet and the sextet are seen as separate, but weak lines (Plate XVI, Fig. 2). In this phase of the phenomenon

the great difference of definiteness of the central and outer regions is very remarkable. This contrast is still more marked with eye observation. In order to obtain good photographs, it was necessary to increase the density of the vapor above the one required for the observation of the very first trace of absorption.

2) Vapor of *intermediate* density. The components in the upper and lower strips are now no more separately visible, or only in the case of the quartet. In the central strip a superposition of the kind mentioned in 6 takes place. In place of the quartet an apparent doublet is seen, the components of which are situated between the outer and inner components of the quartet. This case is very clearly represented in Plate XVI, Fig. 3. The phenomena exhibited by the sextet (D_2 line) become rather complicated. The superposition phenomenon is often very distinct. The D_2 line on Plate XVI, Fig. 3, shows the appearance sufficiently.

3) With still *denser* vapor, the components become very broad and the magnetic change hardly visible. The *polarization of the edges* of the broad line may be recognized. This phase is represented in Plate XVI, Fig. 4. It corresponds to the emission effect as it was first discovered: a slight change of broad lines in a weak field. With still greater absorption the influence of the field becomes imperceptible. All these phases appear with great regularity. If the intensity of the field is known, it seems possible, the resolving power of the spectroscope being given, to deduce the density of the vapor from the nature of the observed phenomena.

The phenomena of magnetic division hitherto observed in sun-spots appear to fall under the second and third phases above mentioned. From measurements of spot lines, compared with laboratory experiments, Hale deduces a maximum intensity of the spot field of 4,500 gausses. Hence one would be inclined to think that the density in the layers which bring about the absorption in the sun-spot spectrum can only be small. Moreover, the non-uniformity of the field of sun-spots produces by itself a widening of the components. Light from a limited portion of the spot would give perhaps very narrow spectral lines. In view, however, of the critical remarks of Kayser[1] concerning our knowledge of the

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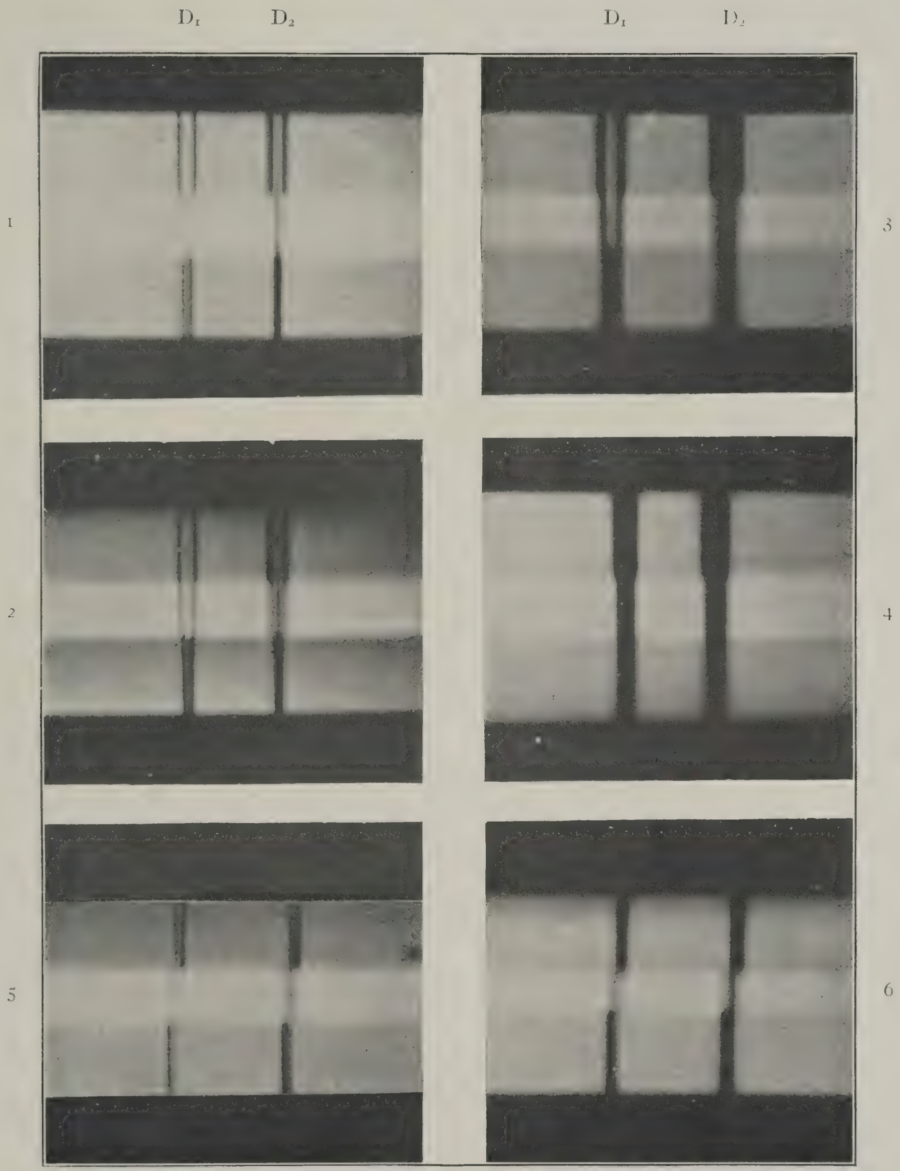
[1] Kayser, *Handbuch der Spectroscopie*, **2**, Kap. v.

PLATE XVI



The Magnetic Separation of Absorption Lines

influence of pressure and of temperature on spectra, all such considerations must be put forward with great diffidence.

11. Observations parallel to the lines of force. In the present experiment the absorbing vapor subjected to magnetic forces is placed between perforated poles.

After putting on the current, one sees in the continuous spectrum two dark bands in the case of D_1, four in the case of D_2, according to the diagrammatical Fig. 1. The absorption is incomplete also now, because for some wave-lengths only the right-handed circularly polarized light is absorbed and the reverse. In order to observe the separation and the polarization a Fresnel rhomb is placed with its principal plane at an azimuth of 45° with the horizon, a horizontal slit being placed in one of the perforated poles. The Fresnel rhomb converts circularly polarized light into plane-polarized light. By means of a calc-spar rhomb three strips are now also obtained. The first phase (very dilute vapor) is represented in Plate XVI, Fig. 5.

Vapor of intermediate density (second phase) exhibits the superposition phenomena mentioned in 7 and 8 and diagrammatically illustrated by Fig. 2. In the central strip *one* line, at the position of the unmodified one, surrounded by feebly absorbing regions, is seen. Plate XVI, Fig. 6, shows these lines for the doublet and the quartet; especially with D_2 the effect is very marked.

12. Observations in directions inclined to the field. According to Lorentz' elementary theory of magnetic division one generally observes in a direction oblique to the lines of force by an angle θ a triplet with elliptically polarized outer components.[1]

The ellipse, which characterizes the state of polarization of the components with period T_0+v, is the projection on the wave-front of the circle perpendicular to the field in which the electron with period T_0+v is moving. v is a small quantity. The direction of the motion of the moving electron also determines the motion in the ellipse. The ratio of the axes is as 1 to cos θ. For the other outer component with period T_0-v, the same reasoning holds, *mutatis mutandis*. The central line with the unmodified period T_0 always remains linearly polarized. The vibrations of the middle component are in the plane determined by the ray and the line of force and the amplitude of the vibrations is proportional to sin θ.

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[1] Cf. Righi, *op. cit.*

If we put $\theta = 0$, i.e., in the case of the longitudinal effect, only circular motions remain.

All this applies to very narrow spectral lines in a strong field, the distance of the components being much greater than their width. According to Voigt and Lorentz we must expect some interesting peculiarities if this restriction be discarded. We return to this point later on. As a general rule the deductions from the elementary theory are verified. Also in the case of the quartet and the sextet the outer components become elliptically polarized, as has been observed already by Righi.[1] In contradiction with the elementary theory, though not strictly applicable to the case, is the very slight diminution of intensity of the middle components of the quartet even for $\theta = 45°$.

13. Observations at $\theta = 60°$. If the observation is made with a calc-spar rhomb, the image remains as with the transversal effect. Yet the presence of elliptic polarization ought to manifest itself by the appearance in the lowest stripe of lines, corresponding to the outer components. With very dilute vapor and with that of intermediate density, practically no trace of it is seen. Plate XVII, Fig. 7, shows the first phase with dilute vapor, Fig. 8 the second phase with denser vapor. Only traces of absorption, indicative of elliptic polarization, can be seen near D_2, Fig. 8. The ellipticity is, however, undoubtedly proved by means of the Fresnel rhomb, placed with its principal plane at an azimuth of 45° with the horizon. Fig. 9 shows the appearance.

The outer components of the quartet toward the red or toward the violet, dependent upon the stripe and the direction of the field, are now considerably weakened; in the case of the sextet they have vanished altogether. All this proves the elliptical polarization of the outer components. For, if the polarization were linear, as might be inferred from observations with the calc-spar alone, then the observation with calc-spar and rhomb combined ought to show

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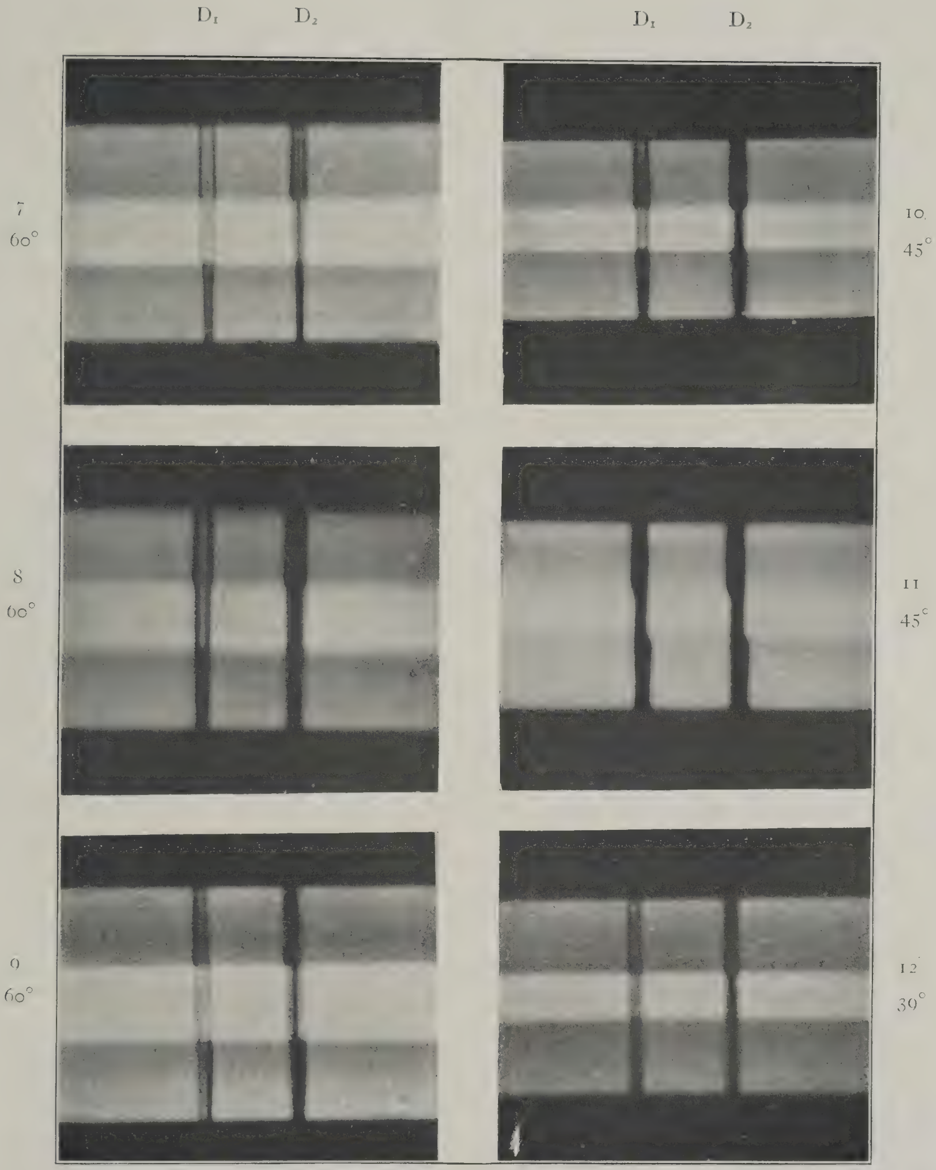
[1] Righi's observations (*op. cit.*) all refer to an angle of nearly 55°, the angle at which according to the elementary theory the three components of the triplet are of equal intensity.

PLATE XVII



The Magnetic Separation of Absorption Lines

no difference between the highest and lowest stripe. The light of all plane-polarized components would issue circularly polarized from the rhomb, and, the calc-spar making no selection between right-handed and left-handed polarizations, the components toward red and toward violet would all be alike. Such a condition is disproved by photographs such as Fig. 9, Plate XVII.

14. One point must be considered somewhat more in detail. What is the reason that the ellipticity is not shown by the calc-spar rhomb alone, whereas its existence is most clearly demonstrated by means of the Fresnel rhomb?

Let an elliptic vibration with vertical axis b, horizontal axis a, be incident upon the rhomb, the principal plane of which is at an azimuth of 45°. It is easily proved that the elliptic vibration issuing from the Fresnel rhomb has its axes in the same direction as the original motion and a ratio of the axes $\frac{a_1}{b_1}=\frac{b-a}{b+a}$, the original ratio being $\frac{a}{b}$. If a be small in relation to b (an elongated ellipse), then the light issues from the Fresnel as a more circular vibration, which is more easily analyzed. It depends upon the magnitude of a whether $\frac{a}{b}$ is *greater* or *less* than $\frac{b-a}{b+a}$.

We distinguish the following cases:

(1) a very small, then $\frac{b-a}{b+a}>\frac{a}{b}$.

(2) $a=0.414\ b$, then $\frac{b-a}{b+a}=\frac{a}{b}$.

(3) $a>0.414\ b$, then $\frac{b-a}{b+a}<\frac{a}{b}$.

We shall apply these results to the interpretation of our observations. Two cases dependent upon the magnitude of a are of principal importance.

In the first case we can observe the effect of both the axes of the ellipse by means of the combination of the Fresnel rhomb and the calc-spar (*this is the case of the quartet*) (D_1, Fig. 9), whereas without Fresnel rhomb no effect of the small axis is visible. In the second case the effect of the small axis becomes apparent by the use of the calc-spar, whereas its existence cannot be demonstrated with the Fresnel, the value of $\frac{b-a}{b+a}$ being too small. *This case is represented by the sextet* (D_2, Fig. 9, Plate XVII).

If the observation is made by means of the calc-spar rhomb, we indeed see with dense vapor new components in the lowest strip (see Fig. 8, D_2). The theoretical import of this result will be discussed on another occasion. After introduction of the Fresnel rhomb the component to the left of the central line (small axis of the ellipse) remains invisible (Fig. 9, D_2, lower strip). Hence we may conclude that at the angle now investigated, the ellipticity of the outer components of the *sextet* (the ratio $\frac{a}{b}$) exceeds that of the quartet (and is also larger than 0.414).

15. Observations at $\theta=45°$. The photographs taken with the calc-spar alone show very clearly the ellipticity of the outer components. With vapor of intermediate density the phenomenon is already very marked, especially in the case of D_2 (Plate XVII, Fig. 10). Very remarkable is the slight diminution of intensity of the inner components of the quartet. According to the elementary theory the intensity of the central component of *a triplet* ought to have diminished already to less than *half* the original value.

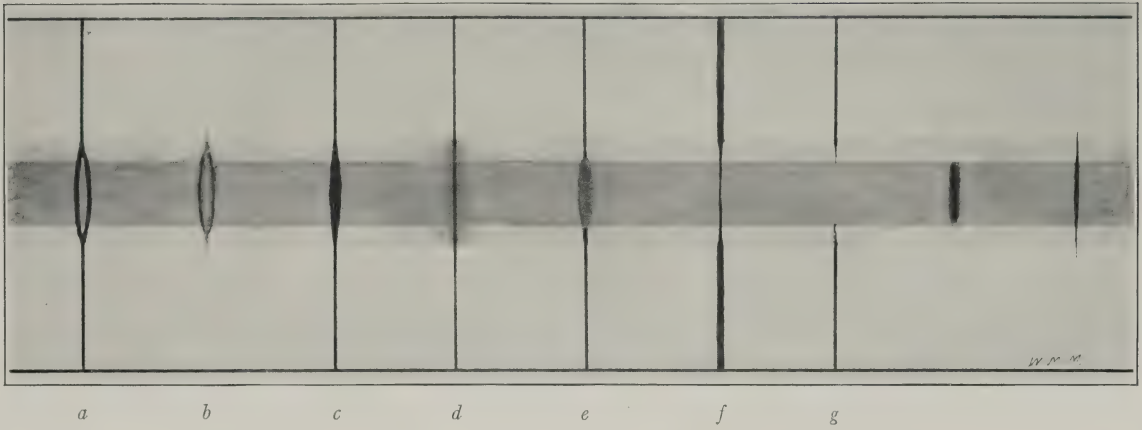
16. If a Fresnel rhomb combined with a calc-spar rhomb is introduced in the beam, one of the components of the quartet also entirely disappears. At an angle of 60° this was the case only with the sextet (Plate XVII, Fig. 11).

17. Observations at $\theta=39°$. The elliptic polarization tested by means of the calc-spar rhomb is very marked, even with dilute vapor (Plate XVII, Fig. 12, Plate XVIII, Fig. 13). The inner components of the quartet are now decidedly less intense than the outer ones. Plate XVIII, Fig. 13, especially shows the smaller intensity of the components of D_1 in the lower strip. Indeed, they are unmistakably thinner than those in the upper strip.

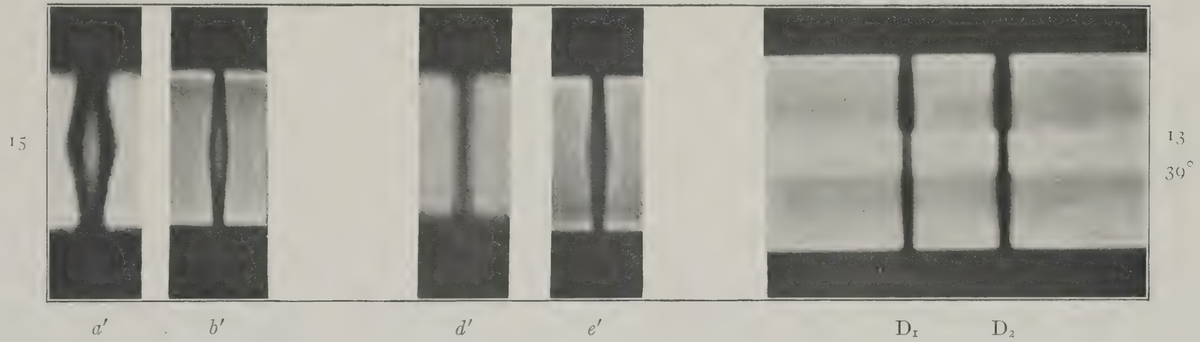
18. According as the angle between the ray and the lines of force is diminished, the intensity of the field must diminish at the

PLATE XVIII

14. Types of Sun-Spot Lines (W. M. Mitchell)



a, *b*, widened lines with centers reversed bright; *e*, widened and weakened line; *d*, winged line



a′, *b′*, *e′*, types of magnetic resolutions in non-uniform fields; *d′*, superposition of magnetic components

same time. In order to make it possible for the rays to traverse the field under smaller angles the vertex semi-angle of the cones must deviate more and more from the theoretical optimum of nearly 55°. The decrease of the magnetic separation is clearly shown in our photographs. We intend to communicate on another occasion experiments under smaller angles θ and to enter upon some details concerning the case in which the components of the triplet are not neatly separated. Some measurements of the ellipticity of the components will also be given. On the present occasion we intended to give only a general survey of the inverse effect, illustrating it by some particular cases.

19. Types of separation in spot and laboratory.—In one direction we shall now enter upon some more details. The magnetic separation of lines in a *non-uniform* field has been treated on a former occasion.[1] The results then obtained and our present observations may be of some interest in connection with certain phenomena observed by Hale. We intend to return to this subject. It seems of interest to allude presently to W. M. Mitchell's descriptions of the various types of spot lines as indicated in the diagram published in the *Astrophysical Journal*, **22**, 6, 1905, and copied with some modifications in the *Transactions of the International Solar Union*.[2]

Our Plate XVIII, Fig. 14, has been copied from Mitchell's paper. The types *a*, *b*, *d*, and *e* of the figure are very characteristic. Type *g* perhaps falls under the type of lines invisible without nicol, mentioned in 7 above. In Fig. 15 are represented some separations observed in the laboratory *without* nicol or other analyzer; a', b', e' have been taken in non-uniform fields. b' is the quartet of D_1 observed across the field; a' the sextet of D_2 observed axially in a non-uniform field, very strong in the central part; e' also refers to D_2 in a weaker field, the observation being made across the lines of force. The type d' refers to the D_2 line, when observed in a direction parallel to the field. The field is uniform. The separation gives an example of the superposition phenomenon mentioned in paragraph 7.

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[1] Zeeman, *Proceedings of the Amsterdam Academy*, April 1906, November 1907.

[2] *Transactions of the International Union for Solar Research*, **2**, 199, 1908.

The analogy of the type *d′*, Fig. 15, and the type of the "winged line" seems very remarkable. Of course observation of the state of polarization would be necessary in order to prove the analogy.

20. Explanation of Plates XVI–XVIII. Figs. 1–13 are about thirteen-fold enlargements of the images given by the grating of the absorption lines D_1 and D_2 in a magnetic field.

The upper and lower of the three strips of these figures relate to (oppositely) polarized light; in the central strip the phenomenon is represented as it is seen in natural light.

Plate XVI, Figs. 1, 2, 3, 4, observations perpendicular to lines of force with different vapor-density. Figs. 5, 6, observations parallel to lines of force with different vapor-density.

Plate XVII, Figs. 7, 8, observation at $\theta=60°$, calc-spar rhomb alone. Fig. 9, $\theta=60°$, calc-spar combined with Fresnel rhomb. Figs. 10, 11, $\theta=45°$. Fig. 12, $\theta=30°$.

Plate XVIII, Fig. 13, $\theta=39°$. Fig. 14, types of sun-spot lines (adopted from Mitchell). Figs. 15, *a′*, *b′*, *e′*, separations in non-uniform laboratory fields; *d′* superposition phenomenon, in paragraph 7.

21. The outer components of a magnetically divided line, if observed in a direction inclined to the lines of force under an angle θ, are elliptically polarized. In our experiments of paragraphs 12–17 we frequently referred to this elliptical polarization. In paragraph 12 the simple rules were summarized which relate to the ellipses characterizing the state of polarization of the outer components, if *very narrow* spectral lines are observed in a *strong* field. The linear vibrations of the central component of a triplet lie, according to the elementary theory, in the plane passing through the ray and a line of force, and the amplitude is proportional to $\sin\theta$. Righi's theoretical considerations in his paper cited in paragraph 1 also agree with this conclusion.

22. In Voigt's[1] theoretical investigation of the magnetic effect in a direction inclined to the lines of force, the remarkable conclusion is drawn that the central component also of a triplet may execute an elliptical vibration. This result is most closely connected

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[1] "Weiteres zur Theorie der magneto-optischen Wirkungen," *Annalen der Physik*, 1, 389, 1900.

with the consideration of the mutual action between neighboring molecules.

Lorentz' considerations concerning our present subject (see paragraph 1 above) give results which we may be permitted to summarize here briefly. For arbitrarily chosen values of the angle θ between the ray and the magnetic force for every frequency two elliptical vibrations of opposite directions can be transmitted. In the case of the *outer* components of a sharp triplet one of the two elliptic vibrations is absorbed. If we are not dealing with a sharp triplet, i.e., three absorption bands that are completely separated, we can still say something about the vibration-ellipses of the outer components. Let axes OY and OX' be chosen, the one normal to the plane passing through the ray and the magnetic force, the other perpendicular to the ray and lying in the plane just mentioned. Then one of the characteristic vibration-ellipses can be considered as the reflected image of the other with respect to a line bisecting the angle $X'OY$. This rule also applies to the direction of motion in the two ellipses.

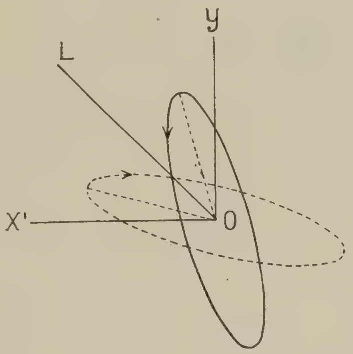

FIG. 5

The nature of the phenomena that will be observed for rays of a frequency corresponding to the *central* line of the triplet depends upon the value of θ being greater or smaller than a certain angle θ_1. This latter is determined by the equation

$$\tan \theta_1 \sin \theta_1 = \frac{g}{\nu}.$$

The quantity g may be regarded as a measure of the width of an absorption line and depends upon the constants of the vapor; ν is determined by the change of the frequency of the free vibrations of the electrons and has a value proportional to the strength of the field.

If $\theta > \theta_1$, then two linearly polarized beams with equal indices of refraction and different absorptive indices can be propagated. The rectilinear vibrations make equal angles with the line *OL*, bisecting the angle $X'OY$. The absorption is stronger for the beam whose vibrations make the smaller angle with the direction of the field. In the figure the more strongly absorbed vibration is indicated by a thicker arrow. As θ decreases, the vibrations of the two principal beams approach more and more to *OL*, so that for $\theta = \theta_1$ both directions coincide with the bisectrix. The two principal beams are now equally absorbed also.

When $\theta < \theta_1$, the state of things is wholly different. In this case two elliptically polarized beams can be propagated; they are equally absorbed, but have different velocities of propagation. For both beams the characteristic ellipses are the same, but described in opposite directions. One of the axes of the ellipses coincides with the line *OL* in Fig. 6. The ellipses become less and less eccentric as the wave becomes less inclined to the direction of the field. For $\theta = 0$ the ellipses become circles described in opposite directions. A further approximation for $\theta = \theta_1$ shows that in this case the two vibrations do not coincide exactly. As in the general case, there are two distinct beams with different characteristic ellipses, both deviating somewhat from the line *OL* of Fig. 6. The regions of the longitudinal and the transverse magnetic effect overlap to a certain extent and are not sharply separated from each other at the angle θ_1.

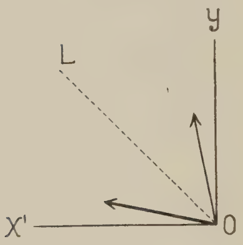

FIG. 6

23. There are three results of Lorentz' theory that probably admit of experimental verification. Let us imagine the absorbing vapor placed in such circumstances that the elementary theory cannot be applied. The components of a divided line are now not neatly separated by practically transparent regions. The vapor-density must be chosen relatively great and the magnetic intensity rather small. As always in the present paper, we suppose the lines of force to be horizontal; we examine the propagation of the light also in a horizontal plane.

The three predictions referred to, which apply, if we exclude the cases of the true longitudinal and transverse effects, are: (1) the major axes of the vibration-ellipses of the outer components deviate from the vertical line; (2) the vibrations of the middle component (c.q. components) are, depending on circumstances, either linear and not horizontal, or elliptic, the axes of the ellipse being inclined to the horizon; (3) there exists an angle θ_1 separating the regions of the longitudinal and the transverse magnetic effect.

OBLIQUE POSITION OF THE VIBRATION-ELLIPSES OF THE OUTER COMPONENTS

24. We succeeded in establishing experimentally the oblique position of the vibration-ellipses in the inverse magnetic effect of the D lines; the amount of the slope of the axes we could measure. The obliquity is far from striking. When θ was already such that the ellipticity was very marked, it was only after some difficulty that we could make sure of the obliquity. Some details of a definite case may be given. With $\theta = 69°$ and a field of about 18,000 gausses the first observations were made. Attention was given to D_2, the vapor-density being regulated so that the outer components of the sextet could not be seen separately. Before the slit of the spectroscope a nicol was placed with its plane of vibration at an azimuth of say 35° with the horizon. The central part of the resolution figure is now very dark; the outer components of the pseudo-triplet, however, are only faintly visible. This has the advantage of increasing the visibility of small changes of the intensity of the outer components.

The direction of the field we denote as field-direction 1. *With the reversed field-direction 2, the outer components became darker.* This experiment was repeated several times with the same result. The nicol then was placed in a position symmetrical to the one just mentioned. Now with field-direction 1, the outer components were darker. From these experiments we must conclude that a vertical line is not an axis of symmetry of the vibration-ellipses of the outer components, hence that the position of these ellipses is oblique.

25. The direction of the smaller axis of the vibration-ellipse we

measured for $\theta = 69°$, the vapor-density being between the first and second phase (13). In front of the slit of the spectroscope was introduced a nicol, mounted upon a divided circle which gives the rotation of the nicol in degrees. The vanishing or reappearing of the outer components gave a good criterion for the determination of the smaller axis and therefore of the major axis of the vibration-ellipse. The measurements gave the result that under the circumstances of the experiment the major axis made an angle of 5 degrees with the vertical. The obliquity was the same in amount and direction for the components toward the red and toward the violet. The diagram, Fig. 7, illustrates the relation between the slope of the ellipses and the direction of the field.

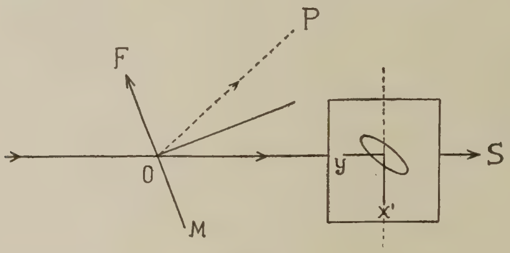

Fig. 7

Let *OS* be the beam, which traverses the source of light placed in *O*, and *OF* the direction of the magnetic force. For an observer looking in the direction *SO*, the upper part of the vibration-ellipse is inclined toward the right. The plane *YX'*, containing the ellipse, is normal to the ray and in the figure has been rotated round the dotted line until brought into coincidence with the plane *SOM*. That side of the plane which was visible from *S* can now be seen. Both the ellipse toward the red, and the ellipse described in the opposite direction toward the violet, have the same slope with a given direction of the magnetic field, as was remarked above.

26. *The same* sodium flame, investigated as to the inverse effect in the direction *OS*, we studied in the direction *OP* (i.e., for an angle $FOP = MOS = 180° - \theta$) for the phenomenon of partial polarization, discovered by Egoroff and Georgiewsky. A small telescope focused upon the flame was used and provided with a Savart plate

and a nicol. This polariscope is mounted upon a divided circle graduated in degrees. The direction in which the fringes were most brilliant was determined in order to detect a possible deviation of the plane of maximum polarization from the vertical. It was easily seen that there was such a deviation. The fringes were most clear if for the observer in *P* their direction was from the upper left to the lower right quadrant, the direction of the field being always as indicated in the figure. After reversal of the magnetic field the fringes became indistinct. They became distinct again if the principal direction of the polariscope was from the upper right to the lower left quadrant. The result of these observations at least proves that the whole phenomenon is asymmetrical with respect to the vertical, and hence proves the presence of oblique vibrations. In a conversation with one of the authors Professor Lorentz had kindly communicated that he observed phenomena of the kind described in this paragraph.

27. In the experiment of the last paragraph the axis of the telescope must be placed carefully in a horizontal plane passing through the poles of the electromagnet. If the observation is made in a plane which is not horizontal, an apparent slope of the axes of the vibration-ellipses becomes manifest, as is easily seen from a geometrical consideration.

28. The position of the plane of maximum polarization can be determined rather accurately. The obliquity of the major axis of the outer ellipses of sextet and quartet in one experiment was 5°; with the very same vapor-density and the same strength of field, the plane of partial polarization made an agle of 21° with the vertical. At first sight it seems rather startling that the polariscope of Savart is so sensitive to the obliquity of the ellipses.

The phenomenon of the partial polarization of the emitted light is very complicated and the complete theory still outstanding. It does not seem doubtful, however, in what direction we have to look for the explanation of the remarkable difference between the indications of the two instruments. They measure different quantities.

As long as the inclination of the vibration-ellipses of the emitted light is zero, the total light also vibrates symmetrically relatively to the vertical. If the inclination is not zero, however, but has the value α, the plane of maximum resultant luminous motion is inclined at an angle $\alpha+\beta$, which may be occasionally much greater.

The light emitted by the sodium flame contains: (1) horizontal vibrations of intensity c^2 (we neglect here a change mentioned in paragraph 30 below); (2) elliptic vibrations, the major axes of which form an angle α with the vertical. Let the principal axes of these ellipses be a and b.

The intensity I_x in a direction OX becomes

$$I_x=c^2 \sin^2 (\alpha+\beta)+a^2 \cos^2 \beta+b^2 \sin^2 \beta. \tag{1}$$

This expression becomes a maximum for a value of β satisfying

$$c^2 \sin 2(\alpha+\beta)+(b^2-a^2) \sin 2\beta=0. \tag{2}$$

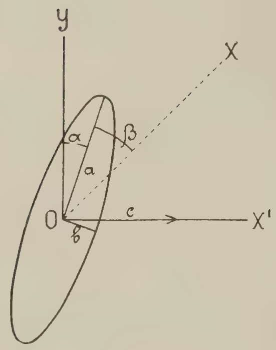

Fig. 8

Hence it follows at once that β cannot be zero, for otherwise α ought to be zero also.

From (2) we obtain

$$\frac{\sin 2(\alpha+\beta)}{\sin 2\beta}=\frac{a^2-b^2}{c^2}. \tag{3}$$

Hence the value of β depends upon the intensities of the horizontal and vertical vibrations. We always have $a>b$; in the emitted light the vertical vibrations generally preponderate, hence also $a>c$. We conclude that β can be positive only.

If we make $\alpha=5°$, $\alpha+\beta=21°$, $b=0.3a$ (see 29), equation (3) gives

$$\frac{a^2}{c^2}=1.4.$$

This is a plausible value. Hence there is no contradiction between the observations made with the polariscope and the results obtained with the nicol alone.

29. We made, with the inverse effect, some measurements of the ellipticity of the outer components at different angles of incidence. We used for this investigation the well-known method of

the quarter-wave plate and nicol. The axes of the quarter-wave plate being placed parallel to the axes of the original ellipse, the resulting light is plane polarized. Let b and a be the horizontal and vertical or the nearly horizontal and the nearly vertical axes, then

$$\frac{b}{a}=\tan \alpha .$$

The mica quarter-plate used proved to be very accurate for light of the refrangibility of the sodium lines, when tried by the method described on a former occasion.[1] Three determinations gave for the deviation from an exact quarter-wave plate the values 1.8, 0.1, 1.0 per cent. For our present determinations this accuracy of the plate is quite superfluous. The measurements are very difficult, relating as they do to the mean of the outer components of the sextet, hence to an extremely narrow part of the spectrum. Moreover the density of the vapor can be defined only approximately (10).

The following table embodies the results concerning the ellipticity of the outer components of the sextet obtained in a somewhat extended series of measurements.

θ	b/a	Remarks
69°.5	$\left.\begin{matrix}0.31\\0.31\\0.28\end{matrix}\right\}0.30$	Vapor of intermediate density (§ 10)
47°	$\left.\begin{matrix}0.45\\0.45\end{matrix}\right\}0.45$	Vapor of intermediate density (§ 10)
	$\left.\begin{matrix}0.47\\0.50\end{matrix}\right\}0.47$	Vapor somewhat denser
39°	$\left.\begin{matrix}0.67\\0.70\\0.70\\0.60\\0.64\\0.67\\0.63\\0.65\\0.65\end{matrix}\right\}0.66$	Very dilute vapor (§ 10)

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[1] Zeeman, *Amsterdam Proceedings*, October 30, 1909.

The ratio of the axes at a certain angle undoubtedly somewhat depends upon the vapor-density. Part of the oscillations of the results obtained at the same angle must be ascribed to this cause.

At $\theta = 69^\circ 5$ and with dense vapor the inclination of the major axis of the ellipse was 6°; with very dilute vapor the value zero was obtained. At $\theta = 47^\circ$ and with vapor of intermediate density the inclination was $4^\circ 5$. The Savart fringes then made an angle of 28° with the vertical.

OBLIQUE POSITION OF THE VIBRATIONS OF THE MIDDLE COMPONENTS

30. Whereas the inclination of the vibration-ellipses of the outer components could be demonstrated first for the sextet, it was for the quartet, on the contrary, that we first succeeded in verifying the second of Lorentz' above-mentioned conclusions (23). The deviation of the vibrations of the middle components of the quartet from the horizontal line can be shown in the same manner as the inclination of the ellipses (24).

The principal section of the nicol before the slit was placed at an angle of about 30° with the horizon. The outer components of the quartet of D_1 are then hardly visible. The inner components are rather dark. The direction of the field is indicated as direction 1. Under the influence of the reverse field 2, the middle components become more black. If the nicol be placed in the symmetrical position, then it is with the field-direction 1 that the middle components are most distinct. The angle θ in this experiment was 47°.

Two different attempts to measure the angle between the vibration and the horizontal gave the results $4^\circ 5$, and $5^\circ 5$. These measurements are very difficult, however, and perhaps indicate only the order of magnitude of the inclination. The vicinity of the outer components largely interferes with the accuracy of the adjustment of the nicol, for while it is moved about near the position of extinction and approaches to a vertical direction the greater intensity of the outer components distracts the eye.

31. We have made yet another experiment which confirms the result of paragraph 30 for both the sodium lines and also exhibits

the relation between the inclinations of the different components. This connection is shown diagrammatically for a triplet in Fig. 9; for the result obtained with the middle components of the quartet and the sextet certainly can be applied qualitatively to the triplet.

The experiment was the following: the principal section of the nicol made an angle of +40° with the vertical; the positive direction in Fig. 9 was counter-clockwise. Then the nicol was placed at 320° (i.e., in the symmetrical position). The last position may be indicated as position *B*, the first mentioned as position *A*. The direction of the field remains unchanged. In position *A* all lines were weaker than in position *B*.

Hence we conclude that the ellipses as well as the vibrations of the middle components are inclined; moreover, that the relative position of the vibrations must be that shown in Fig. 9.

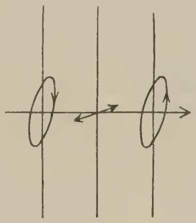

Fig. 9

32. In the important paper already frequently mentioned (note p. 291 of the paper cited in paragraph 1 above) Righi says that Voigt's theoretical investigation of the general case of propagation of light in a direction inclined to the lines of force was published too late to guide him in his investigation. Righi expresses the opinion that it is rather improbable that in the course of his numerous observations peculiarities in the behavior of the middle components as indicated by Voigt could have escaped him, and that Lorentz' elementary theory is in accordance with all the observed phenomena.

This seems in contradiction with our experiments. This contradiction vanishes, however, if we assume that the vapor in Righi's experiments was very dilute, or the field so intense that the components were neatly separated. Under such circumstances our observations are also in complete accordance with the elementary theory, at least as to the polarization of the components and the direction of the vibrations. Neither was it in Righi's experiments a matter of course to reverse the direction of the magnetic field, the procedure which most easily exhibits any obliquity of the vibrations.

APPLICATION OF THE RESULTS OF PARAGRAPHS 24–31 TO THE INTERPRETATION OF SUN-SPOT SPECTRA

33. The vibrations of the middle component of a triplet are parallel to the lines of force. The outer components vibrate linearly at right angles to the field. These rules also apply to dense vapors, if only the pure transverse magnetic effect be under consideration. If we assume that the direction of observation is oblique to the lines of force, then only in the case of very dilute vapors can the projection of the magnetic force on a plane normal to the line of vision be found according to the rules of the elementary theory from the direction of the vibrations. If, however, the components of an inverse triplet are not neatly separated by practically transparent parts—and the sun-spot lines seem to belong to this class of lines—the particulars diagrammatically illustrated by Fig. 9 are to be taken into consideration.

In drawing charts of the magnetic fields in sun-spots, showing the intensity, the direction, and the polarity of the magnetic force, the determination of the direction of the force will give some difficulties. The value of the correction to the indications of the elementary theory necessary in some cases will be given on another occasion.

The rule which determines the direction of the deviation may be indicated here. The direction of rotation in the vibration-ellipses of the outer components toward the red and toward the violet shows whether θ is acute or obtuse. If θ is obtuse (Fig. 7), then the relative position of the directions of the magnetic force, of the major axis of the vibration-ellipses, and of the vibration of the middle component is shown in Fig. 9.

From any point O draw a line OB parallel to the major axis of the vibration-ellipses of the outer components and a line OM parallel to the vibration of the middle component, the angle BOM being always chosen acute. The projection OF of the magnetic force on a plane normal to the line of sight then makes a positive acute angle with OB, the angle BOF being greater than BOM, the positive direction being reckoned from OB to OM.

By ascertaining whether or not the major axes of the ellipses and the vibrations of the middle component are perpendicular to each other, we can make sure whether the elementary theory may be applied or not.

DEMONSTRATION OF OBLIQUE POSITION OF VIBRATIONS BY MEANS OF HALF-WAVE-LENGTH PLATE

34. The observations in the preceding portion of our communication relate to the region between $\theta = 90^\circ$ and $\theta = 39^\circ$, including the two principal directions. We now intend to describe experiments relative to the remaining region between $\theta = 39^\circ$ and 0°.

This region seemed very interesting because under suitably chosen circumstances it probably would contain the angle θ_1 of Lorentz, separating the regions of the longitudinal and the transverse magnetic effect. The principal object we had in view in undertaking this third part of our investigation was to prove experimentally the existence of an angle of the kind mentioned. We think that we attained our purpose.

Before proceeding to describe these experiments, we shall mention a method for verifying the results (paragraphs 24–32) relating to the oblique position of the vibration-ellipses of the outer components and that of the vibrations of the inner components, but without commutation of the current in the electromagnet.

Whereas in our former experiments the *difference* of the intensity of the components by *commutation* of the current gives the proof for the obliquity of the components, the half-wave-length plate demonstrates it at once.

A half-wave-length plate, with one of its principal directions situated horizontally and limited by a horizontal line, is placed near the source of light. Vibrations from the source, making a definite angle with the edge of the plate, after traversing it are rotated through twice that angle. The plate covers only half of the field of view. The directions of the emergent vibrations make the same angles with the horizontal edge as at first, but upon the farther side. An image of the edge is focused upon the slit of the spectroscope; before the slit a nicol is placed.

In one of our experiments, θ being 39°, the plane of vibration of the nicol was at an angle of 35° with the horizon. The magnetic components are now seen unequally dark in the two halves of the field of view. It appeared possible to photograph the phenomenon; small variations of vapor-density, which may possibly introduce errors with other methods of observation, are now without influence. Reversal of the direction of the current changes the sign of the difference of intensity of the two halves of the field of view.

CONNECTION BETWEEN THE INCLINATION OF THE ELLIPSES IN PARTICULAR CASES

35. The direction of the magnetic field, and that of propagation of the beam traversing the magnetized source of light determine the sense of the inclination of the vibration-ellipses (paragraph 25). If the direction of the field be reversed, the sign of the inclination of the vibration-ellipses also changes. In Fig. 7 (paragraph 25) the connection established by our experiments between the three mentioned directions is given.

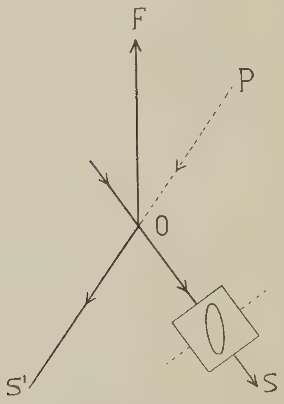

FIG. 10

Let *OF* be the magnetic force, and let the beam, traversing the magnetized flame *O*, be propagated in the direction from *O* to *S*. The inclination of the ellipses in this case is indicated in Fig. 10. The plane normal to the ray and containing the ellipse has been rotated round the dotted line until brought into coincidence with the plane of the paper.

What is the inclination, if the source of light be traversed by the beam in the direction *OS'*?

This question is easily answered by applying the well-known method of reflected images. The geometrical outlines of all things composing a given system, together with the physical processes in the system, which we suppose may be all represented by geometrical figures, we imagine reflected at every instant in a plane *V*. The new system obtained by reflection, which we call the image of the original system, is a possible one, as soon as the last-mentioned one has an objective existence.

Applying this to our experiment (Fig. 11) and placing the plane

V parallel to *OF* and perpendicular to the plane of the paper, we obtain from system I the system II.

The magnetic field in the second system is the inverted image of the field in the first one; indeed, before taking the image of the field we have to substitute it by the equivalent ampere currents. Hence in II the arrow *F′O′* is drawn from *F′* to *O′*. The field in system II being afterward reversed, the inclination of the ellipse changes its sign. Hence we conclude that (Fig. 12), if *OF* be the direction of the magnetic field, the inclination of the major axes of the ellipses, as observed from *S* as well as from *S′*, is always from the lower left to the upper right quadrant.

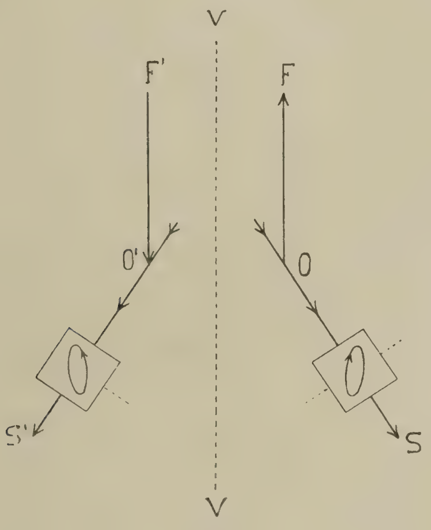

FIG. 11

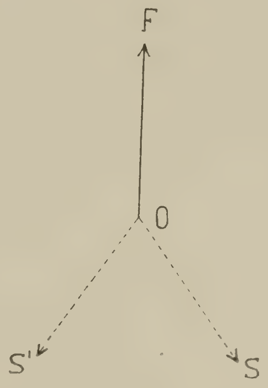

FIG. 12

By means of Savart's polariscope all this could be experimentally verified. We come to the same conclusion by using the experimental result of paragraph 26, concerning the inclination of the ellipses in the beam emitted in the direction *OP* (see Fig. 10).

The close connection existing between emission and absorption enables us to predict the phenomena to be seen if light traverses the source in the direction *OS′* (see paragraph 44).

INVESTIGATION CONCERNING THE EXISTENCE OF AN ANGLE θ_1 (36–46)

36. It seems possible to give by different ways experimental proof of the existence of an angle θ_1, separating the regions of the longitudinal and the transverse effect.

The most direct proof would be given if, with a chosen magnetic force, the vapor-density could be changed in such a degree that at last the direction of the vibrations in the issuing beam were inclined at an angle of 45° with the vertical. Then one would observe at the angle θ_1 itself the values of density (width) and magnetic intensity corresponding. The execution of this plan gives rise, however, to serious difficulties.

The significance and the distinctness of the angle θ_1 become manifest also, however, if it be possible to establish the existence of the characteristic phenomena only observable for a direction of observation which forms an angle with the lines of force lying between 0° and θ_1. We have experimentally verified the theoretical inference. We made many experiments belonging to each of the two classes of experiments mentioned and intend to give a few examples of each.

37. *Observations at* $\theta=32°$.—Soft iron cones with a vertex semi-angle of 32° were made and adapted to a Du Bois electromagnet. The intensity of the magnetic field proved sufficient to establish the character of the resolution in the first-order spectrum of the large Rowland grating.

The middle components were especially watched. It is easily established that the vibrations of these components deviate from the horizontal. In order to demonstrate an inclination of 45°, a quartz plate, cut perpendicularly to the axis, and exactly 2 mm thick, was introduced in the beam. This plate rotates the plane of polarization for sodium light $2\times 21.7=43^\circ\!.4$. Vibrations at azimuth 45°, after traversing the plate, become either horizontal or vertical.

Between the plate and the spectroscope slit a calc-spar rhomb was inserted and a horizontal slit placed near the source; two contiguous horizontal images of this slit are now formed on the slit. The one contains the vertical, the other the horizontal constituents of the beam. The middle components, which at the angle θ under consideration are rather weak, are dependent upon the direction of the current, and are visible either only in the upper or only in the lower of the two strips, if the vapor-density be properly chosen.

This experiment does not prove definitely, however, that the middle components may vibrate under an angle of 45° with the vertical. The rather limited sensitiveness of the method must be taken into account. The experiment certainly proves that the vibrations are inclined relatively to the horizon, at an angle of perhaps 20° or 30°.

It is shown by an observation with the calc-spar rhomb alone, after removal of the quartz plate, that the vibrations are not performed under 45°. A difference between the upper and the lower image is now manifest. This would be impossible if the inclination of the vibrations were 45°. The difference of intensity in the two strips decreases with increased density of the vapor.

All experiments undertaken in order to measure more accurately the inclination gave no decisive results. The weak intensity of the middle components, the feeble separation (to be expected for the observations in view, according to the theory), the perturbation by the vicinity of the outer components, and also the fact that the vibrations become probably slightly elliptic, account for the difficulty of the measurements.

We also investigated the emitted light without the aid of the spectroscope, with a Savart polariscope alone; the emitted light appeared to be nearly unpolarized. The fringes in the polariscope were very weak. This is clearly due to the light containing equal portions of right-handed and left-handed nearly circularly polarized light; the intensity of the light of the middle components is relatively very small and therefore scarcely perceptible in the resulting total intensity. The indistinctness of the fringes made only inaccurate determinations of the position of the plane of polarization possible. An inclination of 42° relatively to the vertical was found.

38. The method of the non-uniform field[1] seemed to open the possibility of a direct reading of the field-intensity corresponding to θ_1, the vapor-density (i.e., the width of the spectral line) being

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[1] Zeeman, *Amsterdam Proceedings*, April 1906, November 1907.

given. At $\theta = 39^\circ$, a diminished image of the cones of the electromagnet was focused upon the slit-plate of the spectroscope. The magnetic separation is different at different heights, and in the spectroscope the spindle-shaped resolution figure, a photograph of which was given on a former occasion, is seen; but now, as the inverse effect is under consideration, rather dark lines on a luminous background are seen. A nicol with its plane of vibration at 45° with the horizontal is placed before the slit. If the vibrations occur at 45° somewhere in the divided lines, the components must become black at such a place. Width and field-intensity, belonging to the part mentioned of the components, correspond to a value of θ_1 equal to 39°.

No clear result was obtained by means of this method, however, which was tried with several vapor-densities. The change of the state of polarization in the resolution figure apparently is too gradual to prove the existence of θ_1 by direct observation. Our experiments following (paragraphs 39–46) seem, indeed, to leave no doubt as to the real existence of such an angle.

39. In order to extend observations to still smaller angles θ, the second-order spectrum of the large Rowland grating was employed for all following observations. The brightness is still amply sufficient and more details are seen. Even with cones with a vertex semi-angle of 26° the characteristic phenomena may now be advantageously observed. With vapor of intermediate density (paragraph 10) only the outer components of the quartet and sextet are now visible, the phenomenon closely resembling the purely longitudinal one. Middle components make their appearance only after the density is largely increased. The nature of these components appears (paragraph 40), however, to have changed, as is proved by an examination of their state of polarization.

The latter is more easily ascertained, if the components are more widely separated. This is the case in the experiments described in the next paragraphs and therefore we prefer to give some details of the observations made with the more efficient arrangement.

40. A still smaller angle between the directions of the beam and of the field may be employed, and moreover wider separation obtained than in paragraph 39, by looking through axial holes and

deviating the beam in the field by means of two small prisms. A remark of Professor Wertheim Salomonson induced us to give prisms a trial.

The arrangement for $\theta = 16°$ is shown in the figure (Fig. 13). The prisms are fixed to copper tubes, which are put into the bored cones of a Du Bois electromagnet and may be turned about their axes. It is therefore possible to adjust the parallelism of the planes of prisms and to arrange the edges vertically.

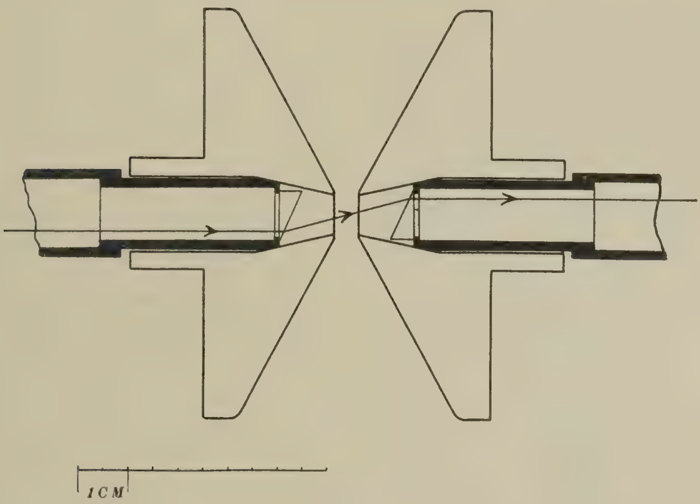

FIG. 13

A drawback inherent to this method is that after some time the interior surfaces of the prisms become covered with some white precipitate. With very dense vapors this inconvenience is rather troublesome. Immediately after introduction of the flame into the interferrum, aqueous vapor condenses upon the prism faces, soon disappearing, however, when the temperature of the prisms has increased. In order to avoid the danger of cracking, the prisms have been placed at some distance from terminal planes of the cones.

Even with very dense vapor (third phase of 10), the field being of the order of 20,000 gausses, the phenomenon closely resembles the purely longitudinal one. No trace of middle components is visible. After an increase, however, of the vapor-density to the limit obtainable by the introduction of a glass rod, charged with melted salt, into the gas-oxygen flame, two new black lines appeared in the vicinity of D_1; they were clearly visible against the rather dark background formed by the broadened outer components.

These new lines, which have the same period as the middle components, are unpolarized (see paragraphs 41–44).

41. We have come to this conclusion after trying in vain to detect any trace of polarization phenomena of the new components.

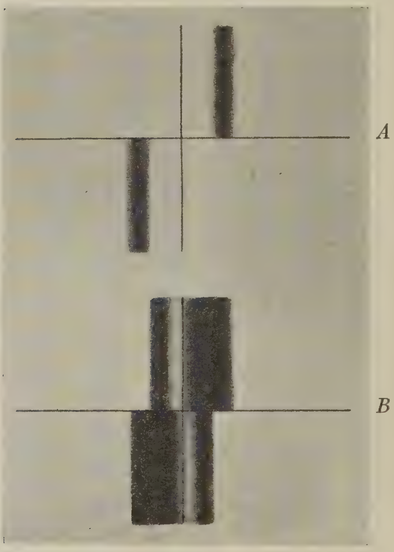

Fig. 14

In the first place, rotation of a nicol placed before the slit of the spectroscope gave no change of intensity of the lines; only the background formed by the nearly, but not accurately, circularly polarized outer components was slightly changed.

42. After removal of the nicol a quarter-wave plate with its principal direction at 45° was inserted in the beam and a broad horizontal slit placed near the field. By means of a calc-spar rhomb two strips are obtained, separating the oppositely polarized circular vibrations.

With vapor of intermediate density Fig. 14*A* gives the appearance for D_1. The vertical line represents the reversed line due to the arc light.

With very dense vapor, we get the phenomenon represented in Fig. 14*B*. New components appear in the initially bright parts of the field of view.

The positions of the new components correspond to those of the inner components of the quartet, at least as far as can be judged by eye observation. This observation is confirmed by measurements made on a photograph, it must be said, of only moderate quality.

As to the polarization of the new lines a few remarks may be made. From an inspection of Fig. 14*B* alone, one might infer a circular polarization of the inner components, of a sign opposite to that of the outer ones. One might be tempted to infer that, under the circumstances of the experiments, the inner components are due to the motion of positive charges. There is no need of discussing the degree of probability of such a conclusion, as it is refuted by the next observation.

42. If the quarter-wave plate be rotated in its own plane so that the principal direction more and more approaches to the horizontal position, the intensity of the outer components decreases. The inner components, which at first are invisible in two of the quadrants, being entirely hidden by the black, broad outer components, are very soon seen as continuous bands crossing at right angles the horizontal separation line. Finally, when the principal direction of the quarter-wave plate has become horizontal, there is, as far as concerns the inner components, no difference at all between the upper and lower fields, and only a slight one as far as concerns the outer components.

43. From the observations recorded in paragraphs 41 and 42 we cannot but conclude that under the circumstances of the experiment the inner components of the *new* quartet are unpolarized. This result seems paradoxical, because one now has become accustomed to expect polarization of all magnetically separated and displaced lines. The result, however, seems to be in perfect accordance with theory, at least if it be permitted to apply to the middle components of the quartet, the theoretical inference drawn for the central component of the triplet.

Lorentz has proved that in the case of a triplet for a frequency $n=n_0$ and $\theta<\theta_1$ two oppositely elliptically polarized beams may be transmitted, having the same index of absorption, but unequal velocities of propagation. The characteristic vibration-ellipses for the two beams are the same, but described in opposite directions (see also 22 above.) Since the indices of absorption of the two beams are equal, we may expect that, under the circumstances mentioned, a magnetized vapor can produce in a continuous, unpolarized spectrum, unpolarized absorption lines only.

44. The consideration in paragraph 35 of the reflected image of

a system was made in order to show that the inclination of the ellipses remains unaffected by a change of sign of the angle between the line of force and the ray.

45. *Quartet for* $\theta=0$.—By increasing still further the vapor-density necessary for the experiment of paragraph 43, we were able to observe, even in the direction $\theta=0$, the two unpolarized lines, corresponding to the inner components of the quartet. The outer components, however, have then become extremely diffuse. It is certainly remarkable that the two new components are still relatively narrow. The theoretical reason for this feature of the phenomenon has still to be worked out.

It is, however, in accordance with theory (always on the supposition that it does apply directly to the quartet) that for $\theta=0°$ the density of the vapor must exceed that for $\theta=16°$, in order to render visible the new lines. Indeed, according to the formulae (42) and (26) of Lorentz' paper (cited in 1 above) the absorptive index decreases with decreasing θ.

The experiments (39–43) seem to give conclusive evidence that an angle θ_1 really exists. Indeed, phenomena of the kind described in the last paragraphs are to be expected in a region only between θ_1 and 0°.

The experimental verification of Lorentz' deductions, formulated in 23 above, gives a new proof of the rational connection established by Voigt's theory of the inverse magnetic effect between diverse phenomena.

A more accurate measurement of θ_1, the vapor-density and the field being chosen, must be postponed.

46. The new type of magnetic separation, with some components polarized, the other ones unpolarized, which returns to the ordinary separation by decrease of vapor-density, we were able to observe also with D_2. Since the density of the vapor must be great in the present experiment, the effects observed with D_2, which splits up into a pseudo-triplet, are less clear and characteristic than with D_1. We therefore restricted the detailed description of our observations to the case of D_1.

PHYSICAL INSTITUTE
THE UNIVERSITY OF AMSTERDAM
1910

MASS-RATIOS IN THE SYSTEMS OF *KRÜGER 60* AND *CASTOR*

By HENRY NORRIS RUSSELL

I. *Krüger 60*

Burnham's General Catalogue of Double Stars, 11761 ($\alpha=22^h\ 24^m5$, $\delta=+57°\ 13'$, 1900)

In Professor Barnard's thorough discussion of the parallax and proper motion of this star,[1] he points out that the motion of the principal component, A ($9^m_.7$), with respect to the distant optical companion C ($10^m_.2$), is distinctly curved, showing that the faint component B ($11^m_.0$) of the binary system has a mass comparable with that of A.

These observations alone would give a fairly good determination of the mass-ratio. The addition of unpublished measures of the last two years, which Professor Barnard has very kindly communicated to the writer, makes it possible to fix this quantity within quite narrow limits.

If the distance and position angle of C, referred to A, are s, p, and those of B are s', p', their apparent rectangular co-ordinates, referred to axes whose position angles are p_0 and $p_0+90°$, will be

$$x=s\cos(p-p_0),\quad x'=s'\cos(p'-p_0),$$
$$y=s\sin(p-p_0),\quad y'=s'\sin(p'-p_0).$$

If k is the ratio of the mass of B to that of A and B combined, the co-ordinates of the center of gravity of the binary system will be kx', ky', and, since the motion of C relative to this point must be uniform, we must have

$$x=a+b(t-t_0)+kx',\quad y=a'+b'(t-t_0)+ky',$$

where t_0 is any convenient epoch. Each set of simultaneous observations of AB and AC gives us an equation of condition for the constants a, b, k, and another for a', b', k.

No knowledge of the period of the binary system, or of the form of the complete orbit, is necessary in forming these equations, which depend only on the relative motion during the interval covered by the observations; and it is obvious that the accuracy with which k can be determined depends on the magnitude of the departures of the motion of B (relative to A) from uniformity, during this interval.

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[1] *Monthly Notices*, **68**, 629, 1908.

The data on which the present discussion is based are as follows. They are taken from Barnard's paper, already mentioned, except for the last two lines which are annual means of the unpublished measures referred to above. The rectangular co-ordinates x and y are computed with $p_0 = 59^{\circ}00$.

The parallax of A is 0″25 (according to three very accordant determinations by Barnard, Schlesinger, and the writer). The observed co-ordinates of C, relative to A, therefore require to be increased by the corrections

$$\Delta x = \Delta s = +0''25 \cos(\odot - 93^{\circ}),$$
$$\Delta y = \quad +0''22 \cos(\odot - 179^{\circ}).$$

The tabular values of s are already corrected for parallax (this having been done by Barnard). Those of p are uncorrected, but the correction has been applied to the tabular y's. The mean of the corrections for the individual dates has been taken when necessary. The position angles have also been corrected for precession, to 1900, at the rate of −0°0042 annually.

	AB					AC				
Date	p	s	n	x	y	p'	s'	n	x'	y'
1890.79.....	178°8	2″32	1	−1″16	+2″02	56°26	26″73	1	+26″70	−1″48
1898.45.....	140.7	3.19	5	+0.46	3.15	58.70	34.64	5	34.64	0.15
1900.84.....	133.6	3.22	8?	0.86	3.11	59.20	36.36	8?	36.36	+0.02
1901.77.....	130.7	3.28	7	1.03	3.12	59.63	37.23	7	37.23	0.19
1902.76.....	127.0	3.36	4	1.26	3.13	59.65	38.25	4	38.25	0.21
1903.63.....	123.3	3.37	15	1.47	3.04	59.58	39.12	32	39.12	0.30
1904.63.....	119.6	3.38	19	1.66	2.95	59.44	40.03	27	40.03	0.19
1905.91.....	115.3	3.31	2	1.84	2.76	59.66	41.21	2	41.21	0.37
1906.56.....	112.2	3.32	7	2.00	2.66	59.44	41.83	7	41.83	0.21
1907.57.....	107.2	3.28	12	2.18	2.44	59.45	42.84	11	42.84	0.21
1908.36.....	104.4	3.18	6	2.24	2.27	59.11	43.66	8	43.66	0.22
1909.63.....	98.8	3.14	16	2.41	2.01	59.39	44.87	17	44.87	0.25
1910.59.....	94.9	3.13	9	+2.54	+1.84	59.49	45.77	9	+45.77	+0.25

The first measure is by Burnham, the second by Eric Doolittle, and the third, the mean of the results of Doolittle and Barnard.

All the rest are Barnard's. As the latter points out (*op. cit.*, p. 638), it is desirable in such an investigation to use the measures of one observer as much as possible. The number of nights' measures combined into each mean place is denoted by n. In one or two cases the measures of AB have been interpolated to the epoch of those of AC (differing by a month or less).

The motion of B, relative to A, is so nearly uniform in x that only the y's can furnish a reliable value of k. An approximate solution gives

$$y = -1''.63 + 0''.092(t-1900) + 0.5\,y'.$$

The residuals from this solution are the absolute terms of the equations of condition given below. The unknowns in these are $\alpha = \Delta a' + 2''.70\Delta k$, $\beta = \Delta b'$, $\gamma = \Delta k$, where $\Delta a'$, $\Delta b'$, Δk are the corrections to be added to the approximate values.

		O.–C.	Wt.			O.–C.	Wt.
$\alpha - 9.2\beta - 0.69\gamma =$	$-0''.01$	$-.03$	1	$\alpha + 5.9\beta + 0.06\gamma =$	$+0''.08$	$+.09$	$\frac{1}{2}$
$\alpha - 1.5\beta + 0.42\gamma =$	$+\ .06$	$+.03$	1	$\alpha + 6.6\beta - 0.04\gamma =$	$-\ .09$	$-.08$	1
$\alpha + 0.8\beta + 0.46\gamma =$	$+\ .01$	$-.01$	1	$\alpha + 7.6\beta - 0.22\gamma =$	$-\ .08$	$-.06$	1
$\alpha + 1.8\beta + 0.43\gamma =$	$+\ .10$	$+.08$	1	$\alpha + 8.4\beta - 0.44\gamma =$	$-\ .06$	$-.03$	1
$\alpha + 2.8\beta + 0.38\gamma =$	$+\ .02$	$+.01$	1	$\alpha + 9.6\beta - 0.65\gamma =$	$-\ .02$	$+.02$	2
$\alpha + 3.6\beta + 0.32\gamma =$	$+\ .08$	$+.07$	3	$\alpha + 10.6\beta - 0.90\gamma =$	$-\ .01$	$+.04$	1
$\alpha + 4.6\beta + 0.22\gamma =$	$-\ .09$	$-.09$	3				

The resulting normal equations are

$$\begin{aligned} 17.5\alpha + 74.5\beta - 0.25\gamma &= -0''.02, \\ +688.\ \beta - 11.9\ \gamma &= -1.91, \\ +\ 3.55\gamma &= +0.200, \end{aligned}$$

whence $\alpha = +0''.011$, $\beta = -0''.0034$, $\gamma = +0.032$. The probable error of unit weight is $\pm 0''.054$, and the final values of a', b', k are

$$\begin{aligned} a' &= -1''.70 \pm 0''.084, \\ b' &= +0.089 \pm 0.003, \\ k &= +0.531 \pm 0.031. \end{aligned}$$

The relatively large probable error of a' is due to the term $2''.70\ \Delta k$ which it contains.

The mass of B appears therefore to be slightly greater than that of the brighter star A, the ratio $\frac{\mathrm{B}}{\mathrm{A}}$ being $\frac{k}{1-k}$, or $\underline{\overline{1.14 \pm 0.14}}$.

Comparison with the probable errors of other determinations of similar ratios—such as those given in the appendix to Boss's *Preliminary General Catalogue*—shows that this may be regarded as a fairly good determination; for which reason it is now published, although the observations of ten years more will confine the uncertainty within much narrower limits.

Correcting the x co-ordinates, with this value of k, the annual motion of C, relative to the center of gravity of AB, is found to be $+0''.861 \pm 0''.006$.

The proper motion of the system, relative to this star, is therefore $0''.866 \pm 0''.006$ in position angle $244^\circ.9 \pm 0^\circ.2$. The orbital motion of the principal star accounts for the difference between this and the value $0''.968$ in $246^\circ.5$, found by Barnard for the latter for the interval 1890–1905. Both values, however, require unknown corrections for the proper motion of star C, which cannot safely be neglected in dealing with observations of such accuracy.

II. *Castor* (*α Geminorum*)

Though this binary has been under observation for nearly 200 years, the elements of its orbit are still very uncertain. Were they accurately known, the spectroscopic observations of H. D. Curtis[1] would enable us to find very reliable values of the parallax and mass.

It may be shown that the existing uncertainty of the elements falls almost entirely upon the determination of the parallax, and does not seriously influence that of the mass of the system.

Let i be the angle which the relative velocity of the two components in space makes with the plane tangent to the celestial sphere, v the component of this velocity parallel to this plane (in seconds of arc per year), and ρ that perpendicular to it (in astronomical units per year). Then $v = \pi\rho \cot i$, where π is the star's parallax.

If now p is the apparent position angle of the velocity v, as projected on the plane of reference, and Ω and γ are the node and inclination of the orbit-plane, $\tan i = \tan \gamma \sin (p - \Omega)$, and we have

$$\pi = \frac{v}{\rho} \sin (p - \Omega) \tan \gamma. \qquad (1)$$

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[1] *Astrophysical Journal*, **23**, 351, 1906.

From Curtis' observations, the value of ρ (for the centers of mass of the rapid spectroscopic binaries which form the components of the visual system) is 7.18 ± 0.23 km/sec or 1.52 ± 0.05 astronomical units per year, at the epoch 1904.9. v and p can be found directly from the micrometer measures of the last few years. These give the mean places.[1]

	Position Angle	O.−C.	Distance	O.−C.
1897.88......	226°4	−0°2	5″69	−0″02
1900.46......	225.3	−0.1	5.66	0.00
1903.43......	224.4	+0.1	5.63	+0.03
1906.03......	224.0	+0.8	5.54	−0.01
1908.33......	221.6	−0.7	5.52	+0.02
1909.32......	221.8	0.0	5.45	−0.03

From these with equal weights, we find:

$$p=(224\overset{\circ}{.}1\pm0\overset{\circ}{.}15)-(0\overset{\circ}{.}415\pm0\overset{\circ}{.}037)\,(t-1904),$$
$$s=(5\overset{''}{.}59\pm0\overset{''}{.}007)-(0\overset{''}{.}0200\pm0\overset{''}{.}0017)\,(t-1904).$$

A check upon the value of $\frac{dp}{dt}$ may be obtained from the principle of the uniform description of areas in the apparent orbit. According to Doberck[2] the projected radius vector swept over 7.658 square seconds of arc between 1832.0 and 1896.0. The corresponding angular velocity at distance $5\overset{''}{.}59$ is $-0\overset{\circ}{.}438$ per annum —agreeing with the observed value within its probable error. The computed value obviously deserves the preference (its probable error being doubtless less than 1 per cent of its value). Combining it with the observed value of $\frac{dr}{dt}$, we find, for 1904:

$$v=0\overset{''}{.}047\pm0\overset{''}{.}001$$
$$p=109^\circ\pm2^\circ,$$

whence, substituting in (1)

$$\pi\ \mathrm{cosec}\ (109^\circ-\Omega)\ \cot\gamma=0\overset{''}{.}0313\pm0\overset{''}{.}0013 \qquad (2)$$

in which the probable errors of both ρ and v are taken into account.

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[1] The first three are derived from measures given by Lewis, *Memoirs R.A.S.*, **56**, 217, 1908; the others from observations published in the *Astronomische Nachrichten*, for 1905–1910.

[2] *Astronomische Nachrichten*, **166**, 145, 1904.

The numerical terms in this equation depend only on modern spectroscopic and micrometric observations, and are independent of any hypotheses about the orbit; but to find the parallax we must know the position of the orbit-plane.

The extent to which the latter is uncertain is shown by comparing the five orbits recently derived by Doberck (*op. cit.*) and by Lohse.[1] All of these represent the observations (including Bradley's estimated position angle of 1720) fairly well; but the first and last give distinct systematic deviations, in opposite senses, from the observed positions so that the range of uncertainty cannot be much greater than the table indicates.

	P	a	e	Ω	γ	π	M
Lohse I........	249y	7″6	0.80	203°1	73°1	0″103	6.3
Doberck III	268	7.3	0.75	209.5	73.0	0.101	5.3
Lohse II	298	5.9	0.58	207.8	65.4	0.068	7.4
Doberck IV.....	347	5.8	0.44	213.9	63.6	0.061	7.1
Doberck V	502	6.5	0.23	222.6	61.9	0.053	7.3

All the elements vary through a wide range; but if they are plotted as functions of the period, the individual points lie in all cases near a smooth curve. They may therefore be regarded as functions of a single ill-determined quantity, e.g., the period. In other words, there is practically only one way of drawing an ellipse of given area so as to represent the observations, but a great deal of latitude as regards this area.

The values of the parallax and mass, computed for each set of elements with the aid of equation (2),[2] will also be functions of the assumed period. The parallaxes vary through a considerable range (though not more than would at present be considered satisfactory in the case of as many direct determinations). The computed masses are remarkably accordant—the variations in period, major axis, and parallax compensating one another almost perfectly. This would not generally happen, and must be due to

[1] *Publikationen des astrophysikalischen Observatoriums zu Potsdam*, **20**, 92, 1908.

[2] As the apparent direction of motion determined from the micrometer observations has been used in all cases (instead of that of the tangent to the individual apparent orbits), these values will differ slightly from the results of direct computation from the radial velocity and the orbital elements. It appears, to the writer at least, that the present process, involving as it does the minimum of uncertain data, is preferable.

some unusually favorable chance as regards the form and position of the orbit in this particular case; but the fact is put beyond question by the numerical calculations.

We may therefore conclude that the parallax of *Castor* is approximately 0″08 ± 0″03, and that the mass of the whole system is about 6.5 ± 1.0 times that of the sun.

Little is yet known concerning the ratio of the masses of the components of the visual system. Crommelin has shown[1] that the path of the bright component, derived from meridian observations, is much more curved than that of the fainter, indicating that the latter has the greater mass. The same conclusion may be deduced from the micrometer measures of the distant companion, which may be summarized as follows (setting $p_0=163°5$):

AC				Differences from Uniform Motion	
	p	$s=x$	y	Δx	Δy
1835.2......	162°5	72″52	−1″27	+0″06	−0″15
1846.0......	162.78	72.42	−0.91	−0.15	−0.11
1861.5......	163.40	72.86	−0.13	+0.13	+0.20
1880.5......	163.98	73.10	+0.61	+0.18	+0.37
1901.3......	164.10	73.14	+0.76	+0.02	−0.10
1909.1......	164.28	72.94	+0.99	−0.26	−0.20
Yearly	motion	+0.010	+0.030		

AB*					Differences from Uniform Motion	
	p	s	x	y	Δx	Δy
1835.2......	257°5	4″78	−0″33	+4″77	−0″10	−0″24
1846.0......	250.9	4.97	+0.22	+4.97	−0.03	−0.04
1861.5......	242.4	5.34	+1.03	+5.24	+0.11	+0.23
1880.5......	234.5	5.63	+1.84	+5.32	+0.09	+0.31
1901.3......	224.9	5.66	+2.71	+4.97	+0.06	−0.04
1909.1......	221.8	5.48	+2.89	+4.77	−0.11	−0.24
Yearly	motion		+0.044	0.000		

* Interpolated from Lewis' table, *Memoirs R.A.S.*, 56, 214, 1906.

The motion of A, relative to both B and C—especially in *y*—shows a distinct curvature, which appears to be nearly equal in amount in the two cases, indicating once more that the center of gravity of the system is near the fainter component. The curvature is at present too small to afford a reliable determination of the mass-ratio; but within twenty years a good determination will be possible. For this reason careful measures of AC, especially of the position angle, should be made every year.

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[1] *Monthly Notices*, 67, 140, 1906.

It might at first appear that our knowledge of the orbits of the spectroscopic binaries might help us here. But as only one component of each of these is bright, all that we can get from this is the value of $\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$ (where m is the mass of the bright star, m_1 is that of its invisible attendant, and i is the inclination of the orbit). This may for convenience be called the "apparent mass." If $\frac{m}{m_1}=c$, the actual mass of the system is $(1+c)^3 \operatorname{cosec}^3 i$ times the "apparent mass." From Curtis' data we have for the stars in question:

	$a \sin i$	P	Apparent Mass
	km	d	
α_2 *Geminorum*	1,485,000	9.219	0.0015 ☉
α_1	1,279,000	2.928	0.0097 ☉

The "apparent mass" of the fainter component is six times that of the brighter; but the sum of the "apparent masses" of the two is only $\frac{1}{600}$ of the real sum of their masses, previously determined.

It is evident that, for one or both of the close pairs, c must be large or i small. Beyond this they are at present unknown, and hence the ratio of the "apparent masses" tells us nothing about the real mass-ratio.

If the planes of the secondary orbits are approximately coincident with that of the visual orbit (which seems plausible) and c is the same for the two close pairs, it must be about 6.5. If less than this for one pair, it must be more for the other.

This is a much greater preponderance of the primary than has appeared in any of the systems susceptible of reasonably exact investigation. The greatest well-established value of c is 3.0

in the case of Procyon.[1] To get as small a value in the present case it would be necessary to assume that the orbits of the close pairs are inclined less than 30° to the plane of projection—that is, that both are inclined at least 35° to the plane of the visual orbit, and that the ascending nodes of both on this plane lie near its descending node on the plane of projection. This seems rather improbable, and the difficulty increases if we assume smaller values of c. It is therefore probable that the "dark" companions in these two spectroscopic binaries are less massive, in comparison with their primaries, than is the case in any systems previously investigated.

However this may be, it is certain that the small "apparent masses" of these pairs are no valid indication of actual smallness of mass. This bears on the interpretation of numerous similar cases. The great range presented by the "apparent masses" of spectroscopic binaries has recently been discussed by Campbell[2] and by Schlesinger.[3] It is easy to show that the number of small "apparent masses" is much too great to be explained by the chance occurrence of small values of i. Schlesinger accounts for them by assuming that the masses of some of these stars are actually insignificant in comparison with that of the sun, and Campbell by assuming that in these instances the companions are of considerably smaller mass than the visible primaries.

The present case, which is so far the only one in which the actual masses of such pairs can be found, confirms the latter explanation and is inconsistent with the former.

Certain similar cases (e.g., *13 Ceti*, *κ Pegasi*) where one component of a visual binary is spectroscopically double, will further test this matter in future.

Meanwhile the clear evidence in this typical case makes it seem reasonable to regard the very small "apparent masses" as evidence of small *relative* mass of the unseen companion, rather than of minute *absolute* mass of the whole system.

PRINCETON UNIVERSITY OBSERVATORY
November 12, 1910

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[1] Boss, *Preliminary General Catalogue*, p. 267, 1910.

[2] *Lick Observatory Bulletin*, No. 181, **6**, 40, 1910.

[3] *Pub. Allegheny Observatory*, **1**, 147, 1910.

PHOTOGRAPHIC DETERMINATIONS OF STELLAR PARALLAX MADE WITH THE YERKES REFRACTOR. I

By FRANK SCHLESINGER

Almost all the determinations of stellar distances that have been published up to the present time have been made with instruments of short focal lengths, surprisingly short when we consider the accuracy that is demanded and has occasionally been attained in this class of work. Thus for example, the Yale heliometer, used by Elkin, Chase, and Smith to such good purpose, is only 2.5 meters long.

In 1902 some correspondence upon this subject passed between Professor Hale (then director of the Yerkes Observatory) and the present writer. It seemed very likely that, with the great focal lengths of modern refractors and the consequent increase of scale, it ought to be possible to bring about a considerable decrease in the accidental errors of observation, or else to reduce greatly the labor necessary to obtain results within a predetermined limit of error. The founding of the Carnegie Institution late in that year afforded an opportunity for putting this idea into practice, for among its first grants was one for this specific purpose to Professor Hale, who invited the writer to take charge of the experiment with the great Yerkes refractor. My connection with the Yerkes Observatory extended from May 1903 until I assumed new duties at the Allegheny Observatory in March 1905. In this interval Professor Hale had been succeeded in the directorship of the observatory by Professor Frost, who was equally interested in the outcome of this work, and who made it possible for me to complete it in its present form.

It is a pleasure to acknowledge here, at the outset of these papers, my obligations to those who have contributed so largely to them in one way or another: first, to the Carnegie Institution, under whose auspices the work was undertaken, the Institution assuming the principal items of expense, such as the salaries of myself and of a computer, and the cost of a measuring engine; to the Yerkes Observatory and its successive directors, for putting at my disposal the facilities of the observatory, including one-quarter of the total nights with the 40-inch refractor, and for defraying all minor expenses, such as the cost of plates and recording blanks; to Miss Louise Ware, now of the Solar Observatory at Pasadena, who proved a most efficient assistant, and to whose conscientiousness the measures and reductions owe much of their merit; to Mr. Frank Sullivan, night assistant at the telescope, who aided in securing almost all the plates; to Mr. Philip Fox and Mr. Frank C. Jordan, who, at Director Frost's request and with Mr. Sullivan's assistance, secured the additional plates necessary to complete the series for certain of the stars; and finally to Mr. Robert H. Baker, who did some computing early in 1907, under a special grant to the writer by the Carnegie Institution.

AT THE TELESCOPE

That excellent photographs may be secured with a large visual refractor was first demonstrated by Mr. G. W. Ritchey,[1] who employed the method of placing a yellow color-screen or filter immediately in front of a plate that is sensitive to the yellow as well as the blue rays. I had expected to make use of this ingenious device for the parallax plates, but upon securing some photographs upon yellow-sensitive plates without a screen, and after comparing them with some of Mr. Ritchey's, I came to the somewhat surprising conclusion that isolated images upon the former are not inferior either in smallness or sharpness to those upon the screened plates.[2]

The reasons for this I have given in some detail in the *Astrophysical Journal*, **20**, 123, 1904. They are in brief that the plates employed, Cramer Instantaneous Isochromatic, act as their own filters for the region λ 4800 to λ 5200, as they are only slightly sensitive to this part of the spectrum. They have a strong maximum at λ 4500, but light of this wave-length is so much out of focus that it is spread into a circle about 2 mm in diameter by the time it reaches the plate, and is consequently much enfeebled if the source of light is a point. For exposures of moderate length the only part of the spectrum that is effective in forming the image is included between λ 5200 and λ 5700, a region that is all in fairly good focus for a proper setting of the plate. If the exposure be prolonged, the brighter stars upon the plate will begin to show halos several millimeters in diameter, due to the blue and violet light. Such an image is not suitable for accurate measurement, even though its nucleus remains well defined. This constitutes a disadvantage in the unscreened plates, since it is not possible, or at least not advisable, to include in the measurements as large a range of magnitudes as upon the screened plates. For parallax work, however, this is of no great consequence, because other considerations make it desirable that the comparison stars should not differ too greatly from each other in brightness, a point that will be referred to later.

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[1] *Astrophysical Journal*, **12**, 352, 1900.

[2] Under the most favorable circumstances either kind of plate, taken with this telescope, will separate the components of a double star that are 0."75 apart.

The use of screened plates raises the question as to the effect of the screen's presence upon precise measures, and the distortions introduced by a plate of glass in front of the sensitive film. It was deemed better to avoid this difficulty rather than to devise methods for overcoming it, and for this reason all the plates here discussed were taken without a screen.

The above remarks refer to the filters that were then available. I had pointed out[1] in 1904 that "the best screen for stellar work would not be one which cuts out only the blue rays, but rather one which prevented the region λ 5200 to λ 5400 from reaching the plate." Since then Mr. R. J. Wallace, who had made the original screens without special reference to the Yerkes refractor, has had an opportunity for studying its color-curve in this connection, and has succeeded in producing screens that leave little to be desired. Not only do they intercept almost completely all that portion of the spectrum that is out of focus, but they introduce practically no absorption for the yellow and orange rays. Stellar images secured with these later screens are superior to the earlier in sharpness, and are more suitable for precise measurement. The

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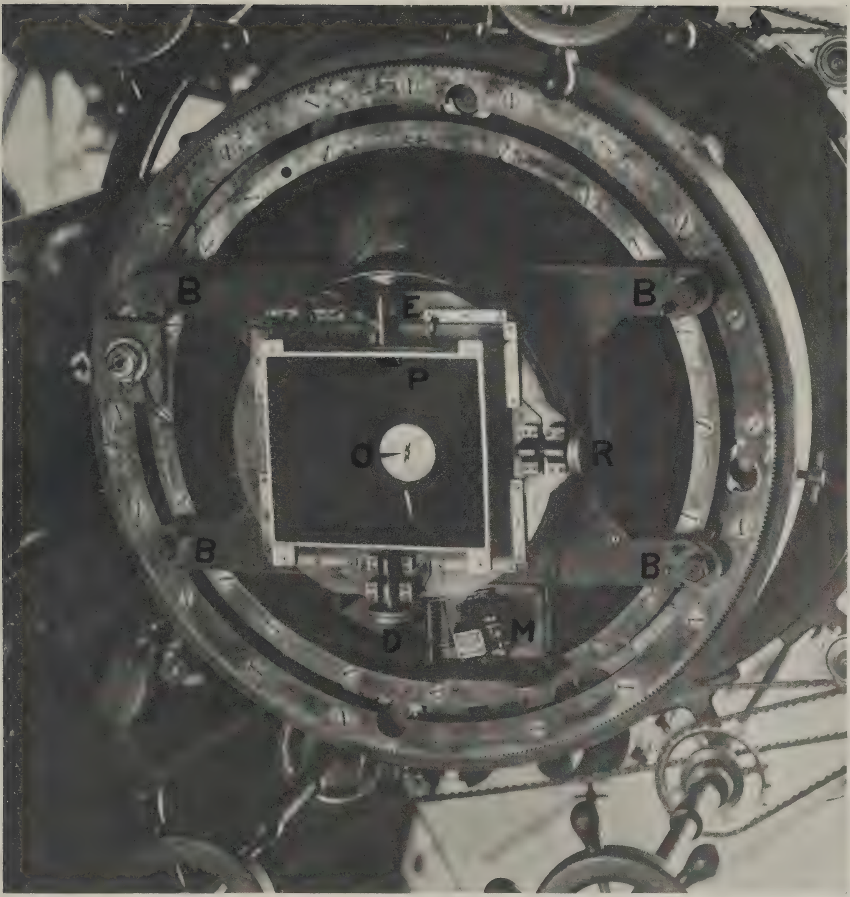
[1] *Astrophysical Journal*, **20**, 125, 1904.

PLATE XIX



Double-Slide Plate-Carrier for 8 × 10-Inch Plates as Attached to Yerkes Refractor

question whether screened plates or unscreened should be used for astrometrical purposes, in connection with a visual refractor, is thus reopened, particularly as I have recently shown that the errors introduced by photographing or measuring through glass are very small, even if ordinary glass has been employed, and are probably altogether negligible if the screen is made of worked glass.[1]

The first few plates obtained for the parallax work were secured with a double-slide plate-carrier for 4×5 inch plates. This had the advantage of being readily attached to the telescope, but it was soon found that the field subtended was too small for practical purposes and the more cumbersome 8×10 inch (20×25 cm) carrier was used instead. A photograph of this instrument appears herewith (Plate XIX), and a brief description of it is as follows: A small diagonal prism *P* is placed just in front of the photographic plate, and near its longer edge. The bundle of rays from a star that would otherwise form an image upon the plate are thus brought to the focus of a positive eyepiece *E*, in which are two fine spider-threads, respectively parallel and at right angles to the diurnal motion. The 40-inch telescope having been pointed by means of the slow motions until the threads exactly bisect the image, the observer keeps the star in this position, and compensates for irregularities in the driving clock and in refraction, by moving the eyepiece with the two screws *D* and *R*. These screws are at right angles to each other, the first moving the eyepiece and upper slide upon a second or lower slide that is controlled with the screw *R*. The upper slide not only supports the guiding eyepiece but the plateholder as well. Consequently the plate follows the apparent fluctuations of the stellar images with the same faithfulness that the guiding star is held at the intersection of the spider-threads in the eyepiece, and all the stellar images upon it remain small and round. In order to afford greater facility in finding a guiding star after any desired object has been placed in the center of the field, the guiding eyepiece is provided with a motion in the direction of the long edge of the plate; and with the same object in view the lower slide is mounted upon a circular casting that may be rotated in position-angle. This circular casting is provided with a coarse circle, graduated at every fifth degree; the index itself is graduated to single degrees. It is thus possible to set the instrument within a fraction of a degree, and this is ample for the present purposes. The whole plate-carrier, which weighs about 43 kilos (96 lbs.), is attached to the tail-piece of the telescope by means of four stout bolts *B*, *B*, and its position is fixed by two dowel pins. The tail-piece itself may be racked toward or from the 40-inch objective, and it is in this way that the film of a plate is adjusted to the focal plane. The plate-holder is of brass, and loads from the back, as every plate-holder should that is to be used in precise work, for then the plate is supported along its entire perimeter by a suitable ledge, against which it is pressed by several weak springs.[1]

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[1] *Publications of the Allegheny Observatory*, **1**, 101, 1909. As early as 1903, Ludendorff (*Publicationen des Astrophysikalischen Observatoriums zu Potsdam*, **15**, No. 49) had occasion to measure some plates through glass and concluded that the errors thus introduced, while appreciable, are not so large as to prohibit measurements made in this way.

The optical center of a photograph may be defined as the foot of a perpendicular let fall from the center of the objective on the plane of the plate. It is necessary (with photographs intended for accurate measurement) that we should know where the optical center lies, and that it should not be too far from the geometrical center of the plate. For this purpose an aluminum plate was made of the same size as the photographic plate, and it was fastened in the holder in precisely the same position that the photograph itself ordinarily occupies. To this aluminum plate a small telescope was fixed, approximately perpendicular to its plane and projecting through a hole in its center. In its focus were two parallel wires, which could be adjusted so as to be tangent to the image of the 40-inch objective. The plate was now removed from the holder and turned 180° around the optical axis of the large telescope. If the little telescope had been accurately perpendicular to the plane of the plate, and if the latter had been perpendicular to the line joining its center with that of the 40-inch objective, the image of the objective would have again been tangent to the two parallel wires in the focus of the little telescope. The amount by which this fails to be the case can be estimated with sufficient accuracy and gives the position of the optical center. This method furnishes an accurate and convenient means for adjusting the "tilt" of the plate, and the writer has used it to advantage with several telescopes.

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[1] The plate-carrier was designed and constructed by Mr. Ritchey, and is the one used by him to obtain his lunar photographs; a few additions for which the writer is responsible were made to meet the requirements of the parallax work, consisting of the graduated scale on the guiding eyepiece slide, the graduated circle, the dowel pins to fix the position of the whole carrier, the metal plate-holder, and a device, to be described presently, for partially occulting the light of a bright star.

In the present instance it was found that the optical center lay about 6 cm (11 minutes of arc) east of the geometrical center, and 5 cm (9 minutes of arc) south of it, the telescope being west of the pier and directed to the intersection of the meridian with the equator. These quantities are somewhat larger than we should expect. No adjustment for tilt had been provided in the design of the plate-carrier, reliance having been placed on the mechanical precision of the tail-piece, etc. A search was made to locate the cause of the divergence and it was found to lie in the inequality of the four tubes by which the tail-piece is racked toward and from the objective. The upper right-hand one of these was found to be nearly 2 mm longer, and the opposite one the same amount shorter, than the two others.[1]

This mechanical defect might have been corrected by removing the tubes and making them the same length; but this would have occasioned considerable inconvenience and expense, and would have necessitated a readjustment of the solar and stellar spectrographs that are in constant use with this telescope. A simpler plan would have been to alter the plate-carrier so as to give it an equal tilt in the opposite direction. However, neither of these changes was carried out, the small field subtended by the photographs making it unnecessary in the present work. With a set of comparison stars as they would ordinarily be distributed upon the plate, the deduced position of the central star would be affected by less than 0."01, if the optical center should change its position by as much as 3 cm. Therefore all that is necessary is that the tilt should not change beyond this limit. The test above described was carried out twice with an interval of two years, and was found to give substantially the same results. It was furthermore found that the tilt remained the same whether the tubes had been racked in or out. The flexure of the massive tube of the telescope may also introduce a slight change in the tilt, but this need be given no further consideration in the present work, since all the plates were taken with the telescope west of the pier and not far from the meridian; and the flexure is therefore always the same for any one star.

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[1] The centers of the tubes form a square that is 64 cm on each side.

As the time just after dark is very valuable in parallax work, especially if the regions are to be photographed near the meridian, it was desirable to devise some rapid method for setting the plate in focus, without suffering the delay incident to securing the usual "focus plate." For this purpose a rigid aluminum plate was fixed in the position ordinarily occupied by the metallic plate-holder. In its center an adapter was mounted, into which fitted a 750-power eyepiece, whose tube was graduated into millimeters. A focus plate having first been secured and developed, the tail-piece was racked in or out so as to place the film in the focal plane. The aluminum plate was now attached and the eyepiece pushed in or out until a star appeared in good focus. The reading of the scale upon the eyepiece was then recorded. These operations were repeated on several evenings and the mean of all the eyepiece readings was adopted as standard. In order to set the plate to focus on a subsequent evening, without securing a focus plate, all that it is necessary to do is to focus the eyepiece upon a star, to obtain the reading on its scale, and then to rack the tail-piece in or out by an amount equal to the difference between this reading and the standard. This not only saves much time but it is actually more accurate than securing a focus plate; for it was found that the standard readings for the eyepiece scale, as deduced from different focus plates, varied by as much as 2 mm; while the eyepiece could be set to focus visually with considerably greater accuracy, if the definition were not too poor.

Later in these papers we will discuss the amount of error that is incurred by a maladjustment of the plate to focus.

Perhaps the most prolific source of error in early parallax determinations, particularly those made by means of photography, has been what Kapteyn calls the "hour-angle error." The complete explanation of its origin has not yet been given, but it is doubtless due, partly at least, to the cause assigned by Rambaut, who first called attention to this error under the name of "Atmospheric Dispersion."[1] Let us suppose that we have two stars in the same field, one of which is white and the other red. If the distance between the two stars is measured at such an hour-angle that the red star appears *above* the other, we shall get this distance too small, because the coefficient of refraction for blue rays, which are relatively richer in the white star, is greater than for the red rays. On the other hand, if we measure the distance between them at an hour-angle for which the red star appears *below* the other, we shall get a measured distance that is too large. In the present work, this source of error is less to be feared than is usually the case, since only a narrow region of the spectrum (λ 5200 to λ 5700) is effective in producing the images, no matter what the color of the star may be. It would be entirely feasible to eliminate this error, as well as any others that may depend upon the hour-angle, by investigating its effect with a properly planned series of plates, for each region under observation. But it is perhaps well to avoid the error altogether rather than to attempt to evaluate it; and this may be done by photographing the region, as Kapteyn has repeatedly urged, always in the same hour-angle and preferably near the meridian. This has accordingly been done for all the stars here discussed. It was of course not practicable to comply strictly with this condition, for the time with the Yerkes refractor is very precious; so that I did not wait until the next region came to just a certain hour-angle, but secured as many plates as I could during the night, even though some were taken, in extreme cases, as much as two hours from the mean hour-angle. An effort was made, however, to keep the mean hour-angle for the morning plates equal to the mean hour-angle for the evening plates of any one region. It can easily be shown that this procedure would almost entirely eliminate this error, if it is proportional to the hour-angle itself; and this must be approximately the case if the plates are taken not too far from the meridian. But even this less rigid condition could not always be complied with, and for a few of the regions the mean of the morning hour-angles differs considerably from that for the evening plates. In order that the reader may see for himself how these hour-angles run, this datum is set down for each plate in the tables of observations to be given later.

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[1] *Monthly Notices*, **55**, 123, 1895.

Another possible source of error is "optical distortion." This may be defined as an error in the place of stellar images, depending upon the orientation of the objective with reference to the telescope tube. If the objective were perfect in shape and homogeneity, of course no such error could exist. This subject has been studied by several astronomers, and with especial care in the case of the two astrographic refractors at Helsingfors and the Cape of Good Hope. In both cases it was concluded by Dr. Furness[1] and Professor Jacoby[2] that no optical distortion of appreciable size exists. But it would be unsafe to extend this conclusion to other objectives, and this remark acquires particular force for the Yerkes refractor. The great weight and diameter of the lenses, and the consequent sag when the telescope is pointed near the zenith, produce their effect upon the photographic images,[3] rendering them very slightly triangular, the vertices corresponding in position-angle to the three supports upon which the objective rests. This triangularity doubtless causes the bisections to be made at a slightly different place than if the image were perfectly round. The error varies with the magnitude of the star, since the triangularity is more pronounced with intense images and is in fact not at all apparent when the image is faint. The effect on the measures is in any case small, but it is very essential in parallax work to take it into the reckoning, where quantities of 0".01 become important if they are systematic. For this reason, it was decided not to reverse the telescope with reference to the mounting, but to observe with the tube always on the west side of the pier. For reversing a telescope of this form is equivalent to rotating the objective and its cell 180° around the optical axis, as referred to any configuration in the sky. If this were done, it is obvious that the distortion would at some times shift the images apparently to the east, say, and at others to the west. As the tendency would be to take all the evening plates with the tube to the east of the pier and all the morning plates with the tube to the west, almost the whole effect of the distortion would enter into the parallax determinations. It has been suggested by Jacoby that the objective be rotated 180°, around the axis of the tube, each time the telescope is reversed. This would eliminate the error, but as no such rotation was provided for in the construction of the telescope, and as it is questionable whether it would be advisable to rotate this objective, the only alternative was to avoid reversing the telescope.

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[1] *Publications of the Vassar College Observatory*, No. 1, p. 73, 1900.

[2] *Contributions from the Observatory of Columbia University*, No. 19, p. 74, 1902.

[3] A trace of the effect of sag seems also to be present in visual observations. See the paper by Keeler in the *Astrophysical Journal*, **3**, 154, 1896.

There are other reasons why I decided to observe with the tube always west of the pier: first, the driving clock was found to perform somewhat better in this position than in the other.[1] Observing from the west side, rather than the east, is in any case preferable, as it interferes with the work in the morning hours less than in the evening, and this is as it should be. Again, restricting operations to one side of the pier and to the neighborhood of the meridian preserves the constancy of conditions (so desirable on general principles in parallax work) as far as it is possible to do so.

Toward the beginning of the work the rule of observing only on one side of the pier was violated for a few plates. Attention will be called to each of these cases in the tables of observations.

As the focal length of the telescope is 19.36 meters, one millimeter on the plate corresponds to 10."6. The dimensions of the plates employed (20 cm by 25 cm) therefore correspond to about 35 and 45 minutes of arc. In order to secure suitably situated comparison stars in sufficient number within so small an area, it is usually necessary to use stars down to about the tenth magnitude, and for this purpose exposures of 5 minutes are ordinarily sufficient. With so long a focal length the size of the images varies greatly with the atmospheric conditions, and it is therefore possible, under the best conditions and when the field is a rich one, to reduce the exposures to 2 minutes. On the other hand, when the transparency is poor, and more especially when the definition is bad, it is necessary to double the normal exposures.

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[1] The remarkable faithfulness of this delicate mechanism, used in connection with so massive an instrument and in temperatures ranging from +30° to −30° C., is a source of constant admiration to all who have had the privilege of working with this telescope.

The duration of exposure is fixed by the brightness of the comparison stars, rather than by that of the parallax star.[1] The latter is usually considerable brighter than the former, occasionally by as much as seven magnitudes for some of the stars here investigated. But even when the difference is only two magnitudes it becomes a matter of difficulty, on a plate for which the comparison stars are sufficiently strong, to measure the overexposed and broad image of the parallax star. Some device for enfeebling the light of a bright star is therefore a necessity. To this end, I suggested several years ago[2] that the film might be rendered less sensitive in the small area upon which the bright image will fall, by local washing with some colored fluid. A somewhat similar but more simple device was suggested to me with some hesitation by Dr. H. M. Reese: to expose the plate in the usual way, to develop first until the image of the bright star just appears, to "fix" the plate at this point only, by the use of a drop or two of hyposulphite, and then after washing to continue the development so as to bring out the images of comparison stars. Dr. H. N. Russell[3] has recently put in front of the photographic plate a sheet of plane-parallel glass, in the center of which, and nearly in contact with the film, is mounted a yellow patch of absorbing material. This appears to have worked well except as to permanency, a matter that there should be no great difficulty in overcoming.

None of these devices is quite satisfactory for the purpose here in view, for it is not only necessary to reduce the image of the parallax star to such an extent as to make it measurable, but it is hardly less desirable to be able to control its brightness within narrow limits, say two or three tenths of a magnitude, in order that its image on the developed plate may be closely equal in intensity to the mean for the comparison stars. The reason for this lies in the "guiding-error," which is perhaps the largest item of accidental error still outstanding in general photographic work. This is due to the impossibility of keeping the images of the stars exactly stationary upon the plate during the exposure. They will all wander more or less from their mean positions, and while these excursions are equal in amount and direction for all the stars upon the plate, they are registered to a different degree for bright and for faint stars. Consequently the position of any image will depend to some extent upon its intensity. Now the parallax computations may roughly be said to consist in subtracting for each plate the mean of the measures upon the comparison stars from that upon the parallax star. It is evident, therefore, that the effect of guiding-error will be almost entirely eliminated if the magnitude of the parallax star can be reduced to the mean of the magnitudes for the comparison stars.

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[1] For the sake of brevity, we shall use this designation for the star in each field whose parallax we desire to determine.

[2] *Astrophysical Journal*, **10**, 243, 1899.

[3] *Ibid.*, **26**, 147, 1910.

With this in mind I decided to try some mechanical method for occulting the light of the parallax star, and mounted an ordinary photographic shutter, about 4 cm in diameter, in the center of the plate-carrier. This was operated in the usual way by a tube and a hand bulb. The exposure began with the shutter closed, so that the light of the parallax star did not reach the plate. The observer then opened the shutter for a few seconds at a time and at intervals symmetrically distributed over the whole exposure. The developed plate showed excellent images for the parallax star, in no way distinguishable in appearance from those of the comparison stars. I had feared that the effect of diffraction due to the shutter might be visible, but no trace of this appeared. When, however, we measured the four plates (each with three exposures) secured in this way, we found that the probable error of the relative position of the parallax star was threefold that for ordinary plates; that is, plates on which the parallax star was, to begin with, about as faint as the comparison stars. The explanation of this is the strong tendency of the observer to avoid opening the shutter to the parallax star unless the guiding star is exactly bisected by the cross-wires; but the guiding star usually shows a tendency to drift off the wires in a definite direction, let us say to the right, for any one plate. Consequently the position of the parallax star, as obtained by means of the shutter, corresponds closely to the intersection of the cross-wires, while the comparison stars correspond to points a little to the right.

This experiment indicated the necessity for occulting the star automatically and continuously without the intervention of the observer. Accordingly a rotating disk with an opening of variable aperture was constructed, something after the fashion of an Abney disk photometer. This is mounted directly in front of the photographic plate, with its center a little below that of the plate itself, as shown at *O* in Plate XIX. The disk is rotated perhaps six or eight times a second, by means of a small electric motor *M* just outside the plate-carrier, the motor being connected with the disk by means of a cord belt. The two halves of the disk can be rotated with respect to each other and then clamped, leaving a clear sector of any angular aperture desired up to nearly 180°. A scale and an index are provided so that the angle of opening can be set with accuracy. The whole device can be put into place or removed in a minute or two.

The method of using the rotating disk is as follows: the telescope is pointed so as to bring the parallax star to the center of the field, just above the axis of the disk. The opening of the latter is then set, let us say, at 36°, and the disk is put into motion. The exposure now begins; it is evident that only one-tenth of the light of the parallax star reaches the plate in several hundred intermittent exposures, lasting a few hundredths of a second each. Since the jaws of the opening are radial, it is obvious that this fraction of one-tenth is independent of the relative positions of the star and the disk, and of the speed at which the latter is rotated. The parallax star will, in this case, be apparently reduced in brightness by 2.5 magnitudes. For each region observed the amount of the opening was determined once for all by experiment, so as to reduce the apparent magnitude of the parallax star to the mean of the comparison stars.

This method was found to be practicable up to reductions of 7 magnitudes, corresponding to an angular opening of a little more than $0^\circ.5$. For smaller openings than this, that is, for greater reductions, the uncertainty in setting becomes a considerable fraction of the whole opening, since the graduated scale is necessarily small on a disk that must not cover too large a portion of the photographic plate. The images thus obtained, like those with the photographic shutter, show no trace of diffraction effects due to the disk, and their measurement proves to be as accurate as upon unobstructed images of fainter stars. Some of the parallax stars in the present program are not very much brighter than the comparison stars, and for these the disk was not used. For similar work in the future, I should use the disk in relatively more cases than I have here, extending it to those fields in which the parallax star is only slightly brighter than the comparison stars. For experience has shown that the uncertainties in the measured positions, doubtless chiefly because of guiding-error, are considerably greater for a field in which the parallax star is as little as one magnitude brighter than the comparison stars, than for a field in which the parallax star was reduced by the rotating disk to approximate equality with them.

The question as to how many exposures should be made upon each plate is an important one. At the outset I decided provisionally upon three; and later, when opportunity was afforded to investigate this matter from a discussion of measures made upon the plates, this number was definitely adopted as being about the most economical. In common with other observers, I found that measures of exposures upon the same plate have a tendency toward better agreement than between exposures on plates secured on different nights. From this point of view alone, therefore, only one exposure should have been impressed upon the plate. But the time consumed in setting the telescope upon a region, finding the guiding star, etc., is large compared with the length of the exposure itself, and consequently three exposures can be secured in less than double the time that a single exposure would require. It consumed an average of about fifteen minutes to change plates in the holder, to point the telescope to a new field, to set the scales on the finding eyepiece slide, the position circle, and the rotating disk, and to find the guiding star. Thus for five-minute exposures the first was obtained after a lapse of about twenty minutes, and the third after a lapse of about thirty-two minutes. For longer exposures the gain in time is of course relatively less.

The complete operation at the telescope was, therefore, as follows: Shortly before dark the focus was obtained by the visual method described. The telescope was now pointed to a star in the program that was then on or near the meridian. The guiding for the first exposure was executed by myself; the guiding eyepiece slide was then moved to the right or left so as to secure a second exposure upon the plate separated from the first by 5 mm, and the guiding for this was done by Mr. Sullivan. At its completion the slide was moved 5 mm to a third position and the guiding was resumed by myself. While Mr. Sullivan was guiding for the second exposure, I made the necessary notes and selected the next region to be photographed. Only one plate-holder was available, and at that time there were no facilities for changing the plates in the dome; so that it was necessary to take the holder to a dark-room in the observatory below, to store the exposed plate, and to put a fresh one in the holder. While I was doing this Mr. Sullivan pointed the telescope to the next star, from the information given on the "observing-card,"[1] rotated the dome, and put the rising floor into proper position. This process was repeated until about ten o'clock, after which time, and until well into the morning, it is not profitable to continue this work. The "parallax factors" of stars that are on the meridian near midnight are small (except in rare cases) and plates taken under these conditions would add little to the observational material. Accordingly the opportunity was taken to develop a few of the plates just secured, in order to make certain that all was going well. The telescope was then used for other photographic work (such, for example, as securing plates of "loose" clusters) until about two o'clock, when the parallax work was resumed. It will be seen that Mr. Sullivan's assistance greatly facilitated these operations; it increased the output of the telescope, as compared with what one alone would have been able to accomplish in the same time, by at least 30 per cent.

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[1] This card gave the position of the parallax star, the settings for the position circle, the guiding eyepiece slide, and the opening in the rotating disk, the normal length of exposure, and other information, such as a diagram of the stars in the field, that would enable the observer to begin the exposure with as little loss of time as possible.

The plates that have been measured and reduced for the purposes of the present papers number 327, and they relate to 25 different regions. Of these plates 201 were secured by myself, and the remaining 126 by Messrs. Jordan, Fox, and Sullivan. In addition, many other parallax plates were obtained successively by myself, by the observers named, and, since September 1909, by Dr. Frederick Slocum. These are to form the basis of subsequent studies of stellar parallax at the Yerkes Observatory.

Allegheny Observatory
December 1910

THE SOLAR ROTATION FROM THE MOTION OF THE FACULAE ON THE DISK (1906–1908)

By STANISLAS CHEVALIER

The history of this subject is short and well known. Only two investigations have to be considered, namely, that of Wilsing of the Astrophysical Observatory of Potsdam, who inferred that the rotation was uniform for all latitudes; and that of Stratonoff who, contrariwise, found that the phenomenon known as the equatorial acceleration extended to the stratum of the faculae. Although the conclusions of the latter were actually beyond doubt, it was clear that a new examination of the problem which should check, and increase the precision, if possible, of the values thus obtained, would not fail to be useful. Moreover, the recent investigations of Messrs. Hale and Adams upon the rotation of the flocculi and that of the reversing layer call for a more accurate measurement of the rotation from the faculae. It is for this reason, on the advice of Mr. Hale, that we have attempted to derive from the photographs of the Observatory of Zô-sè a new determination of that rotation.

Preparation of the photographs.—We began by examining all the negatives of the years 1906–1908, in order to select those which would be able to furnish data for this work. It is necessary to find two photographs separated by an interval of about twenty-four hours, both of which are as sharp as possible, to permit of recognizing the same faculae with certainty on both of the plates. It does not suffice to find on them one and the same group, but it is necessary to be able to determine the same small facula and the same detail distinctly marked in the group. For this work of reconnaissance it is necessary to examine the two photographs side by side very carefully, with the assistance of eyepieces having greater or less power. These identical details once found must be marked in red ink on the varnished film of celluloid with which we cover all our negatives of the sun. This point should be marked as accurately as possible and in the same manner upon the two photographs, for the measurer has to determine the heliographic position of the point. The method of pointing is certainly not ideal, and it introduces a new source of error, which will be of the same order as errors of measurement. But with objects so delicate and difficult of recognition, it was absolutely indispensable.

This work of identification is important and difficult, for the observer has no guide except the similarity of the form, and occasionally the relative position of the spot. The mobility of the faculae is furthermore such that it is generally impossible to be absolutely certain of the identity of the two faculae. It is necessary to content oneself with a greater or less degree of probability. It is, therefore, inevitable that one must make some errors of identification. Sometimes the faulty identification of the faculae marked appears on the first attempt at measurement: then there is nothing to do but to discard it without giving it further attention. But there are cases where the error remains doubtful, where one does not know whether he is dealing with a facula having a very large proper motion, or whether there are two faculae of the same group more or less closely resembling each other. The observer is in the well-known situation where the extreme residuals must be rejected.

Measurement of the plates.—The sources of error, and very appreciable ones, are numerous and inevitable in the measurement of the rotation of the sun from faculae. The proper motion of the faculae, their real changes of form, which occur often and rapidly, their changes of form due to perspective, the fact that their position is always so near the limb, which interferes with the precise determination of their longitude—these are all necessary causes, to which we are obliged to add our errors of pointing. One therefore cannot hope to obtain anything more than a tolerable accuracy in the measures, except by making a large number of observations in each zone.

It accordingly becomes indispensable to adopt some method of measurement which permits the determination of many thousands of heliographic positions without excessive labor. Since the beginning of our work on the sun we have employed a very simple procedure in measuring heliographic positions on our photographs. It consists in placing upon the image of the sun a transparent chart of the solar meridians and parallels, such as would be imprinted upon the image if they really existed. With the chart selected, and accurately fitted upon the solar image, the heliographic position of any object whatever upon the surface is obtained by a single reading. As our first charts seemed to be too imperfect, a series of new ones on a larger scale (disk of 40 cm diameter) were prepared for the work. The series includes fourteen charts, each differing from the preceding one by increasing the latitude of the center by half a degree. From these charts the parallels are traced by alternate degrees from zero to 42°, and the meridians are traced at the same interval up to 80°. At the center of the chart one degree is equivalent to 3.5 mm, and the error of the ruling does not exceed 0.1 mm, or 0°03. Near the edges the relative error is naturally somewhat larger. These charts, drawn by our draftsman, Mr. F. Tsang, have appeared to be very satisfactory.[1] Every one of them has been photographed several times, to conform to the size of the solar image at different dates.

The maximum difference which is permitted to occur between the latitude of the center of the solar disk and that of the center of the chart employed is 15′. Its effect, which is nearly zero on the longitude, is more sensible on the latitude, but it is possible to cancel it by a double measurement with two charts between which the true latitude of the center is comprised. Mr. Hale, to whose examination I submitted one of the charts, as well as the plan of employing them in our work, replied as follows:

> I requested Professor Seares, superintendent of our computing division, to make a critical examination of the method of measurement and to report the results to me. . . . . Professor Seares believes that if a sufficient number of measuring scales is available, the error of measurement of a position should not exceed 0°3 to 0°5. It seems probable to me that results obtained by this method will be sufficiently precise, especially in view of the nature of the faculae and their rapid change in form.

The sole difficulty of the measurement consists in accurately fitting the chart to the solar image so that the equator of the chart coincides with that of the sun, and the central meridians and the edges are accurately superposed. The two photographs are then placed between the clamps of a support which keeps them pressed one upon the other, before the eye of the observer. Using a magnifier, he has only to verify the coincidence and read the heliographic position. To diminish errors, but particularly to avoid the introduction of an error of 2° in the reading, which might easily be made, all the measures are done in duplicate and are verified in case of doubt. All these measures were made by Mr. Sinow Zeng, chief of the bureau of solar measures, and upon examination of the double series of independent measures, it is manifest that the readings were very well made.

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[1] An example of these charts and a description of the method used in drawing them may be found in Vol. IV of the *Annales* of the Observatory of Zô-sè.

Calculation and formation of the tables of observation.—Five hundred and seventy-two plates combined in pairs have been utilized in the work, from which 5216 heliographic positions of faculae have been derived. The positions of the same facula picked out on two plates separated by an interval of time θ, generally about 24 hours, give by a simple difference the observed motion of the facula during the time θ. The observed motion in longitude multiplied by the ratio $24:\theta$ gives the synodic rotation of the sun in 24 hours. To pass from this to sidereal rotation, it is only necessary to add the projection on the solar equator of the motion of the earth during the twenty-four hours. The 5216 observations of the faculae have furnished 2608 measures of the diurnal sidereal rotation of the sun. All the observations have been reduced to tables of nine columns, containing: the date of the photographs and the factor $24:\theta$; the successive number of the observed faculae, arranged by dates of the plates and upon each plate according to the latitude; the mean latitude and the observed variation in latitude; the distances to the central meridian measured on the two photographs, and the motion in longitude, which is deduced from it; the synodic rotation in 24 hours, and, finally, the diurnal sidereal rotation, which we shall designate by ξ, as is customary. The examination of the tables has led us to eliminate 33 values, for which the variation in latitude, combined with the variation in longitude, implied a proper motion of the facula equal or superior to 3°. The reason for adopting that limit was that a summary examination gave $0\overset{\circ}{.}5$ as the mean error of a measure.

Procedure, and result of observations.—To make use of the 2576 measures of the rotation which remain, it was necessary to construct new tables which would be arranged, not chronologically, but according to latitude. For these abridged tables, which will be published in the *Annales* of the Observatory of Zô-sè, we have adopted the division into zones of 5°, from the equator to the two poles. Besides furnishing a number of zones, limited but sufficient, this division has the further advantage of being more readily adapted to a comparison between our results and those of other observers. We shall give here only the final results of the observations, i.e., the rotation ξ obtained for each zone of five degrees, the number of observations made in the zone, the mean latitude of the faculae measured, the mean error of one measure of rotation, and the probable error of the mean. For completeness, we shall add the mean of the variations of latitude observed in each zone.

Latitude Zones	Number of Observations	Mean Latitude	ξ	Probable Error of the Mean	Mean Error of an Observation	Mean Variation in Latitude
+30° to +25°.....	42	26°.6	13°.900	±0°.113	±0°.495	0°.50
25 to 20......	97	22.2	14.168	0.088	0.582	0.55
20 to 15......	246	17.6	14.209	0.049	0.521	0.50
15 to 10......	371	12.4	14.279	0.039	0.508	0.50
10 to 5......	312	7.9	14.451	0.044	0.530	0.45
5 to 0......	101	3.0	14.471	0.071	0.479	0.55
0 to −5......	97	3.3	14.521	0.076	0.505	0.49
−5 to −10......	249	7.9	14.418	0.050	0.531	0.51
−10 to −15......	375	−12.5	14.390	0.036	0.467	0.43
−15 to −20......	359	−17.4	14.251	0.040	0.508	0.44
−20 to −25......	222	−22.2	14.209	0.051	0.507	0.53
−25 to −30......	104	−26.8	14.042	0.063	0.432	0.54

In order to give a still clearer idea of the solar rotation from faculae in the above table, we have drawn a curve of the values in the column under ξ. The second curve of that figure, which is entirely regular, was drawn from the means of the same observations, but grouped by zones of 10°.

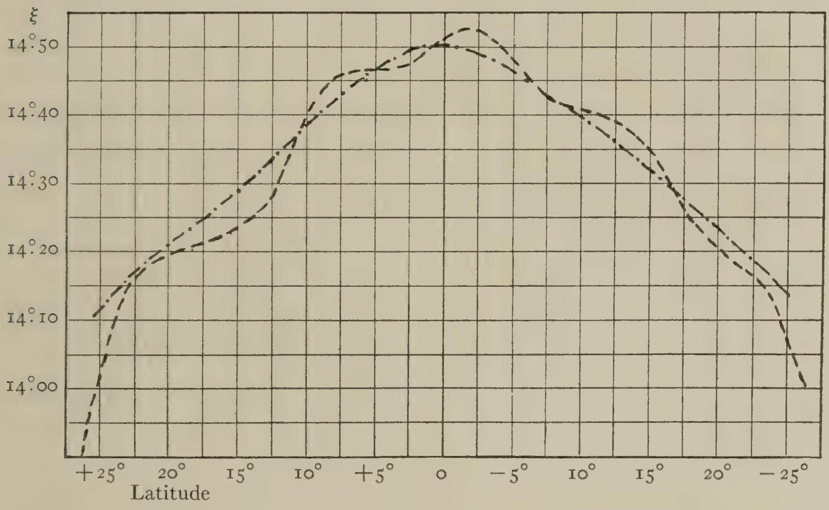
Briefly summarized, the conclusions which may be drawn from the curves are as follows:

1. The equatorial acceleration is so clearly indicated in the whole zone from +30° to −30°, over which our observations extended, that it is absolutely impossible to fail to recognize it.

2. The acceleration is very regular, even on the curve of five-degree zones, and its irregularities may properly be assigned to errors of observation.

3. The variations seem to be slightly less in the vicinity of the equator than in higher latitudes. This is furthermore entirely natural, since the curve passes through a maximum at about 0°.

4. The difference between the two hemispheres is so clearly indicated that it is difficult to attribute it to an accidental arrangement of errors of observation. All the values of ξ in the southern hemisphere are larger than the corresponding ones in the northern hemisphere. There is but a single exception for the zone $+5°$ to $+10°$, and further, the general course of the curve leads us to think that the value of the zone $+5°$ to $+10°$ is a little too large. It therefore seems highly probable that during the period of three years, 1906–1908, the mean rotation has been actually a little more rapid in the southern hemisphere than in the northern.



Solar Rotation from motion of the faculae (1906–1908)
— — — from 5° zones; —·—·— from 10° zones

5. There is another very remarkable difference between the two hemispheres. The variation of velocity as a function of the latitude is more rapid in the northern hemisphere than in the southern.

6. According to the curve based on the observations by five-degree zones, the maximum velocity would not be found exactly at the equator, but a little south of it.

The last point is, however, altogether doubtful; for it depends only upon the values obtained for the equatorial zone, which are less certain on account of the more limited number of observations.

In grouping the observed values of ξ by zones of 10°, we obtain in each hemisphere three points which are nearly on a right line. Their arrangement entirely contradicts the remark under 6, although it confirms the others.

Despite the fact that the differences between the two hemispheres seem to be real, as they are neither certain nor constant, they ought not to prevent us from combining the zones of the same latitude north and south to obtain a mean value which should be more accurate and more assured. This mean is given in the following table. We have included the mean latitude corresponding to the mean value of ξ, and the number of days necessary for a complete solar rotation with the velocity assigned.

Zones	ξ	Probable Error	Latitude	Period of Rotation in Days
0° to 5°	14°495	±0°033	3°2	24°836
5 to 10	14.436	0.021	7.9	24.937
10 to 15	14.335	0.017	12.4	25.129
15 to 20	14.236	0.020	17.3	25.288
20 to 25	14.197	0.032	22.2	25.357
25 to 30	14.001	0.041	26.8	25.712

Two formulae are almost equally well adapted to express this result, the one a function of the square of the sine, and the other a function of the cosine of the latitude, viz.:

$$\xi = 14^\circ\!.470 - 2^\circ\!.268 \sin^2\lambda = 868'\!.2 - 136'\!.1 \sin^2\lambda;$$
$$\xi = 10^\circ\!.140 + 4^\circ\!.331 \cos\lambda = 608'\!.4 + 259'\!.9 \cos\lambda.$$

These two formulae give sensibly the same velocity of rotation for the different latitudes, within the limits of observation, but it is clear that they could not be extended to a higher latitude.

Comparison of our results with those of other observers.—As there is only one other attempt at the measurement of solar rotation by the faculae, that of Stratonoff of the Observatory of Pulkowa, we must first of all compare our results with his. Since Stratonoff's memoir is not available to me, I will copy his values from the volume by Messrs. Hale and Fox, *The Rotation Period of the Sun as Determined from the Motions of the Calcium Flocculi*, No. 93 of the *Publications of the Carnegie Institution of Washington*. The following table gives a comparison of the two series of determinations at Pulkowa and at Zô-sè. We have added the number of observations made in each zone to the value of the rotation found for the zone. To the table of mean values based on the same observations for the two hemispheres we have given a column for the probable error of the mean.

ZONES	NORTHERN HEMISPHERE				SOUTHERN HEMISPHERE			
	Pulkowa		Zô-sè		Pulkowa		Zô-sè	
	Number	ξ	ξ	Number	Number	ξ	ξ	Number
0° to 5°	9	14°62	14°471	101	...	.....	14°521	97
5 to 10.....	39	14.61	14.451	312	9	14°63	14.418	249
10 to 15.....	125	14.34	14.279	371	67	14.26	14.390	375
15 to 20.....	110	14.14	14.209	246	124	14.21	14.251	359
20 to 25.....	124	14.21	14.168	97	137	14.17	14.209	222
25 to 30.....	109	13.97	13.900	42	101	14.20	14.042	104
30 to 35.....	15	13.50	......	...	34	13.65	......	...
35 to 40.....	...	.....	......	...	24	13.61	......	...

ZONES	MEANS OF BOTH HEMISPHERES			
	Pulkowa		Zô-sè	
	ξ	ϵ	ξ	ϵ
0° to 5°	14°62	±0°127	14°495	±0°033
5 to 10.....	14.61	0.061	14.436	0.021
10 to 15.....	14.31	0.044	14.335	0.017
15 to 20.....	14.18	0.036	14.296	0.020
20 to 25.....	14.19	0.036	14.197	0.032
25 to 30.....	14.08	0.040	14.001	0.041
30 to 35.....	13.60	0.059	......	.....
35 to 40.....	13.61	0.086	......	.....

It seems evident, owing to the much larger number of observations, that our series marks an actual progress in the study of the question. The superiority of our series follows less from the comparison of probable errors than from the regularity of the values obtained. If the accordance of the two series is not complete, it is nevertheless very satisfactory, at least for the zones where the Pulkowa series has a sufficient number of observations. The discordance is hardly worth considering, except in the two equatorial zones from 10° to 0°, but for those two zones Pulkowa has only a very small number of measures, forty-eight for one and nine for the other. The value $14^{\circ}.61$ for zone 5° to 10° is in accordance neither with those of a higher latitude, nor with those of the zones 0° to 5°.

Comparison with the rotation derived from spots.—We now should compare the rotation we have determined from the faculae with that from the motion of spots. The spots lie in the photosphere, while the faculae float at a great distance above them. Do these two strata share in the same rotation? This comparison ought to settle the question. Unfortunately in spite of all the work on the spots and the solar rotation according to the motion of the spots, we have no expression of that rotation of which we may be truly certain. The formulae proposed, following that of Carrington, are numerous, but they are in very poor agreement among themselves, as well as with those which we have derived from the movement of the faculae. They are:

Carrington, $\xi = 865' - 165' \sin \tfrac{7}{4} \lambda.$
Faye, $\xi = 862' - 186' \sin^2 \lambda.$
Spoerer, $\xi = 512'.9 + 347'.9 \cos \lambda.$
Tisserand, $\xi = 857'.6 - 157' \sin^2 \lambda.$

The most extensive work on the subject, and that which has involved the most numerous observations, seems to be that of Mr. and Mrs. Maunder, entitled "Solar Rotation Period from Greenwich Sun-Spot Measures," *Monthly Notices*, **65**, 813, 1905. Out of 4700 groups of spots observed from 1879 to 1901, during two eleven-year cycles, the Maunders have retained only 1872 groups, which were observed at least for six days. They obtained two different expressions for the rotation: the one derived from all the groups, each of which was treated individually with regard to its transit across the disk; and the other from the spots of long duration observed at least on two successive transits:

From the first, $\xi = 875'.7 - 164' \sin^2 \lambda$.
From the second, $\xi = 866'.6 - 128' \sin^2 \lambda$.

The second of these formulae is in almost complete accord with that which we have derived from the movement of the faculae. It seems furthermore to deserve more confidence than the first formula, for, in the first place, it is less compromised than the first by the objections which may be raised against Mr. Maunder's method; in the second place, as that author remarked, "there is no doubt that these groups [long-lived groups] are much more free from the effect of accidental motions than the groups when considered separately at each apparition." But, in view of such different formulae to represent the same phenomenon, the observer is in much perplexity. Each formula represents the mean rotation of a certain number of spots, which may differ from the mean rotation of a certain number of other spots, and differs from the rotation of the photosphere. The last fact is the most important one to consider, for to make deductions from the rotation of a certain number of spots, it is necessary that the sum of their proper motions should be zero. We are consequently under the necessity of employing in this research only those spots which have small proper motions, under penalty of vitiating the result, and of comparing among themselves the heliographic positions of points that are well determined.

The conclusion of the comparison which we have attempted to make seems to us clearly to indicate that the rotation of the faculae does not differ more from the rotation of the spots than the rotations of different spots differ among themselves. To make a more accurate comparison of the rotations of these two superposed strata, it would be necessary to know both with a greater degree of certainty.

Comparison with the rotation determined from the flocculi.—There is only one preliminary work upon this rotation, that of Messrs. Hale and Fox, already cited. We are therefore able to compare with provisional results only. It is sufficient to say that it is necessary that we should not be hasty in drawing conclusions. It will moreover be useful to place before the eyes of the reader the two results as we have arranged them in the following table:

ZONES	NORTHERN HEMISPHERE				SOUTHERN HEMISPHERE			
	Flocculi		Faculae		Flocculi		Faculae	
0° to 5° . .	14°72	±0°031	14°47	±0°071	14°57	±0.045	14°52	±0°076
5 to 10. . .	14.50	.027	14.45	.044	14.55	.030	14.42	.050
10 to 15. . .	14.34	.024	14.28	.039	14.39	.020	14.39	.036
15 to 20. . .	14.14	.025	14.21	.049	14.30	.028	14.25	.040
20 to 25. . .	14.13	.035	14.17	.088	14.11	.038	14.21	.051
25 to 30. . .	13.74	.060	13.90	.113	14.03	.073	14.04	.063
30 to 35. . .	13.64	.073	. . . . .	. . . . . .	13.93	.120	. . . . .	. . . . .

ZONES	MEANS OF BOTH HEMISPHERES			
	Flocculi		Faculae	
0° to 5°	14°66	±0°026	14°50	±0°033
5 to 10.	14.52	.020	14.44	.021
10 to 15.	14.37	.016	14.34	.017
15 to 20.	14.22	.019	14.24	.020
20 to 25.	14.12	.026	14.20	.032
25 to 30.	13.90	.049	14.00	.041
30 to 35.	13.76	.067	. . . . .	. . . .

According to these preliminary results the rotation of the flocculi would seem to be more rapid at the equator than that of the faculae; but their variation of velocity as a function of the latitude would be more much rapid than for the faculae.

Comparison with the rotation of the reversing layer.—The memoir by N. C. Dunér is still the most complete work upon the rotation of the reversing layer, at least to our knowledge. According to Dunér, the rotation of the absorbing stratum is well represented by the following formula:

$$\xi=14^{\circ}.807-4^{\circ}.172\sin^2\lambda,$$

while we have found for the faculae

$$\xi=14^{\circ}.470-2^{\circ}.268\sin^2\lambda.$$

The rotation of the reversing layer would therefore be still more rapid than that of the flocculi. As for the flocculi, the variation as a function of the latitude is still more rapid than that which we have found.

The last two comparisons would favor the supposition that the acceleration derived from our measures may be a little too small, but its agreement with the formula which Mr. and Mrs. Maunder have derived from a study of spots of long duration is, on the contrary, favorable to its correctness. This question, therefore, still requires new investigations before it can be fully cleared up.

Observatoire de Zô-sè
China

Reviews

Physik der Sonne. Von E. PRINGSHEIM. Leipzig and Berlin: B. G. Teubner, 1910. 8vo, pp. 435, with 235 diagrams and 7 plates; cloth, M. 18.

Never in the history of astronomy has the interest in solar research been so great as at the present time. The perfection of the spectroscope, the invention of the spectroheliograph, the construction of great astrophysical observatories, and the activity of the International Union for Co-operation in Solar Research have all contributed largely toward this end. Great discoveries and new theories have followed each other so rapidly that it has been difficult to keep pace with them. Under these conditions any book on the subject must necessarily be somewhat out of date by the time it leaves the press. However, in the *Physik der Sonne* we find an excellent résumé of the status of the scientific knowledge of the sun down to the year 1909.

For many years Professor Pringsheim gave a course of lectures under the above title at the University of Berlin, and the book is the outcome of these lectures. They were open to students of all the faculties and were therefore popular in nature. The book likewise will appeal not only to astrophysicists and physicists, but to all who are interested in general science.

In the introduction the author deals with the sun as the source of all terrestrial energy and considers its influence upon the life on the earth. In successive chapters he discusses the distance, size, and mass of the sun; the photosphere; solar rotation and periodicity of solar activity; spectrum of the sun and its chemical composition; solar eclipses; chromosphere and prominences; solar theories; corona and solar atmosphere; flocculi, vortices, and Zeeman effect; and radiation and temperature of the sun.

For many readers the most interesting chapters of the book will be those that deal with the theories of the sun. "Die Theorien entstehen und vergehen wie die Blätter der Bäume und die Geschlechter der Menschen," as the author says. On account of their historic interest, several of the older theories are given in outline; e.g., the theories of Cossini, Wilson, Bode, Herschel, Kirchhoff, Zöllner, Respighi, and Lockyer; while the more modern theories of Young, Schmidt, Schwarzschild, Julius, Emden, and Oppolzer are discussed more in detail. The anomalous dispersion theory of Julius is given more attention than any other. The author says that this theory has always been taken more seriously by physicists than by astrophysicists. This is perhaps quite natural. It is very difficult for one who has observed sun-spots, faculae, flocculi, and prominences to accept a theory which explains them as mere illusions.

Interesting summaries of numerical data appear throughout the book. The latest value of the sun-spot period is given as 11.125 years, with sub-periods of 8.344 and 4.768 years. From a discussion of the values of the solar constant from the determination of Pouillet in 1837 down to that of Abbot and Fowle in 1908, a mean of 2.2 is derived, and from this an effective temperature of 6033° absolute.

Tables of the distribution of light, heat, spectral and chemical intensity over the sun's disk are given, as well as the transmission coefficients for different wave-lengths. From a combination of the determinations by different methods of the daily velocity of rotation, mean values are derived ranging from 14°7 at the equator to 11°8 at about latitude 75°. These values will be somewhat changed by the more recent investigations of Adams, Perot, and others.

The illustrations are for the most part poor, partly due to the poor paper upon which they are printed. Figs. 113 and 114 have been so retouched in the process of reproduction that they bear little resemblance to the originals.

Attention may be called to a few mistakes. The date of Fig. 101, p. 181, is May 28, 1900, not 1893 as stated on p. 182. To preserve the historic interest of Fig. 103, p. 182, the exact date July 9, 1891, should be added. Referring to the working of the Rumford spectroheliograph, the two slits are fixed and the exposure is made by moving the sun's image over the first slit and the photographic plate over the second, and not as stated at the foot of p. 187. Figs. 108, 109, 110, and 111 are all from photographs taken at the Kenwood Observatory, not at the Yerkes Observatory as stated on p. 189. In Fig. 116, p. 195, the use of C between D_3 and F, apparently to denote coronium, is misleading, if not actually incorrect. Pringsheim himself calls attention to the fact that the chromospheric line 1474 K has a wave-length of 5316.8 and is not identical with the chief coronal line, the wave-length of which is now given as 5303.3. In Riccò's original plate in the *Comptes Rendus* this line is marked *b*. Fig. 183, p. 311, is from a drawing by Professor William Harkness based on twelve photographs.

Throughout the work original sources are cited, forming an excellent bibliography of the whole subject.

FREDERICK SLOCUM

Les théories modernes du soleil. Par J. BOSLER. (*Encyclopédie scientifique.*) Paris: Octave Doin & Fils, 1910. Pp. 370, with 49 diagrams. Fr. 5.

It is a question whether the observations of any natural phenomena have ever been explained in a greater variety of ways than the observations made on the sun. According to the various theories the same phenomena may be explained by the principles of conventional circulation, chemical dissociation, thermodynamics, electrodynamics, refraction, radioactivity, anomalous dispersion, etc.

M. Bosler has set forth most of the different theories in a very elementary way, and, in some cases, has added comments on the merits of the theory. It is interesting to note how the development of the theories has paralleled the development of science, and the reader will undoubtedly feel that the development must proceed farther before a satisfactory theory is found.

The illustrations are, with only a few exceptions, very poor. Fig. 45, p. 321, should be inverted, the legends remaining as they are.

FREDERICK SLOCUM

The Spectroscope and Its Work. By H. F. NEWALL. London: Society for Promoting Christian Knowledge (New York Agent, E. S. Gorham), 1910. 12 mo, pp. 163, with 58 figures, eight half-tone plates, and a frontispiece of colored spectra. 2*s.* 6*d.*

This appears as one of the "Manuals of Elementary Science" of the society named above. It should fulfil its purpose admirably, for it gives a sketch of nearly the whole field in a clear and simple manner. Difficult as is the task of writing a treatise on a given specialized department of science, that of selecting the main points and then compressing them into a primer of 160 pages is in some ways a greater one.

The first thing that strikes the reader is the fresh method of presentation of what is to some a familiar subject: the author has used his own original ideas, without being over-influenced therein by the classical works of earlier days. The figures are new and appropriate: one misses the old-time woodcuts, some of which have done good service for nearly threescore years since their first appearance in memoirs of the pioneers of spectroscopy.

The book is written also to encourage personal observation in a fascinating subject. The historical facts are not omitted, however, and room is found for quotations from Newton's memoir on optics of 1675. After briefly describing the essentials of the spectroscope in an early chapter, the author later reverts to the subject in a particularly useful chapter entitled "Increase of the Power of a Spectroscope." This will be of value to many besides beginners in spectroscopy.

Five chapters are devoted to the work of the spectroscope in its astronomical applications, and these necessarily have to be brief. Two valuable chapters are entitled "Law in the Spectrum," one dealing with the continuous spectrum, the other with bright-line spectra. An excellent brief exposition of diffraction and the measurement of wave-length is given in chap. xiv. The final chapter treats of variations in the spectra of gases and vapors.

The plates are good, and the colored frontispiece should help the general reader in understanding the fundamental differences of spectra. The book deserves a wide circulation and is further recommended as a good work for collateral reading by classes in general astronomy.

E. B. F.

INDEX TO VOLUME XXXII

SUBJECTS

PAGE

ABSORPTION in the Red of the Acetate, Nitrate, and Sulphate of Cobalt. *Frank L. Cooper* 309

Algol, Measurement of Light of Stars with Selenium Photometer, with Application to Variations of. Joel Stebbins . . . 185

ALKALI Metals and Helium, Formulae for Spectral Series for. *R. T. Birge* 112

BARIUM, Series in Spectra of Calcium, Strontium, and. *F. A. Saunders* 153

BINARIES, Measures on Nineteen New Spectroscopic. *Oliver J. Lee* . 300

BROMINE at High Temperatures, Absorption Spectrum of. *E. J. Evans* 291

CALCIUM, Strontium, and Barium, Series in Spectra of. *F. A. Saunders* 153

Castor, Mass-Ratios in Systems of *Krüger 60* and. *Henry Norris Russell* 363

COBALT, Absorption in Red of Acetate, Nitrate, and Sulphate of. *Frank L. Cooper* 309

COLLIMATION of the Correcting Lens. *J. S. Plaskett* . . . 243

COLORS, Note on Bell's Paper on Star. *R. T. A. Innes* . . . 321

COMET as Measured with Selenium Photometer, Brightness of Halley's. *Joel Stebbins* 179

FACULAE on Disk, Solar Rotation from Motion of. *Stanislas Chevalier* 388

GASES, Studies on Emission of. II. *H. Konen* and *W. Jungjohann* 141

HELIUM, Formulae for Spectral Series for Alkali Metals and. *R. T. Birge* 112

INTERNATIONAL Union for Co-operation in Solar Research. *Edwin B. Frost* 258

IODINE Vapor at High Temperatures, Absorption Spectrum of. *E. J. Evans* 1

IRON, Secondary Standards, International System, in Arc Spectrum of. *H. Kayser*, *Ch. Fabry*, and *J. S. Ames* 215

Krüger 60 and *Castor*, Mass-Ratios in Systems of. *Henry Norris Russell* 363

LENS, Collimation of Correcting. *J. S. Plaskett* 243

MASS-RATIOS in Systems of *Krüger 60* and *Castor*. *Henry Norris Russell* 363

MERCURY Lines, On Apparent Periodicity in Spacing of Satellites of. *G. F. Hull* 226

PAGE
Metals and Helium, Formulae for Spectral Series for Alkali. *R. T. Birge* 112
  in Infra-red Spectrum, Dispersion of. *L. R. Ingersoll* 265
Moon's Theoretical Spectrographic Velocity. *Kurt Laves* 17
Mount Wilson Solar Observatory, Some Methods and Results in Direct Photography with 60-Inch Reflecting Telescope of. *G. W. Ritchey* 26
Orion Stars, Velocity of Sun's Motion through Space as Derived from Radial Velocity of. *J. C. Kapteyn* and *Edwin B. Frost* 83
Parallax of Stars of Fourth Type as Compared with That of Stars of Other Types. *J. C. Kapteyn* 91
  Photographic Determinations of Stellar, Made with the Yerkes Refractor. I. *Frank Schlesinger* 372
Photometer, Brightness of Halley's Comet as Measured with Selenium. *Joel Stebbins* 179
  with Application to Variations of *Algol*, Measurement of Light of Stars with Selenium. *Joel Stebbins* 185
Prominences, Two Solar. *Frederick Slocum* 125
Reversal of Spectral Lines. *H. Konen* 249
Reviews:
  Huggins, Sir William and Lady, *The Scientific Papers of Sir William and Lady Huggins* (Edwin B. Frost) 323
  Planck, Max, *Acht Vorlesungen über theoretische Physik* (Edwin B. Frost) 325
  Planck, Max, *Die Einheit des physikalischen Weltbildes* (Edwin B. Frost) 325
  Tables du Bulletin de la Société belge d'astronomie (1895–1909), Auguste Collard (S. B. Barrett) 326
  Natural History and Scientific Book Circular, No. 145 (R. W. Willson) 326
  Davis, William Morris, *Geographical Essays* (Frederick Slocum) 327
  Pringsheim, E., *Physik der Sonne* (Frederick Slocum) 400
  Bosler, J., *Les théories modernes du soleil* (Frederick Slocum) 402
  Newall, H. F., *The Spectroscope and Its Work* (Edwin B. Frost) 402
Schiaparelli, Giovanni Virginio. *Antonio* and *Giorgio Abetti* 313
Solar Atmosphere, General Circulation of Mean and High-Level Calcium Vapor in. *Charles E. St. John* 36
  Disturbances and Terrestrial Temperatures. *W. J. Humphreys* 97
  Research, International Union for Co-operation in. *Edwin B. Frost* 258
  Rotation from Motion of Faculae. *Stanislas Chevalier* 388
Spectra of Calcium, Strontium, and Barium, Series in. *F. A. Saunders* 153
  Magnetic Separation of Absorption Lines in Connection with Sun-Spot. *P. Zeeman* and *B. Winawer* 329

PAGE

SPECTRAL Series for Alkali Metals and Helium, Formulae for. *R. T. Birge* 112
Lines, On the Reversal of. *H. Konen* 249
SPECTROSCOPIC Binaries, Measures on Nineteen New. *Oliver J. Lee* 300
SPECTRUM of Iodine Vapor at High Temperatures, Absorption. *E. J. Evans* 1
Dispersion of Metals in Infra-red. *L. R. Ingersoll* . . . 265
of Bromine at High Temperatures, Absorption. *E. J. Evans* . 291
STANDARDS of Wave-Length, International System, in Arc Spectrum of Iron, Secondary. *H. Kayser*, *Ch. Fabry*, and *J. S. Ames* . 215
of Third Order of Wave-Lengths on International System. *H. Kayser* 217
STAR Colors, Note on Bell's Paper on. *R. T. A. Innes* . . . 321
STARS of Fourth Type, Average Parallax of, as Compared with That of Stars of Other Types. *J. C. Kapteyn* . . . 91
Remarks on Wilsing and Scheiner's Memoir on Temperature of 109. *J. Wilsing* 130
with Selenium Photometer, with Application to Variations of *Algol*, Measurement of Light of. *Joel Stebbins* . . . 185
On the Temperatures of. *C. G. Abbot* 319
STRONTIUM, Barium, and Calcium, Series in Spectra of. *F. A. Saunders* 153
SUN on May 18 and 19, 1910, Observations of. *Frederick Slocum* . 24
SUN'S Motion through Space as Derived from Radial Velocity of *Orion* Stars, On Velocity of. *J. C. Kapteyn* and *Edwin B. Frost* 83
SUN-SPOT Spectra, Magnetic Separation of Absorption Lines in Connection with. *P. Zeeman* and *B. Winawer* 329
Taurus Stream, Variable Radial Velocities of Two Stars in. *Storrs B. Barrett* 183
TELESCOPE of Mount Wilson Solar Observatory, On Some Methods and Results in Direct Photography with 60-Inch Reflecting. *G. W. Ritchey* 26
TEMPERATURE of 109 Stars, Remarks on Wilsing and Scheiner's Memoir on. *J. Wilsing* 130
Solar Disturbances and Terrestrial. *W. J. Humphreys* . . 97
VELOCITIES of Two Stars in *Taurus* Stream, Variable Radial. *Storrs B. Barrett* 183
VELOCITY of Sun's Motion through Space as Derived from Radial Velocity of *Orion* Stars. *J. C. Kapteyn* and *Edwin B. Frost* 83
Determinations, Probable Errors of Radial. *J. S. Plaskett* . 230
WAVE-LENGTH, International System, in Arc Spectrum of Iron, Secondary Standards of. *H. Kayser*, *Ch. Fabry*, and *J. S. Ames* 215
WAVE-LENGTHS on International System, Standards of Third Order of. *H. Kayser* 217
YERKES Refractor, Photographic Determinations of Stellar Parallax Made with. *Frank Schlesinger* 372

INDEX TO VOLUME XXXII

AUTHORS

PAGE

Abbot, C. G. On the Temperature of Stars 319
Abetti, Antonio and Giorgio. Giovanni Virginio Schiaparelli . 313
Ames, J. S., H. Kayser, and Ch. Fabry. Secondary Standards of Wave-Length, International System, in the Arc Spectrum of Iron 215
Barrett, Storrs B. Variable Radial Velocities of Two Stars in the *Taurus* Stream 183
  Review of: *Tables du Bulletin de la Société belge d'astronomie (1895–1909)*, August Collard. 326
Birge, R. T. Formulae for the Spectral Series for the Alkali Metals and Helium 112
Chevalier, Stanislas. The Solar Rotation from the Motion of the Faculae on the Disk 388
Cooper, Frank L. The Absorption in the Red of the Acetate, Nitrate, and Sulphate of Cobalt 309
Evans, E. J. The Absorption Spectrum of Iodine Vapor at High Temperatures 1
  The Absorption Spectrum of Bromine at High Temperatures . 291
Fabry, Ch., H. Kayser, and J. S. Ames. Secondary Standards of Wave-Length, International System, in the Arc Spectrum of Iron 215
Frost, Edwin B. The International Union for Co-operation in Solar Research 258
  Review of: *The Scientific Papers of Sir William and Lady Huggins*, Sir William and Lady Huggins 323
  Review of: *Acht Vorlesungen über theoretische Physik*, Max Planck 325
  Review of: *Die Einheit des physikalischen Weltbildes*, Max Planck 325
  Review of: *The Spectroscope and Its Work*, H. F. Newall . . 402
Frost, Edwin B., and J. C. Kapteyn. On the Velocity of the Sun's Motion through Space as Derived from the Radial Velocity of *Orion* Stars 83
Hull, G. F. On the Apparent Periodicity in the Spacing of the Satellites of Some of the Mercury Lines 226
Humphreys, W. J. Solar Disturbances and Terrestrial Temperatures 97
Ingersoll, L. R. The Dispersion of Metals in the Infra-red Spectrum 265
Innes, R. T. A. Note on Bell's Paper on Star Colors . . . 321
Jungjohann, W., and H. Konen. Studies on the Emission of Gases. II 141

PAGE

Kapteyn, J. C. On the Average Parallax of the Stars of the Fourth Type as Compared with That of Stars of Other Types . . 91

Kapteyn, J. C., and Edwin B. Frost. On the Velocity of the Sun's Motion through Space as Derived from the Radial Velocity of *Orion* Stars 83

Kayser, H. Standards of Third Order of Wave-Lengths on the International System 217

Kayser, H., Ch. Fabry, and J. S. Ames. Secondary Standards of Wave-Length, International System, in the Arc Spectrum of Iron 215

Konen, H. On the Reversal of Spectral Lines 249

Konen, H., and W. Jungjohann. Studies on the Emission of Gases. II 141

Laves, Kurt. The Moon's Theoretical Spectrographic Velocity . 17

Lee, Oliver J. Measures on Nineteen New Spectroscopic Binaries . 300

Plaskett, J. S. Probable Errors of Radial Velocity Determinations 230

The Collimation of the Correcting Lens 243

Ritchey, G. W. On Some Methods and Results in Direct Photography with the 60-Inch Reflecting Telescope of the Mount Wilson Solar Observatory 26

Russell, Henry Norris. Mass-Ratios in the Systems of *Krüger 60* and *Castor* 363

St. John, Charles E. The General Circulation of the Mean and High-Level Calcium Vapor in the Solar Atmosphere . . 36

Saunders, F. A. Series in the Spectra of Calcium, Strontium, and Barium 153

Schlesinger, Frank. Photographic Determinations of Stellar Parallax Made with the Yerkes Refractor. I . . . 372

Slocum, Frederick. Observations of the Sun on May 18 and 19, 1910 24

Two Solar Prominences 125

Review of: *Geographical Essays*, William Morris Davis . . 327

Review of: *Physik der Sonne*, E. Pringsheim 400

Review of: *Les théories modernes du soleil*, J. Bosler . . . 402

Stebbins, Joel. The Brightness of Halley's Comet as Measured with a Selenium Photometer 179

The Measurement of the Light of Stars with a Selenium Photometer, with an Application to the Variations of *Algol* . . 185

Willson, R. W. Review of: *The Natural History and Scientific Book Circular, No. 145*. 326

Wilsing, J. Remarks on Wilsing and Scheiner's Memoir on the Temperature of 109 Stars 130

Winawer, B., and P. Zeeman. The Magnetic Separation of Absorption Lines in Connection with Sun-Spot Spectra . . . 329

Zeeman, P., and B. Winawer. The Magnetic Separation of Absorption Lines in Connection with Sun-Spot Spectra . . . 329

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VOL. XXXII DECEMBER 1910 NO. 5

THE MAGNETIC SEPARATION OF ABSORPTION LINES IN CONNECTION WITH SUN-SPOT SPECTRA - - - - P. ZEEMAN AND B. WINAWER 329

MASS-RATIOS IN THE SYSTEMS OF *KRÜGER 60* AND *CASTOR* HENRY NORRIS RUSSELL 363

PHOTOGRAPHIC DETERMINATIONS OF STELLAR PARALLAX MADE WITH THE YERKES REFRACTOR. I - - - - FRANK SCHLESINGER 372

THE SOLAR ROTATION FROM THE MOTION OF THE FACULAE ON THE DISK (1906–1908) - - - - - - - - - STANISLAS CHEVALIER 388

REVIEWS:

Physik der Sonne, E. PRINGSHEIM (Frederick Slocum), 400; *Les théories modernes du soleil*, J. BOSLER (Frederick Slocum), 402; *The Spectroscope and Its Work*, H. F. NEWALL (E. B. F.), 402.

INDEX TO VOLUME XXXII - - - - - - - - - - - - - - 404

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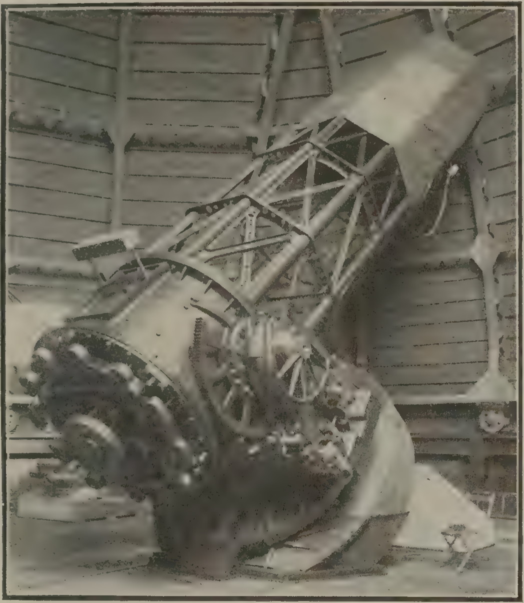

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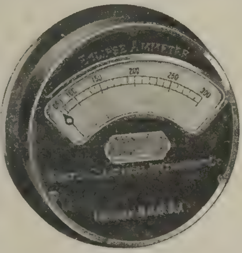

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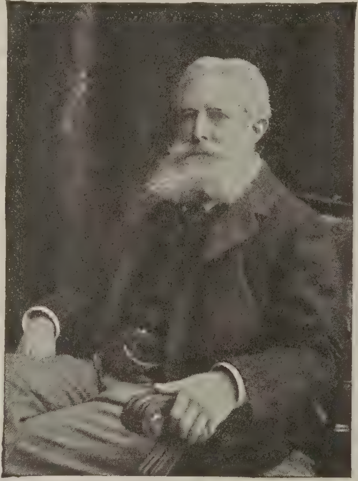

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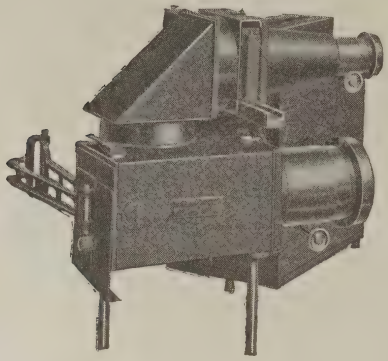

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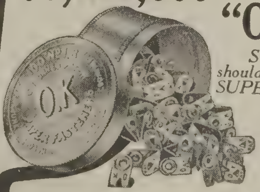

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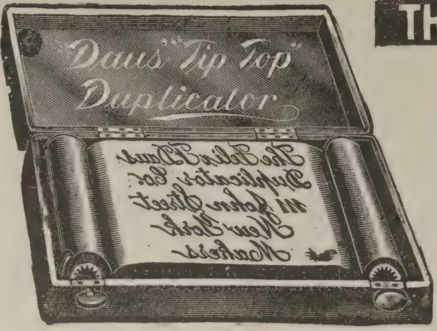
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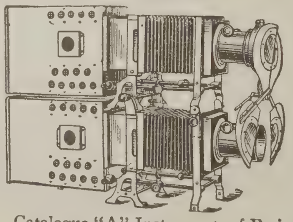

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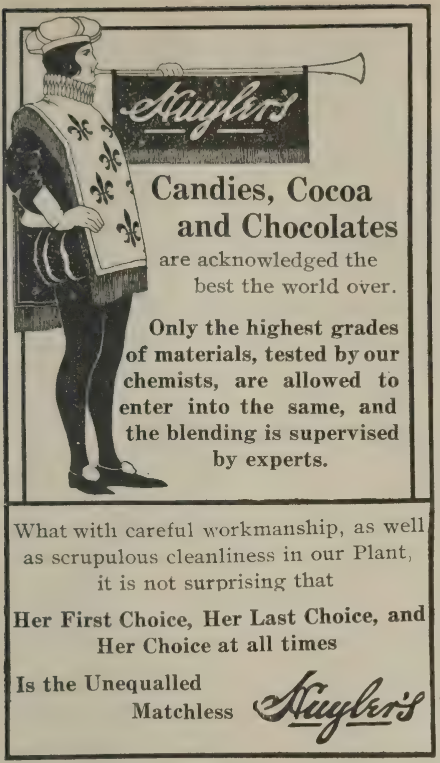

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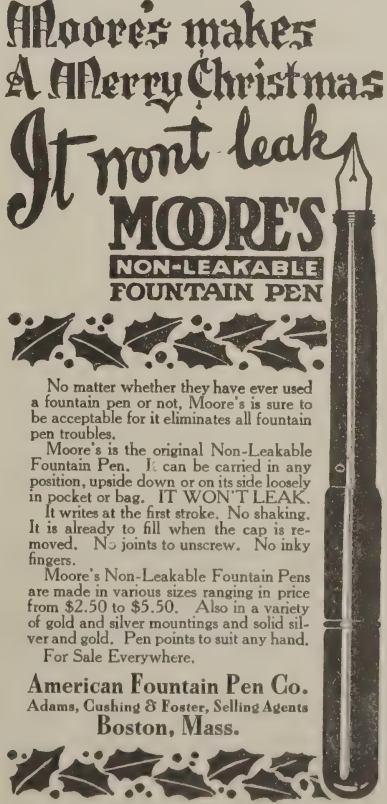

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